Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2014, Article ID 658252, 6 pages
http://dx.doi.org/10.1155/2014/658252

Research Article

Generalized Rough Γ-Hyperideals in Γ-Semihypergroups

Naveed Yaqoob[1,2] and Shamsul Haq[3]

[1] *Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan*
[2] *Department of Mathematics, College of Science in Al-Zulfi, Majmaah University, Al-Zulfi, Saudi Arabia*
[3] *Natural Resource and Engineering Department, University of Kurdistan Hewlr, Kurdistan Region, Iraq*

Correspondence should be addressed to Shamsul Haq; shams_ulhaq@yahoo.com

Received 1 May 2014; Accepted 17 June 2014; Published 8 July 2014

Academic Editor: Ashraf Zenkour

Davvaz (2008) introduced the concept of set-valued homomorphism and T-rough sets in a group. In this paper, we consider the set-valued homomorphism T on Γ-semihypergroup H to interpret the lower and upper approximations. We study the roughness of (m, n) bi-Γ-hyperideals and (m, n) quasi-Γ-hyperideals in terms of set-valued homomorphisms, which are extended notions of (m, n) bi-Γ-hyperideals and (m, n) quasi-Γ-hyperideals of Γ-semihypergroups.

1. Introduction

Hyperstructure, in particular hypergroups, was introduced in 1934, by Marty [1]. Nowadays, hyperstructures have a lot of applications to several domains of mathematics and computer science and they are studied in many countries in the world. Recently, Anvariyeh et al. [2] introduced the notion of Γ-semihypergroup as a generalization of a semigroup, a generalization of a semihypergroup, and a generalization of a Γ-semigroup. Heidari et al. [3] studied the structure further and added some useful results to the theory of Γ-semihypergroups. Abdullah et al. [4–6] studied some properties of M-hypersystems and bi-Γ-hyperideals in Γ-semihypergroups; also Hila et al. [7] studied the structures of Γ-semihypergroups.

In 1982, Pawlak [8] introduced the notion of rough sets as a tool to model uncertainty and vague and incomplete information system. The theory of rough sets is an extension of set theory, in which a subset of a universe is described by a pair of ordinary sets called the lower and upper approximations. At present, this concept has been applied to many directions, such as groups, probability theory, graph theory, automata theory, topology, cognitive sciences, machine learning, knowledge acquisition, and pattern recognition. The algebraic approach to rough sets has been studied by some authors; for instance, Biswas and Nanda [9] studied the classical group theory in terms of rough sets and introduced the notion of rough subgroups. Xiao and Zhang [10] introduced the notion of rough prime ideals and rough fuzzy prime ideals in a semigroup. Kuroki [11] introduced the notion of a rough ideal in a semigroup. In [12] the authors investigated Pawlak's approximations in Γ-semihypergroups. Aslam et al. [13] introduced the concept of rough M-hypersystems and fuzzy M-hypersystems in Γ-semihypergroups. Yaqoob et al. [14–16] applied rough set theory to (m, n) bi-Γ-hyperideals and (m, n) quasi-Γ-hyperideals in Γ-semihypergroups. Fotea [17] and Leoreanu-Fotea and Davvaz [18] discussed the lower and upper approximations of hypergroups and n-ary hypergroups. Jun [19] studied the roughness of Γ-subsemigroups/ideals in Γ-semigroups.

In 2008, Davvaz [20] introduced the concept of set-valued homomorphism and T-rough sets in a group. This idea attracted several mathematicians. Xiao [21] studied the properties of T-roughness in semigroups. Yamak et al. [22] introduced generalized lower and upper approximations in a ring. Yaqoob and Aslam [23] applied generalized rough set theory (in terms of set-valued homomorphisms) to the theory of Γ-semihypergroups. Hosseini et al. [24] applied T-rough set theory to semigroups.

In this paper, we study the roughness of (m, n) bi-Γ-hyperideals and (m, n) quasi-Γ-hyperideals in terms of set-valued homomorphisms.

2. Preliminaries and Basic Definitions

In this section, we will recall some concepts related to Γ-semihypergroups and generalized rough sets. Throughout the paper, S denotes a Γ-semihypergroup unless otherwise specified.

Definition 1. A map $\circ : S \times S \to \mathscr{P}^*(S)$ is called hyperoperation or join operation on the set S, where S is a nonempty set and $\mathscr{P}^*(S)$ denotes the set of all nonempty subsets of S. A hypergroupoid is a set S together with a (binary) hyperoperation. A hypergroupoid $(S, \circ)$, which is associative, that is, $x \circ (y \circ z) = (x \circ y) \circ z$, for all $x, y, z \in S$, is called a semihypergroup.

Definition 2 (see [2]). Let S and Γ be two nonempty sets. S is called a Γ-semihypergroup if every $\gamma \in \Gamma$ is a hyperoperation on S, that is, $x\gamma y \subseteq S$, for every $x, y \in S$, and, for every $\gamma, \beta \in \Gamma$ and $x, y, z \in S$, we have $x\gamma(y\beta z) = (x\gamma y)\beta z$.

Let A and B be two nonempty subsets of S. Then, we define

$$A\Gamma B = \bigcup_{\gamma\in\Gamma} A\gamma B = \bigcup \{a\gamma b \mid a \in A, b \in B \text{ and } \gamma \in \Gamma\}. \quad (1)$$

Let $(S, \circ)$ be a semihypergroup and let $\Gamma = \{\circ\}$. Then, S is a Γ-semihypergroup. So, every semihypergroup is Γ-semihypergroup.

Definition 3 (see [2]). Let S be a Γ-semihypergroup and $\gamma \in \Gamma$. A nonempty subset A of S is called a sub-Γ-semihypergroup of S if $x\gamma y \subseteq A$ for every $x, y \in A$.

Definition 4 (see [2]). A subset A of a Γ-semihypergroup S is called an interior Γ-hyperideal of S if $S\Gamma A\Gamma S \subseteq A$.

Let A be a nonempty subset of S and $n \in N$. A set A^n is defined to be the set

$$A^n = \underbrace{A\Gamma A\Gamma \cdots \Gamma A}_{n\text{-times}}. \quad (2)$$

For example, $A^2 = A\Gamma A$ and $A^3 = A\Gamma A\Gamma A$.

Definition 5 (see [14]). A subset A of a Γ-semihypergroup S is called an $(m, 0)$ Γ-hyperideal (resp., $(0, n)$ Γ-hyperideal) of S if $A^m\Gamma S \subseteq A$ (resp., $S\Gamma A^n \subseteq A$).

Definition 6 (see [14]). A sub-Γ-semihypergroup A of a Γ-semihypergroup S is called an (m, n) bi-Γ-hyperideal of S, if $A^m\Gamma S\Gamma A^n \subseteq A$, where m, n are nonnegative integers (A^m is suppressed if $m = 0$).

Definition 7 (see [15]). An (m, n) bi-Γ-hyperideal B of a Γ-semihypergroup S is called prime if for $x, y \in S$, $x^m\alpha S\beta y^n \subseteq B$ (or $x^m\alpha z\beta y^n \subseteq B$, for all $z \in S$) implies $x \in B$ or $y \in B$, for all $\alpha, \beta \in \Gamma$.

Definition 8 (see [15]). An (m, n) bi-Γ-hyperideal B of a Γ-semihypergroup S is called semiprime if for $x \in S$, $x^m\alpha S\beta x^n \subseteq B$ (or $x^m\alpha z\beta x^n \subseteq B$, for all $z \in S$) implies $x \in B$, for all $\alpha, \beta \in \Gamma$.

Definition 9 (see [6]). Let S be a Γ-semihypergroup and L a sub-Γ-semihypergroup of S. Then L is called an m-left Γ-hyperideal of S if $S^m\Gamma L \subseteq L$ where m is any positive integer. Dually, $R\Gamma S^n \subseteq R$, and then R is called an n-right Γ-hyperideal of S, where n is any positive integer.

Definition 10 (see [6]). Let S be a Γ-semihypergroup and Q a nonempty subset of S. Then Q is called an $(m.n)$ quasi-Γ-hyperideal of S if $S^m\Gamma Q \cap Q\Gamma S^n \subseteq Q$.

Now, we will recall some notions in generalized rough sets.

Definition 11. Let X and Y be two nonempty universes. Let T be a set-valued mapping given by $T : X \to P(Y)$. Then, the triple (X, Y, T) is referred to as a generalized approximation space or generalized rough set. Any set-valued function from X to $P(Y)$ defines a binary relation from X to Y by setting $R_T = \{(x, y) \mid y \in T(x)\}$. Obviously, if R is an arbitrary relation from X to Y, then it can be defined as a set-valued mapping $T_R : X \to P(Y)$ by $T_R(x) = \{y \in Y \mid (x, y) \in R\}$ where $x \in X$. For any set $A \subseteq Y$, the lower and upper approximations $\underline{T}(A)$ and $\overline{T}(A)$ are defined by

$$\begin{gathered}\underline{T}(A) = \{x \in X \mid T(x) \subseteq A\}, \\ \overline{T}(A) = \{x \in X \mid T(x) \cap A \neq \emptyset\}.\end{gathered} \quad (3)$$

The pair $(\underline{T}(A), \overline{T}(A))$ is referred to as a generalized rough set, and $\underline{T}$, $\overline{T}$ are referred to as lower and upper generalized approximation operators, respectively.

If a subset $A \subseteq Y$ satisfies that $\underline{T}(A) = \overline{T}(A)$, then A is called a definable set of (X, Y, T). We denote all the definable sets of (X, Y, T) by $\mathrm{Def}(T)$.

Theorem 12 (see [22]). *Let (X, Y, T) be a generalized approximation space; its lower and upper approximation operators satisfy the following properties: for all $A, B \in P(Y)$,*

$$\begin{aligned}
&(L1)\ \ \underline{T}(A) = \left(\overline{T}(A^c)\right)^c, \\
&(L2)\ \ \underline{T}(Y) = X, \\
&(L3)\ \ \underline{T}(A \cap B) = \underline{T}(A) \cap \underline{T}(B), \\
&(L4)\ \ A \subseteq B \Longrightarrow \underline{T}(A) \subseteq \underline{T}(B), \\
&(L5)\ \ \underline{T}(A \cup B) \supseteq \underline{T}(A) \cup \underline{T}(B), \\
&(U1)\ \ \overline{T}(A) = \left(\underline{T}(A^c)\right)^c, \\
&(U2)\ \ \overline{T}(\emptyset) = \emptyset, \\
&(U3)\ \ \overline{T}(A \cup B) = \overline{T}(A) \cup \overline{T}(B), \\
&(U4)\ \ A \subseteq B \Longrightarrow \overline{T}(A) \subseteq \overline{T}(B), \\
&(U5)\ \ \overline{T}(A \cap B) \subseteq \overline{T}(A) \cap \overline{T}(B),
\end{aligned} \quad (4)$$

where A^c is the complement of the set A.

Theorem 13 (see [22]). *Let (X, X, T) be a generalized approximation space; its lower and upper generalized approximation operators satisfy the following properties: for all $A \in P(X)$,*

$$\begin{aligned} &(1)\ R_T \textit{ is serial} && \Longleftrightarrow (L0)\ \underline{T}(\emptyset) = \emptyset \\ & && \Longleftrightarrow (U0)\ \overline{T}(X) = X \\ & && \Longleftrightarrow (LU0)\ \underline{T}(A) \subseteq \overline{T}(A) \\ &(2)\ R_T \textit{ is reflexive} && \Longleftrightarrow (L6)\ \underline{T}(A) \subseteq A \\ & && \Longleftrightarrow (U6)\ A \subseteq \overline{T}(A) \\ &(3)\ R_T \textit{ is symmetric} && \Longleftrightarrow (L7)\ \overline{T}(\underline{T}(A)) \subseteq A \\ & && \Longleftrightarrow (U7)\ A \subseteq \underline{T}(\overline{T}(A)) \end{aligned} \tag{5}$$

$$\begin{aligned} &(4)\ R_T \textit{ is transitive} && \Longleftrightarrow (L8)\ \underline{T}(A) \subseteq \underline{T}(\underline{T}(A)) \\ & && \Longleftrightarrow (U8)\ \overline{T}(\overline{T}(A)) \subseteq \overline{T}(A) \end{aligned} \tag{6}$$

If R is an equivalence relation on X, then the pair (X, R) is the Pawlak approximation space. Therefore, a generalized rough set is an extended notion of Pawlak's rough sets.

Definition 14. Let S be a Γ-semihypergroup. An equivalence relation ρ on S is called regular on S if

$$(a, b) \in \rho \quad \text{implies} \ (a\gamma x, b\gamma x) \in \rho, \qquad (x\gamma a, x\gamma b) \in \rho, \tag{7}$$

for all $x \in S$ and $\gamma \in \Gamma$.

If ρ is a regular relation on S, then, for every $x \in S$, $[x]_\rho$ stands for the class of x with the representation ρ. A regular relation ρ on S is called complete if $[a]_\rho\gamma[b]_\rho = [a\gamma b]_\rho$ for all $a, b \in S$ and $\gamma \in \Gamma$. In addition, ρ on S is called congruence if, for every $a, b \in S$ and $\gamma \in \Gamma$, we have $c \in [a]_\rho\gamma[b]_\rho \Rightarrow [c]_\rho \subseteq [a]_\rho\gamma[b]_\rho$. It is obvious that, for a regular relation ρ on S, $[a]_\rho\gamma[b]_\rho \subseteq [a\gamma b]_\rho$ for all $a, b \in S$ and $\gamma \in \Gamma$.

3. Generalized Rough Sets in Γ-Semihypergroups

In this section, we will present some results on generalized rough sets in Γ-semihypergroups.

Definition 15 (see [23]). A set-valued homomorphism T from a Γ-semihypergroup S to a $\dot{\Gamma}$-semihypergroup $\dot{S}$ is a mapping from S to $\wp^*(\dot{S})$ which preserves the operation; that is, $T(a)\dot{\beta}T(b) \subseteq T(a\beta b)$ for all $a, b \in S$, $\beta \in \Gamma$, and $\dot{\beta} \in \dot{\Gamma}$. T is called a strong set-valued homomorphism, if $T(a)\dot{\beta}T(b) = T(a\beta b)$ for all $a, b \in S$, $\beta \in \Gamma$, and $\dot{\beta} \in \dot{\Gamma}$.

Theorem 16 (see [23]). *Let S be a Γ-semihypergroup, let $\dot{S}$ be a $\dot{\Gamma}$-semihypergroup, and let $T : S \to \wp^*(\dot{S})$ be a set-valued homomorphism. If A, B are two nonempty subsets of $\dot{S}$, then*

(1) $\overline{T}(A)\Gamma\overline{T}(B) \subseteq \overline{T}(A\dot{\Gamma}B)$;

(2) if T is strong, then $\underline{T}(A)\Gamma\underline{T}(B) \subseteq \underline{T}(A\dot{\Gamma}B)$.

Theorem 17 (see [23]). *Let S be a Γ-semihypergroup, let $\dot{S}$ be a $\dot{\Gamma}$-semihypergroup, and let $T : S \to \wp^*(\dot{S})$ be a set-valued homomorphism.*

(1) If A is a sub-$\dot{\Gamma}$-semihypergroup of $\dot{S}$, then $\overline{T}(A)$ is, if it is nonempty, a sub-Γ-semihypergroup of S.

(2) If A is a left (resp., right) $\dot{\Gamma}$-hyperideal of $\dot{S}$, then $\overline{T}(A)$ is, if it is nonempty, a left (resp., right) Γ-hyperideal of S.

Theorem 18 (see [23]). *Let S be a Γ-semihypergroup, let $\dot{S}$ be a $\dot{\Gamma}$-semihypergroup, and let $T : S \to \wp^*(\dot{S})$ be a strong set-valued homomorphism.*

(1) If A is a sub-$\dot{\Gamma}$-semihypergroup of $\dot{S}$, then $\underline{T}(A)$ is, if it is nonempty, a sub-Γ-semihypergroup of S.

(2) If A is a left (resp., right) $\dot{\Gamma}$-hyperideal of $\dot{S}$, then $\underline{T}(A)$ is, if it is nonempty, a left (resp., right) Γ-hyperideal of S.

Theorem 19. *Let S be a Γ-semihypergroup, let $\dot{S}$ be a $\dot{\Gamma}$-semihypergroup, and let $T : S \to \wp^*(\dot{S})$ be a set-valued homomorphism. If A is an interior $\dot{\Gamma}$-hyperideal of $\dot{S}$, then*

(1) $\overline{T}(A)$ is, if it is nonempty, an interior Γ-hyperideal of S;

(2) if T is strong, then $\underline{T}(A)$ is, if it is nonempty, an interior Γ-hyperideal of S.

Proof. The proof is straightforward. □

Lemma 20. *Let S be a Γ-semihypergroup, let $\dot{S}$ be a $\dot{\Gamma}$-semihypergroup, and let $T : S \to \wp^*(\dot{S})$ be a set-valued homomorphism. Then, for a nonempty subset A of $\dot{S}$,*

(1) $(\overline{T}(A))^n \subseteq \overline{T}(A^n)$ for all $n \in \mathbb{N}$;

(2) if T is strong, then $(\underline{T}(A))^n \subseteq \underline{T}(A^n)$ for all $n \in \mathbb{N}$.

Proof. The proof is straightforward. □

4. Generalized Rough (m,n) (Bi-)Quasi-Γ-Hyperideals

We will study here some properties of generalized lower and upper approximations of (m, n) bi-Γ-hyperideals in Γ-semihypergroup.

A subset A of a $\dot{\Gamma}$-semihypergroup $\dot{S}$ is called a generalized upper (resp., generalized lower) rough (m, n) bi-Γ-hyperideal of S if $\overline{T}(A)$ (resp., $\underline{T}(A)$) is an (m, n) bi-Γ-hyperideal of S.

Theorem 21. *Let S be a Γ-semihypergroup, let $\dot{S}$ be a $\dot{\Gamma}$-semihypergroup, and let $T : S \to \wp^*(\dot{S})$ be a set-valued homomorphism. If A is an (m, n) bi-$\dot{\Gamma}$-hyperideal of $\dot{S}$, then*

(1) $\overline{T}(A)$ is, if it is nonempty, an (m, n) bi-Γ-hyperideal of S;

(2) if T is strong, then $\underline{T}(A)$ is, if it is nonempty, an (m, n) bi-Γ-hyperideal of S.

Proof. (1) Let A be an (m, n) bi-$\dot{\Gamma}$-hyperideal of $\dot{S}$. Then, by Theorem 16(1) and Lemma 20(1), we have

$$\begin{aligned} \left(\overline{T}(A)\right)^m \Gamma S \Gamma \left(\overline{T}(A)\right)^n &= \left(\overline{T}(A)\right)^m \Gamma \overline{T}(\dot{S}) \Gamma \left(\overline{T}(A)\right)^n \\ &\subseteq \overline{T}(A^m) \Gamma \overline{T}(\dot{S}) \Gamma \overline{T}(A^n) \end{aligned}$$

$$\subseteq \overline{T}\left(A^m\dot{\Gamma}\dot{S}\right)\Gamma\,\overline{T}\left(A^n\right)$$
$$\subseteq \overline{T}\left(A^m\dot{\Gamma}\dot{S}\dot{\Gamma}A^n\right) \subseteq \overline{T}(A). \tag{8}$$

From this and Theorem 17(1), we obtain that $\overline{T}(A)$ is an (m,n) bi-Γ-hyperideal of S.

(2) Let A be an (m,n) bi-$\dot{\Gamma}$-hyperideal of $\dot{S}$. Then, by Theorem 16(2) and Lemma 20(2), we have

$$\begin{aligned}(\underline{T}(A))^m\Gamma S\Gamma(\underline{T}(A))^n &= (\underline{T}(A))^m\Gamma\,\underline{T}\left(\dot{S}\right)\Gamma(\underline{T}(A))^n\\ &\subseteq \underline{T}\left(A^m\right)\Gamma\,\underline{T}\left(\dot{S}\right)\Gamma\underline{T}\left(A^n\right)\\ &\subseteq \underline{T}\left(A^m\dot{\Gamma}\dot{S}\right)\Gamma\,\underline{T}\left(A^n\right)\\ &\subseteq \underline{T}\left(A^m\dot{\Gamma}\dot{S}\dot{\Gamma}A^n\right) \subseteq \underline{T}(A).\end{aligned} \tag{9}$$

From this and Theorem 18(1), we obtain that $\underline{T}(A)$ is, if it is nonempty, an (m,n) bi-Γ-hyperideal of S. This completes the proof. □

Corollary 22. *Let S be a Γ-semihypergroup, let $\dot{S}$ be a $\dot{\Gamma}$-semihypergroup, and let $T : S \to \wp^*(\dot{S})$ be a set-valued homomorphism. If A is an $(m,0)$ $\dot{\Gamma}$-hyperideal (resp., $(0,n)$ $\dot{\Gamma}$-hyperideal, m-left $\dot{\Gamma}$-hyperideal, and n-right $\dot{\Gamma}$-hyperideal) of $\dot{S}$, then*

(1) $\overline{T}(A)$ is, if it is nonempty, an $(m,0)$ Γ-hyperideal (resp., $(0,n)$ Γ-hyperideal, m-left Γ-hyperideal, and n-right Γ-hyperideal) of S;

(2) if T is strong, then $\underline{T}(A)$ is, if it is nonempty, an $(m,0)$ Γ-hyperideal (resp., $(0,n)$ Γ-hyperideal, m-left Γ-hyperideal, and n-right Γ-hyperideal) of S.

Proof. The proof is straightforward. □

A subset A of a $\dot{\Gamma}$-semihypergroup $\dot{S}$ is called a generalized upper (resp., generalized lower) rough prime (m,n) bi-Γ-hyperideal of S if $\overline{T}(A)$ (resp., $\underline{T}(A)$) is a prime (m,n) bi-Γ-hyperideal of S.

Theorem 23. *Let S be a Γ-semihypergroup, let $\dot{S}$ be a $\dot{\Gamma}$-semihypergroup, and let $T : S \to \wp^*(\dot{S})$ be a strong set-valued homomorphism. If A is a prime (m,n) bi-$\dot{\Gamma}$-hyperideal of $\dot{S}$, then*

(1) $\overline{T}(A)$ is, if it is nonempty, a prime (m,n) bi-Γ-hyperideal of S;

(2) $\underline{T}(A)$ is, if it is nonempty, a prime (m,n) bi-Γ-hyperideal of S.

Proof. Since A is an (m,n) bi-$\dot{\Gamma}$-hyperideal of $\dot{S}$. by Theorem 21, we know that $\overline{T}(A)$ and $\underline{T}(A)$ are (m,n) bi-Γ-hyperideals of S.

(1) Let w be any element of S. Let $x, y \in S$ and $\beta,\gamma \in \Gamma$ such that $x^m\beta w\gamma y^n \subseteq \overline{T}(A)$. Thus,

$$T\left(x^m\beta w\gamma y^n\right)\cap A = \left(T\left(x^m\right)\dot{\beta}T(w)\,\dot{\gamma}T\left(y^n\right)\right)\cap A \neq \emptyset, \tag{10}$$

where $\dot{\beta},\dot{\gamma} \in \dot{\Gamma}$. Thus, there exist $a^m \subseteq T(x^m) = T(x)^m$, $w' \in T(w)$, and $b^n \subseteq T(y^n) = T(y)^n$ such that $a^m\dot{\beta}w'\dot{\gamma}b^n \subseteq A$. Since A is a prime (m,n) bi-$\dot{\Gamma}$-hyperideal, we have $a \in A$ or $b \in A$. Now,

$$a^m \subseteq T(x)^m \Longrightarrow a \in T(x) \text{ also } b^n \subseteq T(y)^n \Longrightarrow b \in T(y). \tag{11}$$

Thus, $a \in T(x)\cap A$ or $b \in T(y)\cap A$. So $x \in \overline{T}(A)$ or $y \in \overline{T}(A)$. Therefore, $\overline{T}(A)$ is a prime (m,n) bi-Γ-hyperideal of S.

(2) We suppose that $\underline{T}(A)$ is not a prime (m,n) bi-Γ-hyperideal; then for $\beta,\gamma \in \Gamma$ there exist $x, y \in S$ and any element $w \in S$, such that $x^m\beta w\gamma y^n \subseteq \underline{T}(A)$, but $x \notin \underline{T}(A)$ and $y \notin \underline{T}(A)$. Thus $T(x) \nsubseteq A$ and $T(y) \nsubseteq A$. Then, there exist

$$a \in T(x) \quad \text{but } a \notin A, \quad b \in T(y) \quad \text{but } b \notin A. \tag{12}$$

Now, for $w' \in S$, $\beta,\gamma \in \Gamma$, and $\dot{\beta},\dot{\gamma} \in \dot{\Gamma}$, we have

$$\begin{aligned}a^m\dot{\beta}w'\dot{\gamma}b^n &\subseteq T(x)^m\dot{\beta}T(w)\,\dot{\gamma}T(y)^n = T\left(x^m\right)\dot{\beta}T(w)\,\dot{\gamma}T\left(y^n\right)\\ &= T\left(x^m\beta w\gamma y^n\right) \subseteq A.\end{aligned} \tag{13}$$

This implies that $a^m\dot{\beta}w'\dot{\gamma}b^n \subseteq A$. Since A is a prime (m,n) bi-$\dot{\Gamma}$-hyperideal of $\dot{S}$, we have $a \in A$ or $b \in A$. It contradicts the supposition. This means that $\underline{T}(A)$ is, if it is nonempty, a prime (m,n) bi-Γ-hyperideal of S. □

The following example shows that the converse of Theorem 23 does not hold.

Example 24. Let $S = \{x, y, z\}$ and $\Gamma = \{\beta,\gamma\}$ be the sets of binary hyperoperations defined as follows:

$$\begin{array}{c|ccc} \beta & x & y & z\\ \hline x & x & x & x\\ y & x & y & \{y,z\}\\ z & x & \{y,z\} & \{y,z\}\end{array}$$
$$\begin{array}{c|ccc} \gamma & x & y & z\\ \hline x & x & x & x\\ y & x & y & \{y,z\}\\ z & x & \{y,z\} & z\end{array}. \tag{14}$$

Clearly S is a Γ-semihypergroup. Let $\dot{S} = \{a,b,c,d,e\}$ and $\dot{\Gamma} = \{\dot{\beta},\dot{\gamma}\}$ be the sets of binary hyperoperations defined as follows:

$$\begin{array}{c|ccccc} \dot{\beta} & a & b & c & d & e\\ \hline a & \{a,b\} & \{b,c\} & c & \{d,e\} & e\\ b & \{b,c\} & c & c & \{d,e\} & e\\ c & c & c & c & \{d,e\} & e\\ d & \{d,e\} & \{d,e\} & \{d,e\} & d & e\\ e & e & e & e & e & e\end{array}$$
$$\begin{array}{c|ccccc} \dot{\gamma} & a & b & c & d & e\\ \hline a & \{b,c\} & c & c & \{d,e\} & e\\ b & c & c & c & \{d,e\} & e\\ c & c & c & c & \{d,e\} & e\\ d & \{d,e\} & \{d,e\} & \{d,e\} & d & e\\ e & e & e & e & e & e\end{array}. \tag{15}$$

Clearly $\dot{S}$ is a $\dot{\Gamma}$-semihypergroup. Assume that $T(x) = \{d, e\}$, $T(y) = \{a, b, c\}$, and $T(z) = \{a, b, c, d, e\}$. Here, T is a strong set-valued homomorphism from S to $\dot{S}$. Now for $A = \{b, d, e\} \subseteq \dot{S}$, $\overline{T}(A) = \{x, y, z\}$ and $\underline{T}(A) = \{x\}$. It is clear that $\overline{T}(A)$ and $\underline{T}(A)$ are prime (m, n) bi-Γ-hyperideals of S. But A is not a sub-$\dot{\Gamma}$-semihypergroup of $\dot{S}$; hence, A is not a prime (m, n) bi-$\dot{\Gamma}$-hyperideal of $\dot{S}$.

A subset Q of a $\dot{\Gamma}$-semihypergroup $\dot{S}$ is called a generalized upper (resp., generalized lower) rough (m, n) quasi-Γ-hyperideal of S if $\overline{T}(A)$ (resp., $\underline{T}(A)$) is an (m, n) quasi-Γ-hyperideal of S.

Theorem 25. *Let S be a Γ-semihypergroup, let $\dot{S}$ be a $\dot{\Gamma}$-semihypergroup, and let $T : S \to \wp^*(\dot{S})$ be a strong set-valued homomorphism. If Q is an (m, n) quasi-$\dot{\Gamma}$-hyperideal of $\dot{S}$, then $\underline{T}(A)$ is, if it is nonempty, an (m, n) quasi-Γ-hyperideal of S.*

Proof. Let Q be an (m, n) quasi-$\dot{\Gamma}$-hyperideal of $\dot{S}$; that is, $\dot{S}^m\dot{\Gamma}Q \cap Q\dot{\Gamma}\dot{S}^n \subseteq Q$. Note that $\underline{T}(\dot{S}) = S$. Then, by Theorem 12(L3), Theorem 16(2), and Lemma 20(2), we have

$$\begin{aligned} S^m\Gamma\underline{T}(Q) \cap \underline{T}(Q)\Gamma S^n &= \left(\underline{T}(\dot{S})\right)^m\Gamma\underline{T}(Q) \cap \underline{T}(Q)\Gamma\left(\underline{T}(\dot{S})\right)^n \\ &\subseteq \underline{T}\left(\dot{S}^m\right)\Gamma\underline{T}(Q) \cap \underline{T}(Q)\Gamma\underline{T}\left(\dot{S}^n\right) \\ &\subseteq \underline{T}\left(\dot{S}^m\dot{\Gamma}Q\right) \cap \underline{T}\left(Q\dot{\Gamma}\dot{S}^n\right) \\ &= \underline{T}\left(\dot{S}^m\dot{\Gamma}Q \cap Q\dot{\Gamma}\dot{S}^n\right) \\ &\subseteq \underline{T}(Q). \end{aligned} \tag{16}$$

This shows that $\underline{T}(Q)$ is an (m, n) quasi-Γ-hyperideal of S. □

The next theorem shows that the intersection of a generalized lower rough m-left Γ-hyperideal and a generalized lower rough n-right Γ-hyperideal of a Γ-semihypergroup S is a generalized lower rough (m, n) quasi-Γ-hyperideal of S.

Theorem 26. *Let S be a Γ-semihypergroup, let $\dot{S}$ be a $\dot{\Gamma}$-semihypergroup, and let $T : S \to \wp^*(\dot{S})$ be a strong set-valued homomorphism. Let L and R be a generalized lower rough m-left Γ-hyperideal and a generalized lower rough n-right Γ-hyperideal of S, respectively. Then $\underline{T}(L \cap R)$ is, if it is nonempty, an (m, n) quasi-Γ-hyperideal of S.*

Proof. Let L and R be a generalized lower rough m-left Γ-hyperideal and a generalized lower rough n-right Γ-hyperideal of S, respectively. Then,

$$S^m\Gamma\underline{T}(L) \subseteq \underline{T}(L), \qquad \underline{T}(R)\Gamma S^n \subseteq \underline{T}(R). \tag{17}$$

Now, we have

$$\begin{aligned} S^m\Gamma\underline{T}(L \cap R) \cap \underline{T}(L \cap R)\Gamma S^n &\subseteq S^m\Gamma\underline{T}(L) \cap \underline{T}(R)\Gamma S^n \\ &\subseteq \underline{T}(L) \cap \underline{T}(R) \\ &= \underline{T}(L \cap R). \end{aligned} \tag{18}$$

Hence, this shows that $\underline{T}(L \cap R)$ is a generalized lower rough (m, n) quasi-Γ-hyperideal of S. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] F. Marty, "Sur une generalization de la notion de group," in *Proceedings of the 8th Congrès des Mathématiciens Scandinaves*, pp. 45–49, Stockholm, Sweden, 1934.

[2] S. M. Anvariyeh, S. Mirvakili, and B. Davvaz, "On γ-hyperideals in γ-semihypergroups," *Carpathian Journal of Mathematics*, vol. 26, no. 1, pp. 11–23, 2010.

[3] D. Heidari, S. O. Dehkordi, and B. Davvaz, "Γ-semihypergroups and their properties," *University of Bucharest. Scientific Bulletin A*, vol. 72, no. 1, pp. 195–208, 2010.

[4] S. Abdullah, M. Aslam, and T. Anwar, "A note on M-hypersystems and N-hypersystems in Γ-semihypergroups," *Quasigroups and Related Systems*, vol. 19, no. 2, pp. 101–104, 2011.

[5] S. Abdullah, K. Hila, and M. Aslam, "On bi-Γ-hyperideals of Γ-semihypergroups," *University of Bucharest. Scientific Bulletin A: Applied Mathematics and Physics*, vol. 74, no. 4, pp. 79–90, 2012.

[6] K. Hila and S. Abdullah, "Quasi-Γ-hyperideals in Γsemihypergroups," in *Proceedings of the 11th International Pure Mathematics Conference*, Islamabad, Pakistan, 2012.

[7] K. Hila, B. Davvaz, and J. Dine, "Study on the structure of Γ-semihypergroups," *Communications in Algebra*, vol. 40, no. 8, pp. 2932–2948, 2012.

[8] Z. Pawlak, "Rough sets," *International Journal of Computer and Information Sciences*, vol. 11, no. 5, pp. 341–356, 1982.

[9] R. Biswas and S. Nanda, "Rough groups and rough subgroups," *Polish Academy of Sciences*, vol. 42, no. 3, pp. 251–254, 1994.

[10] Q. Xiao and Z. Zhang, "Rough prime ideals and rough fuzzy prime ideals in semigroups," *Information Sciences*, vol. 176, no. 6, pp. 725–733, 2006.

[11] N. Kuroki, "Rough ideals in semigroups," *Information Sciences*, vol. 100, no. 1–4, pp. 139–163, 1997.

[12] S. M. Anvariyeh, S. Mirvakili, and B. Davvaz, "Pawlak's approximations in Γ-semihypergroups," *Computers & Mathematics with Applications*, vol. 60, no. 1, pp. 45–53, 2010.

[13] M. Aslam, S. Abdullah, B. Davvaz, and N. Yaqoob, "Rough M-hypersystems and fuzzy M-hypersystems in Γ-semihypergroups," *Neural Computing and Applications*, vol. 21, no. 1, pp. 281–287, 2012.

[14] N. Yaqoob, M. Aslam, B. Davvaz, and A. B. Saeid, "On rough (m, n) bi-Γ-hyperideals in Γ-semihypergroups," *University Politehnica of Bucharest Scientific Bulletin A*, vol. 75, no. 1, pp. 119–128, 2013.

[15] N. Yaqoob and M. Aslam, "Prime (m,n) bi-Γ-hyperideals in Γ-semihypergroups," *Applied Mathematics & Information Sciences*, vol. 8, no. 5, pp. 2243–2249, 2014.

[16] N. Yaqoob and M. Aslam, "On rough Quasi- Γ -hyperideals in Γ -semihypergroups," *Afrika Matematika*, 2013.

[17] V. L. Fotea, "The lower and upper approximations in a hypergroup," *Information Sciences*, vol. 178, no. 18, pp. 3605–3615, 2008.

[18] V. Leoreanu-Fotea and B. Davvaz, "Roughness in n-ary hypergroups," *Information Sciences*, vol. 178, no. 21, pp. 4114–4124, 2008.

[19] Y. B. Jun, "Roughness of Γ-subsemigroups/ideals in gamma-semigroups," *Bulletin of the Korean Mathematical Society*, vol. 40, no. 3, pp. 531–536, 2003.

[20] B. Davvaz, "A short note on algebraic T-rough sets," *Information Sciences*, vol. 178, no. 16, pp. 3247–3252, 2008.

[21] Q. M. Xiao, "T-roughness in semigroups," in *Proceedings of the IEEE International Conference on Computer Science and Automation Engineering (CSAE '11)*, vol. 2, pp. 391–394, Shanghai, China, June 2011.

[22] S. Yamak, O. Kazanci, and B. Davaz, "Generalized lower and upper approximations in a ring," *Information Sciences*, vol. 180, no. 9, pp. 1759–1768, 2010.

[23] N. Yaqoob and M. Aslam, "Generalized rough approximations in Γ-semihypergroups," *Journal of Intelligent & Fuzzy Systems*, 2014.

[24] S. B. Hosseini, N. Jafarzadeh, and A. Gholami, "T-rough ideal and T-rough fuzzy ideal in a semigroup," *Advanced Materials Research*, vol. 433–440, pp. 4915–4919, 2012.