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Research Article

Robust Adaptive Dynamic Surface Control for a Class of Nonlinear Dynamical Systems with Unknown Hysteresis

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The output tracking problem for a class of uncertain strict-feedback nonlinear systems with unknown Duhem hysteresis input is investigated. In order to handle the undesirable effects caused by unknown hysteresis, the properties in respect to Duhem model are used to decompose it as a nonlinear smooth term and a nonlinear bounded "disturbance-like" term, which makes it possible to deal with the unknown hysteresis without constructing inverse in the controller design. By combining robust control and dynamic surface control technique, an adaptive controller is proposed in this paper to avoid "the explosion complexity" in the standard backstepping design procedure. The negative effects caused by the unknown hysteresis can be mitigated effectively, and the semiglobal uniform ultimate boundedness of all the signals in the closed-loop system is obtained. The effectiveness of the proposed scheme is validated through a simulation example.

1. Introduction

With the development of smart materials, some smart materials-based actuators, such as piezoceramic actuators [1], magnetostrictive actuators, and shape memory alloys, are becoming increasingly important in the application areas of aerospace, manufacturing, defense, and civil infrastructure systems [2–5], because of their excellent performance, for example, high precision, fast response, and flexible actuating ability [6–8]. However, a class of nonsmooth nonlinearities, hystereses, with multibranching and nondifferential properties, widely occur in these smart materials-based actuators. When the system is preceded by these actuators, the existence of the hysteresis behaviour in these actuators will degrade the system performance, causing undesirable inaccuracy. The hysteresis nonlinearities are the nature properties of these smart materials, which cannot be cancelled by the improvement of the smart materials. Therefore, how to mitigate the negative effects caused by the hysteresis nonlinearities from control view becomes one important research topic in this area. Due to the nonsmooth nature of hysteresis, most common control approaches developed for nonlinear systems may not be applicable to hysteretic systems directly, which attracted significant attention in the modeling of hysteresis nonlinearities and the hysteretic systems controller design.

For the modeling method of the hysteresis, it can be roughly classified as differential equation-based hysteresis models, such as Backlash-like model [9], Bouc-Wen model [10, 11], and Duhem model [10, 12], and operator-based hysteresis models, such as Preisach model, Krasnosel'skii-Pokrovskii model, and Prandtl-Ishlinskii model [13–15]. As a differential-equation based hysteresis model, Duhem model can represent numerous hysteresis shapes including saturation and asymmetric properties by choosing different shape functions. However, the output analytical expression of Duhem model is difficult to obtain directly since the output depends on the solution of the differential equation, which may cause a new difficulty for the controller design.

So far, the control design work for the systems in presence of hysteresis nonlinearities has also been paid more attention [16–19]. Generally, two control approaches are used to mitigate the negative effects of hysteresis in the literature. The common one is to construct a hysteresis inverse model to cancel the adverse effects of hysteresis completely or approximately, such as [20, 21]. The main advantage of this inverse control approach is to compensate the effects of hysteresis nonlinearities directly. However, the construction of the inverse hysteresis will increase the complexity of the control systems and may limit the application in the industrial systems. Also, the compensation error depends on the hysteresis modeling parameters; therefore, it is difficult to get the analytical expression of the compensation error. Alternatively, another method is to fuse the hysteresis models with control methods without constructing the hysteresis inverse [9, 22–24], which can be applied in the real-time systems conveniently. For this control structure without inverse, the key point is to explore the characteristics of the hysteresis model and then investigate the suitable control methods to mitigate the effects caused by hysteresis.

Synthesizing the hysteresis modeling methods and control approaches, the output tracking problem for a class of uncertain nonlinear systems in strict-feedback form with unknown Duhem hysteresis is discussed. For the Duhem model, one adaptive robust controller for a class of nonlinear systems was discussed in [25]. Still following the line, the robust adaptive control method for a class of uncertain nonlinear systems in strict-feedback form is investigated in this paper. In order to mitigate the design difficulty caused by the smooth function term in the uncertain nonlinear systems, the mean value theorem and a Nussbaum function lemma are used. The proposed dynamic surface control (DSC) approach [26] without hysteresis inverse avoids "the explosion complexity" in the standard backstepping design, mitigates the negative effects arising from the unknown hysteresis, and ensures the semiglobal uniform ultimate boundedness of all the signals in the closed-loop system.

The rest of this paper is organized as follows. In Section 2, the control problem is formulated. Duhem hysteresis model is introduced in Section 3. In Section 4, an adaptive dynamic surface controller is developed for a class of nonlinear systems in strict-feedback form with unknown Duhem hysteresis, and the stability analysis is given as well. Computer simulations are shown to verify the effectiveness of the proposed scheme in Section 5. Section 6 concludes the paper.

2. Problem Statement

Consider the following class of uncertain nonlinear systems in strict-feedback form with unknown hysteresis input:

$$\begin{aligned} \dot{x}_i &= \theta_i f_i(\overline{x}_i) + g_i x_{i+1} + d_i(x,t), \quad i = 1,\dots,n-1, \\ \dot{x}_n &= \theta_n f_n(x) + g_n w(u) + d_n(x,t), \\ y &= x_1, \end{aligned} \tag{1}$$

where $\overline{x}_i = [x_i,\dots,x_i]^T \in R^i$, $i = 1,\dots,n$; $x = \overline{x}_n = [x_1,\dots,x_n]^T \in R^n$ and $y \in R$ are the system states and output; g_i, θ_i are unknown constant parameters, $d_i(x,t)$ denote the unknown disturbances, $f_i(\cdot)$ are known smooth functions, $i = 1,\dots,n$; and $w \in R$ is the output of the hysteresis nonlinearity with the actual input u.

The control objective is to design a control law u in (1), forcing the output y to track a given desired trajectory y_d, while all the signals of closed-loop system are uniformly bounded.

The following assumptions of the system (1) are made.

Assumption 1. The desired trajectory y_d is continuous and its first-order derivative $\dot{y}_d$ and second-order derivative $\ddot{y}_d$ are bounded and available; that is, there exists a positive constant B_0, such that $\Omega_d = \{[y_d, \dot{y}_d, \ddot{y}_d]^T : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le B_0\} \subset R^3$.

Assumption 2. The signs of g_i are known, and there exist unknown positive constants g_{i0} and g_{i1} such that $0 < g_{i0} \le |g_i| \le g_{i1} < \infty$. Without loss of generality, it can be assumed that $0 < g_{i0} \le g_i$, $i = 1,\dots,n$.

Assumption 3. The disturbances $d_i(x,t)$, $i = 1,\dots,n$, satisfy

$$|d_i(x,t)| \le b_i \rho_i(\overline{x}_i), \tag{2}$$

where $\rho_i(\overline{x}_i)$ are known nonnegative smooth functions and b_i are unknown nonnegative constants.

Remark 4. It should be mentioned that the knowledge of g_{i0} and g_{i1} is not required to be known, which is only used in the analysis of the latter stability proof.

3. Hysteresis Model

In this paper, the Duhem model is used to describe the hysteresis nonlinearity, which is defined by [14]

$$\frac{dw}{dt} = \alpha \left|\frac{du}{dt}\right| (\lambda(u) - w) + \frac{du}{dt}\psi(u), \tag{3}$$

where u and w are the hysteresis input and output, respectively; α is a constant; and $\lambda(u)$ and $\psi(u)$ are shape functions of u.

In order to get the analytic expression of the hysteresis output w, the following three conditions [10, 27, 28] are used for Duhem model.

Condition 1. $\lambda(u)$ is a piecewise smooth, monotone increasing, odd function of u, with a derivative $\dot{\lambda}(u)$, that obtains a finite limit $\lim_{u\to\infty}\dot{\lambda}(u)$.

Condition 2. $\psi(u)$ is a piecewise continuous, even function of u, with a finite limit satisfying

$$\lim_{u\to\infty}\psi(u) = \lim_{u\to\infty}\dot{\lambda}(u). \tag{4}$$

Condition 3. $\dot{\lambda}(u) > \psi(u) > \alpha e^{\alpha u}\int_u^\infty |\dot{\lambda}(\zeta) - \psi(\zeta)| e^{-\alpha\zeta} d\zeta$ for all finite u.

Remark 5. By selecting suitable shape functions, Duhem model can describe the different characteristics of the hysteresis nonlinearities. For example, choose $\alpha = 5$ and $\psi(u) = \dot{\lambda}(u)(1 - 0.85e^{(-0.1|u|)})$ with different shape function $\lambda(u)$

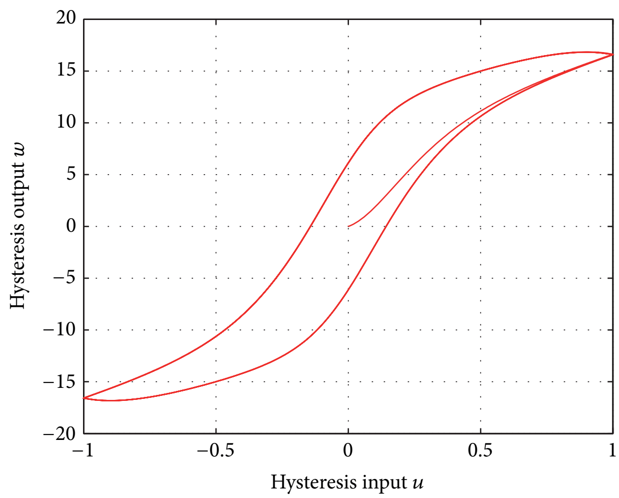

Figure 1: Hysteresis curves described by $\lambda(u) = 10\tanh 5u + 8u$.

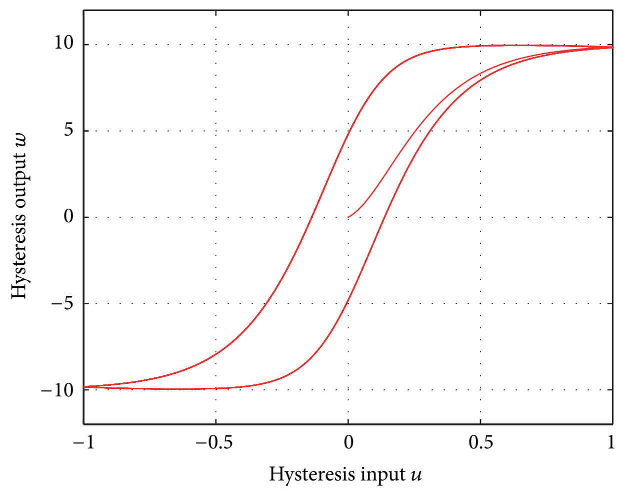

Figure 2: Hysteresis curves described by $\lambda(u) = 10\tanh 5u$.

satisfying three properties; the described hysteresis curves are shown in Figures 1 and 2.

Under the previous three conditions, the Duhem model (3) can be solved explicitly for u piecewise monotone as [14]

$$w = \lambda(u) + \eta(u), \tag{5}$$

where

$$\begin{aligned}\eta(u) = {} & [w_0 - \lambda(u_0)] e^{-\alpha(u-u_0)\operatorname{sgn}(\dot{u})} \\ & + e^{-\alpha u \operatorname{sgn}(\dot{u})} \int_{u_0}^{u} \left[\psi(\zeta) - \dot{\lambda}(\zeta)\right] e^{\alpha\zeta \operatorname{sgn}(\dot{u})} d\zeta.\end{aligned} \tag{6}$$

For $\eta(u)$, if $w(u; u_0, w_0)$ is the solution of (5) with initial values (w_0, u_0), one has

$$\begin{aligned}\lim_{u\to+\infty} \eta(u) &= \lim_{u\to+\infty} [w(u; u_0, w_0) - \lambda(u)] = 0, \\ \lim_{u\to-\infty} \eta(u) &= \lim_{u\to-\infty} [w(u; u_0, w_0) - \lambda(u)] = 0;\end{aligned} \tag{7}$$

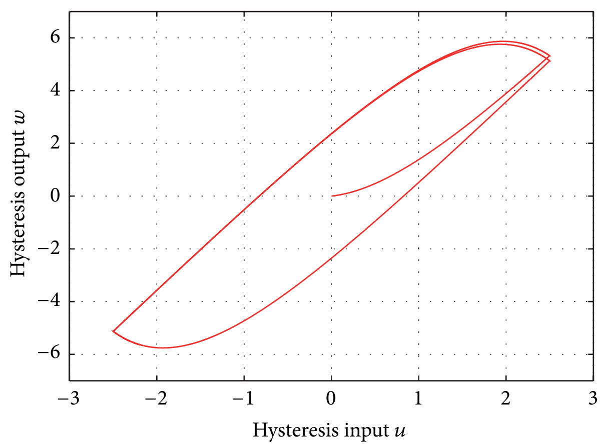

Figure 3: Hysteresis curves described by Backlash-like model.

then it can be deduced that $\eta(u)$ is bounded [14] easily. For simplicity, let D denote the bound of $\eta(u)$, where D is a positive constant.

Remark 6. When $f(v) = cu$ and $g(u)$ is a constant B, the Duhem model can be expressed as

$$\frac{dw}{dt} = \alpha \left|\frac{du}{dt}\right| [cu - w] + \frac{du}{dt} B. \tag{8}$$

When $c > B$, the Duhem model becomes the Backlash-like model defined in [9]. According to the above analysis, it is obvious that the Backlash-like model is a special case of the Duhem model. However, it should be noted that when $f(u) = cu$ and $g(u) = B$, Conditions 1 and 2 are not satisfied necessarily for the Duhem model. Similarly, (8) can be solved explicitly for the Backlash-like model:

$$w = cu + \eta(u) \tag{9}$$

with

$$\begin{aligned}\eta(u) = {} & [w_0 - cu_0] e^{-\alpha(u-u_0)\operatorname{sgn}(\dot{u})} \\ & + e^{-\alpha u \operatorname{sgn}(\dot{u})} \int_{u_0}^{u} [B - c] e^{\alpha\zeta \operatorname{sgn}(\dot{u})} d\zeta.\end{aligned} \tag{10}$$

According to the analysis in [9], it has

$$\begin{aligned}\lim_{u_0\to+\infty} \eta(u) &= \lim_{u_0\to+\infty} [w(u; u_0, w_0) - f(u)] = -\frac{c-B}{\alpha}, \\ \lim_{u_0\to-\infty} \eta(u) &= \lim_{u_0\to-\infty} [w(u; u_0, w_0) - f(u)] = \frac{c-B}{\alpha};\end{aligned} \tag{11}$$

so the disturbance term $\varphi(v)$ is still bounded.

According to the previous proof, D still can be used to denote the bound of $\eta(u)$ defined for Backlash-like model. As a comparison, when $\lambda(u) = cu = 3.1635u$, $\eta(u) = B = 0.345$, and the input $u(t) = 2.5\sin(2.3t)$, the Duhem model can be reexpressed as the Backlash-like model; then the curve of the Backlash-like model is shown in Figure 3.

4. Adaptive DSC Design and Stability Analysis

In this section, the procedure for the design of adaptive dynamic surface controller and system stability will be given. Considering the characteristics of the hysteresis nonlinearities existing in the actual controlled plant, the following assumption is made for the hysteresis model (3).

Assumption 7. The function $\lambda(u)$ of Duhem hysteresis (3) is a smooth and strictly increasing function.

According to Condition 1 of Duhem model, $\lambda(0) = 0$. Combining the derivative form of mean value theorem and Assumption 7, there exists $\vartheta \in (\min(0,u), \max(0,u))$ such that

$$\lambda(u) = \lambda(u) - \lambda(0) = \left.\frac{d\lambda(u)}{du}\right|_{u=\vartheta}(u-0) = \dot{\lambda}(\vartheta)u. \quad (12)$$

Then w is rewritten as

$$w = \dot{\lambda}(\vartheta)u + \eta(u); \quad (13)$$

then the system (1) is expressed as

$$\begin{aligned} \dot{x}_i &= \theta_i f_i(\overline{x}_i) + g_i x_{i+1} + d_i(x,t), \quad i = 1,\dots,n-1, \\ \dot{x}_n &= \theta_n f_n(x) + g_n\left[\dot{\lambda}(\vartheta)u + \eta(u)\right] + d_n(x,t), \\ y &= x_1. \end{aligned} \quad (14)$$

Since the sign of the control gain $g_n\dot{\lambda}(\vartheta)$ is unknown, one useful lemma is given as follows.

Lemma 8 (see [29]). *Let $V(\cdot)$, $\zeta(\cdot)$ be the smooth functions defined on $[0,t_f)$ with $V(t) \ge 0$, for all $t \in [0,t_f)$, and let $N(\cdot)$ be an ever smooth Nussbaum-type function. If the following inequalities hold*

$$V(t) \le c_0 + e^{-c_1 t}\int_0^t \left[G(\cdot)N(\zeta) + 1\right]\dot{\zeta}e^{c_1\tau}d\tau, \quad (15)$$

where c_0 represents some suitable constant, c_1 is a positive constant, and $G(\cdot)$ is a time-varying parameter which takes values in the unknown closed intervals $I = [l^-, l^+]$, with $0 \notin I$, then $V(t)$, $\zeta(t)$, and $\int_0^t G(\cdot)N(\zeta)\dot{\zeta}d\tau$ must be bounded on $[0,t_f)$.

4.1. Adaptive DSC Design. Following the DSC procedure, the coordinate transformation is made as follows:

$$z_1 = x_1 - y_d, \qquad z_i = x_i - s_{i-1}, \quad i = 2,\dots,n, \quad (16)$$

where s_{i-1} are output of the filter (17).

The first-order low pass filters and the boundary filter errors e_i are defined as

$$\tau_i\dot{s}_i + s_i = \alpha_i, \qquad s_i(0) = \alpha_i(0), \quad (17)$$

$$e_i = s_i - \alpha_i, \quad (18)$$

where τ_i are the filter time constant and α_i are the filter input, which are also the virtual control law for the ith subsystem specified hereinafter, $i = 1,\dots,n-1$.

Step 1. Considering the first equation in (14) and invoking (16)–(18), the time derivative of z_1 is given by

$$\begin{aligned} \dot{z}_1 &= \theta_1 f_1(\overline{x}_1) + g_1 x_2 + d_1(x,t) - \dot{y}_d \\ &= \theta_1 f_1(\overline{x}_1) + g_1\left[z_2 + \alpha_1 + e_1\right] + d_1(x,t) - \dot{y}_d. \end{aligned} \quad (19)$$

Define the Lyapunov function candidate

$$V_1 = \frac{1}{2}\left(\frac{1}{g_1}z_1^2 + \frac{1}{\gamma_{\theta_1}}\widetilde{\theta}_{g_1}^2 + \frac{1}{\gamma_{b_1}}\widetilde{b}_{g_1}^2 + \frac{1}{\gamma_{\overline{g}_1}}\widetilde{\overline{g}}_{g_1}^2\right), \quad (20)$$

where $\widetilde{\theta}_{g_1} = \theta_{g_1} - \widehat{\theta}_{g_1}$, $\widetilde{b}_{g_1} = b_{g_1} - \widehat{b}_{g_1}$, and $\widetilde{\overline{g}}_{g_1} = \overline{g}_{g_1} - \widehat{\overline{g}}_{g_1}$ with $\widehat{\theta}_{g_1}$, $\widehat{b}_{g_1}$, and $\widehat{\overline{g}}_{g_1}$ as the estimates of $\theta_{g_1} = \theta_1/g_1$, $b_{g_1} = b_1/g_1$, and $\overline{g}_{g_1} = 1/g_1$, respectively. γ_{θ_1}, γ_{b_1}, and $\gamma_{\overline{g}_1}$ are positive design parameters.

Note that the following inequalities hold [30]:

$$\begin{aligned} z_i d_i(x,t) &\le b_i\left|z_i\right|\rho_i(\overline{x}_i) \\ &\le b_i z_i\rho_i(\overline{x}_i)\tanh\left(\frac{z_i\rho_i(\overline{x}_i)}{\omega}\right) + 0.2785\omega b_i, \end{aligned} \quad (21)$$

where $\omega > 0$, $i = 1,\dots,n$.

Based on (19) and (21), it has

$$\begin{aligned} \dot{V}_1 &= \frac{z_1}{g_1}\left(\theta_1 f_1(\overline{x}_1) + g_1\left[z_2 + \alpha_1 + e_1\right] + d_1(x,t) - \dot{y}_d\right) \\ &\quad + \frac{1}{\gamma_{\theta_1}}\widetilde{\theta}_{g_1}\dot{\widetilde{\theta}}_{g_1} + \frac{1}{\gamma_{b_1}}\widetilde{b}_{g_1}\dot{\widetilde{b}}_{g_1} + \frac{1}{\gamma_{\overline{g}_1}}\widetilde{\overline{g}}_{g_1}\dot{\widetilde{\overline{g}}}_{g_1} \\ &\le z_1\Bigg(\theta_{g_1} f_1(\overline{x}_1) + \alpha_1 \\ &\qquad + b_{g_1}\rho_1(\overline{x}_1)\tanh\left(\frac{z_1\rho_1(\overline{x}_1)}{\omega}\right) - \overline{g}_{g_1}\dot{y}_d\Bigg) \\ &\quad + z_1 z_2 + z_1 e_1 + 0.2785\omega b_{g_1} + \frac{1}{\gamma_{\theta_1}}\widetilde{\theta}_{g_1}\dot{\widetilde{\theta}}_{g_1} \\ &\quad + \frac{1}{\gamma_{b_1}}\widetilde{b}_{g_1}\dot{\widetilde{b}}_{g_1} + \frac{1}{\gamma_{\overline{g}_1}}\widetilde{\overline{g}}_{g_1}\dot{\widetilde{\overline{g}}}_{g_1} \\ &\le z_1\Bigg(\widehat{\theta}_{g_1} f_1(\overline{x}_1) + \alpha_1 + \widehat{b}_{g_1}\rho_1(\overline{x}_1)\tanh\left(\frac{z_1\rho_1(\overline{x}_1)}{\omega}\right) \\ &\qquad - \widehat{\overline{g}}_{g_1}\dot{y}_d\Bigg) \\ &\quad + z_1 z_2 + z_1 e_1 + 0.2785\omega b_{g_1} \\ &\quad + \widetilde{\theta}_{g_1}\left(z_1 f_1(\overline{x}_1) - \frac{1}{\gamma_{\theta_1}}\dot{\widehat{\theta}}_{g_1}\right) \\ &\quad + \widetilde{b}_{g_1}\left(z_1\rho_1(\overline{x}_1)\tanh\left(\frac{z_1\rho_1(\overline{x}_1)}{\omega}\right) - \frac{1}{\gamma_{b_1}}\dot{\widehat{b}}_{g_1}\right) \\ &\quad + \widetilde{\overline{g}}_{g_1}\left(-z_1\dot{y}_d - \frac{1}{\gamma_{\overline{g}_1}}\dot{\widehat{\overline{g}}}_{g_1}\right). \end{aligned} \quad (22)$$

The virtual control law α_1 and the adaptive laws $\hat{\theta}_{g_1}$, $\hat{b}_{g_1}$, and $\hat{\bar{g}}_{g_1}$ are designed as

$$\begin{aligned}\alpha_1 &= -k_1 z_1 - \hat{\theta}_{g_1} f_1(\bar{x}_1) - \hat{b}_{g_1}\rho_1(\bar{x}_1)\tanh\left(\frac{z_1\rho_1(\bar{x}_1)}{\omega}\right) + \hat{\bar{g}}_{g_1}\dot{y}_d,\\ \dot{\hat{\theta}}_{g_1} &= \gamma_{\theta_1}\left(z_1 f_1(\bar{x}_1) - \varpi_1\hat{\theta}_{g_1}\right),\\ \dot{\hat{b}}_{g_1} &= \gamma_{b_1}\left(z_1\rho_1(\bar{x}_1)\tanh\left(\frac{z_1\rho_1(\bar{x}_1)}{\omega}\right) - \mu_1\hat{b}_{g_1}\right),\\ \dot{\hat{\bar{g}}}_{g_1} &= \gamma_{\bar{g}_1}\left(-z_1\dot{y}_d - \nu_1\hat{\bar{g}}_{g_1}\right),\end{aligned} \tag{23}$$

where k_1, ϖ_1, μ_1, and ν_1 are positive design parameters.

Substituting (23) into (22), we obtain

$$\begin{aligned}\dot{V}_1 \le& -k_1 z_1^2 + z_1 z_2 + z_1 e_1 + 0.2785\omega b_{g_1} + \varpi_1\tilde{\theta}_{g_1}\hat{\theta}_{g_1}\\ &+ \mu_1\tilde{b}_{g_1}\hat{b}_{g_1} + \nu_1\tilde{\bar{g}}_{g_1}\hat{\bar{g}}_{g_1}.\end{aligned} \tag{24}$$

By using the following inequalities

$$\begin{aligned}\varpi_1\tilde{\theta}_{g_1}\hat{\theta}_{g_1} &\le \frac{\varpi_1}{2}\left(-\tilde{\theta}_{g_1}^2 + \theta_{g_1}^2\right),\\ \mu_1\tilde{b}_{g_1}\hat{b}_{g_1} &\le \frac{\mu_1}{2}\left(-\tilde{b}_{g_1}^2 + b_{g_1}^2\right),\\ \nu_1\tilde{\bar{g}}_{g_1}\hat{\bar{g}}_{g_1} &\le \frac{\nu_1}{2}\left(-\tilde{\bar{g}}_{g_1}^2 + \bar{g}_{g_1}^2\right),\end{aligned} \tag{25}$$

we have

$$\begin{aligned}\dot{V}_1 \le& -k_1 z_1^2 - \frac{\varpi_1}{2}\tilde{\theta}_{g_1}^2 - \frac{\mu_1}{2}\tilde{b}_{g_1}^2 - \frac{\nu_1}{2}\tilde{\bar{g}}_{g_1}^2 + z_1 z_2 + z_1 e_1\\ &+ 0.2785\omega b_{g_1} + \frac{\varpi_1}{2}\theta_{g_1}^2 + \frac{\mu_1}{2}b_{g_1}^2 + \frac{\nu_1}{2}\bar{g}_{g_1}^2.\end{aligned} \tag{26}$$

Step i ($2 \le i \le n-1$). Considering (17) and (18), and $z_i = x_i - s_{i-1}$, it has

$$s_i = e_i + \alpha_i, \quad \dot{s}_i = -\frac{e_i}{\tau_i}, \quad i = 1, \ldots, n-1, \tag{27}$$

$$\begin{aligned}\dot{z}_i &= \theta_i f_i(\bar{x}_i) + g_i x_{i+1} + d_i(x,t) + \frac{e_{i-1}}{\tau_{i-1}}\\ &= \theta_i f_i(\bar{x}_i) + g_i\left[z_{i+1} + \alpha_i + e_i\right] + d_i(x,t) + \frac{e_{i-1}}{\tau_{i-1}}.\end{aligned} \tag{28}$$

Define the Lyapunov function candidate

$$V_i = V_{i-1} + \frac{1}{2}\left(\frac{1}{g_i}z_i^2 + \frac{1}{\gamma_{\theta_i}}\tilde{\theta}_{g_i}^2 + \frac{1}{\gamma_{b_i}}\tilde{b}_{g_i}^2 + \frac{1}{\gamma_{\bar{g}_i}}\tilde{\bar{g}}_{g_i}^2\right), \tag{29}$$

where $\tilde{\theta}_{g_i} = \theta_{g_i} - \hat{\theta}_{g_i}$, $\tilde{b}_{g_i} = b_{g_i} - \hat{b}_{g_i}$, and $\tilde{\bar{g}}_{g_i} = \bar{g}_{g_i} - \hat{\bar{g}}_{g_i}$ with $\hat{\theta}_{g_i}$, $\hat{b}_{g_i}$, and $\hat{\bar{g}}_{g_i}$ as the estimates of $\theta_{g_i} = \theta_i/g_i$, $b_{g_i} = b_i/g_i$, and $\bar{g}_{g_i} = 1/g_i$, respectively. γ_{θ_i}, γ_{b_i}, and $\gamma_{\bar{g}_i}$ are positive design parameters.

Based on (21) and (28), the time derivative of V_i is given by

$$\begin{aligned}\dot{V}_i =& \dot{V}_{i-1} + \frac{z_i}{g_i}\left(\theta_i f_i(\bar{x}_i) + g_i\left[z_{i+1} + \alpha_i + e_i\right] + d_i(x,t) + \frac{e_{i-1}}{\tau_{i-1}}\right)\\ &+ \frac{1}{\gamma_{\theta_i}}\tilde{\theta}_{g_i}\dot{\tilde{\theta}}_{g_i} + \frac{1}{\gamma_{b_i}}\tilde{b}_{g_i}\dot{\tilde{b}}_{g_i} + \frac{1}{\gamma_{\bar{g}_i}}\tilde{\bar{g}}_{g_i}\dot{\tilde{\bar{g}}}_{g_i}\\ \le& \dot{V}_{i-1} + z_i\left(z_{i-1} + \theta_{g_i} f_i(\bar{x}_i) + \alpha_i + b_{g_i}\rho_i(\bar{x}_i)\tanh\left(\frac{z_i\rho_i(\bar{x}_i)}{\omega}\right) + \bar{g}_{g_i}\frac{e_{i-1}}{\tau_{i-1}}\right)\\ &+ z_i z_{i+1} - z_i z_{i-1} + z_i e_i + 0.2785\omega b_{g_i}\\ &+ \frac{1}{\gamma_{\theta_i}}\tilde{\theta}_{g_i}\dot{\tilde{\theta}}_{g_i} + \frac{1}{\gamma_{b_i}}\tilde{b}_{g_i}\dot{\tilde{b}}_{g_i} + \frac{1}{\gamma_{\bar{g}_i}}\tilde{\bar{g}}_{g_i}\dot{\tilde{\bar{g}}}_{g_i}\\ \le& \dot{V}_{i-1} + z_i\left(z_{i-1} + \hat{\theta}_{g_i} f_i(\bar{x}_i) + \alpha_i + \hat{b}_{g_i}\rho_i(\bar{x}_i)\tanh\left(\frac{z_i\rho_i(\bar{x}_i)}{\omega}\right) + \hat{\bar{g}}_{g_i}\frac{e_{i-1}}{\tau_{i-1}}\right)\\ &+ z_i z_{i+1} - z_i z_{i-1} + z_i e_i + 0.2785\omega b_{g_i}\\ &+ \tilde{\theta}_{g_i}\left(z_i f_i(\bar{x}_i) - \frac{1}{\gamma_{\theta_i}}\dot{\hat{\theta}}_{g_i}\right)\\ &+ \tilde{b}_{g_i}\left(z_i\rho_i(\bar{x}_i)\tanh\left(\frac{z_i\rho_i(\bar{x}_i)}{\omega}\right) - \frac{1}{\gamma_{b_i}}\dot{\hat{b}}_{g_i}\right)\\ &+ \tilde{\bar{g}}_{g_i}\left(z_i\frac{e_{i-1}}{\tau_{i-1}} - \frac{1}{\gamma_{\bar{g}_i}}\dot{\hat{\bar{g}}}_{g_i}\right).\end{aligned} \tag{30}$$

The virtual control law α_i and the adaptive update laws $\hat{\theta}_{g_i}$, $\hat{b}_{g_i}$, and $\hat{\bar{g}}_{g_i}$ are designed as

$$\begin{aligned}\alpha_i =& -k_i z_i - z_{i-1} - \hat{\theta}_{g_i} f_i(\bar{x}_i) - \hat{b}_{g_i}\rho_i(\bar{x}_i)\tanh\left(\frac{z_i\rho_i(\bar{x}_i)}{\omega}\right)\\ &- \hat{\bar{g}}_{g_i}\frac{e_{i-1}}{\tau_{i-1}},\end{aligned} \tag{31}$$

$$\dot{\hat{\theta}}_{g_i} = \gamma_{\theta_i}\left(z_i f_i(\bar{x}_i) - \varpi_i\hat{\theta}_{g_i}\right), \tag{32}$$

$$\dot{\hat{b}}_{g_i} = \gamma_{b_i}\left(z_i\rho_i(\bar{x}_i)\tanh\left(\frac{z_i\rho_i(\bar{x}_i)}{\omega}\right) - \mu_i\hat{b}_{g_i}\right), \tag{33}$$

$$\dot{\hat{\bar{g}}}_{g_i} = \gamma_{\bar{g}_i}\left(z_i\frac{e_{i-1}}{\tau_{i-1}} - \nu_i\hat{\bar{g}}_{g_i}\right), \tag{34}$$

where k_i, ϖ_i, μ_i, and ν_i are positive design parameters.

Considering the following inequalities

$$\varpi_i \tilde{\theta}_{g_i} \hat{\theta}_{g_i} \le \frac{\varpi_i}{2}\left(-\tilde{\theta}_{g_i}^2 + \theta_{g_i}^2\right),$$
$$\mu_i \tilde{b}_{g_i} \hat{b}_{g_i} \le \frac{\mu_i}{2}\left(-\tilde{b}_{g_i}^2 + b_{g_i}^2\right), \quad (35)$$
$$\nu_i \tilde{\bar{g}}_{g_i} \hat{\bar{g}}_{g_i} \le \frac{\nu_i}{2}\left(-\tilde{\bar{g}}_{g_i}^2 + \bar{g}_{g_i}^2\right),$$

we have

$$\dot{V}_i \le -\sum_{j=1}^{i} k_j z_j^2 - \sum_{j=1}^{i}\left(\frac{\varpi_1}{2}\tilde{\theta}_{g_1}^2 + \frac{\mu_1}{2}\tilde{b}_{g_1}^2 + \frac{\nu_i}{2}\tilde{\bar{g}}_{g_1}^2\right)$$
$$+ z_i z_{i+1} + \sum_{j=1}^{i}\left(z_j e_j + 0.2785\omega b_{g_j} + \frac{\varpi_j}{2}\theta_{g_j}^2 + \frac{\mu_i}{2} b_{g_j}^2 + \frac{\nu_i}{2}\bar{g}_{g_j}^2\right). \quad (36)$$

Step n. The actual control law u will be designed in this step. Considering $z_n = x_n - s_{n-1}$ and $\dot{s}_{n-1} = -e_{n-1}/\tau_{n-1}$, the time derivative of z_n is given by

$$\dot{z}_n = \theta_n f_n(x) + g_n\left[\dot{\lambda}(\vartheta) u + \eta(u)\right] + d_n(x,t) + \frac{e_{n-1}}{\tau_{n-1}}. \quad (37)$$

Define the Lyapunov function candidate

$$V_n = V_{n-1} + \frac{1}{2}\left(z_n^2 + \frac{1}{\gamma_{\theta_n}}\tilde{\theta}_{g_n}^2 + \frac{1}{\gamma_{b_n}}\tilde{b}_{g_n}^2 + \frac{1}{\gamma_{\bar{g}_n}}\tilde{\bar{g}}_{g_n}^2\right), \quad (38)$$

where $\tilde{\theta}_{g_n} = \theta_{g_n} - \hat{\theta}_{g_n}$, $\tilde{b}_{g_n} = b_{g_n} - \hat{b}_{g_n}$, and $\tilde{\bar{g}}_{g_n} = \bar{g}_{g_n} - \hat{\bar{g}}_{g_n}$ with $\hat{\theta}_{g_n}$, $\hat{b}_{g_n}$, and $\hat{\bar{g}}_{g_n}$ as the estimates of $\theta_{g_n} = \theta_n$, $b_{g_n} = b_n$, and $\bar{g}_{g_n} = g_n D$, respectively. γ_{θ_n}, γ_{b_n}, and $\gamma_{\bar{g}_n}$ are positive design parameters.

Based on (37), we have

$$\dot{V}_n = \dot{V}_{n-1} + z_n\left(\theta_n f_n(x) + g_n\left[\dot{\lambda}(\vartheta) u + \eta(u)\right] + d_n(x,t) + \frac{e_{n-1}}{\tau_{n-1}}\right)$$
$$+ \frac{1}{\gamma_{\theta_i}}\tilde{\theta}_{g_n}\dot{\hat{\theta}}_{g_n} + \frac{1}{\gamma_{b_n}}\tilde{b}_{g_n}\dot{\hat{b}}_{g_n} + \frac{1}{\gamma_{\bar{g}_n}}\tilde{\bar{g}}_{g_n}\dot{\hat{\bar{g}}}_{g_n} \quad (39)$$
$$= \dot{V}_{n-1} + z_n\theta_n f_n(x) + z_n g_n \dot{\lambda}(\vartheta) u + z_n g_n \eta(u) + z_n d_n(x,t)$$
$$+ z_n\frac{e_{n-1}}{\tau_{n-1}} + \frac{1}{\gamma_{\theta_n}}\tilde{\theta}_{g_n}\dot{\hat{\theta}}_{g_n} + \frac{1}{\gamma_{b_n}}\tilde{b}_{g_n}\dot{\hat{b}}_{g_n} + \frac{1}{\gamma_{\bar{g}_n}}\tilde{\bar{g}}_{g_n}\dot{\hat{\bar{g}}}_{g_n}.$$

Similar to (21), the following inequalities are used:

$$z_n g_n \le \bar{g}_{g_n}|z_n| \le \bar{g}_{g_n} z_n \tanh\left(\frac{z_n}{\omega}\right) + 0.2785\omega\bar{g}_{g_n}; \quad (40)$$

we have

$$\dot{V}_n \le \dot{V}_{n-1} + z_n\theta_n f_n(x) + z_n g_n \dot{\lambda}(\vartheta) u + \bar{g}_{g_n} z_n \tanh\left(\frac{z_n}{\omega}\right) + 0.2785\omega\bar{g}_{g_n} + 0.2785\omega b_n$$
$$+ b_n z_n \rho_n(x)\tanh\left(\frac{z_n\rho_n(x)}{\omega}\right) + z_n\frac{e_{n-1}}{\tau_{n-1}} + \frac{1}{\gamma_{\theta_n}}\tilde{\theta}_{g_n}\dot{\hat{\theta}}_{g_n} + \frac{1}{\gamma_{b_n}}\tilde{b}_{g_n}\dot{\hat{b}}_{g_n} + \frac{1}{\gamma_{\bar{g}_n}}\tilde{\bar{g}}_{g_n}\dot{\hat{\bar{g}}}_{g_n}$$
$$\le \dot{V}_{n-1} + z_n z_{n-1} + z_n\hat{\theta}_n f_n(x) + z_n g_n \dot{\lambda}(\vartheta) u + \hat{\bar{g}} z_n \tanh\left(\frac{z_n}{\omega}\right)$$
$$+ 0.2785\omega\bar{g}_{g_n} + 0.2785\omega b_n + \hat{b}_n z_n \rho_n(x)\tanh\left(\frac{z_n\rho_n(x)}{\omega}\right) + z_n\frac{e_{n-1}}{\tau_{n-1}} \quad (41)$$
$$+ \tilde{\theta}_{g_n}\left(z_n f_n(x) - \frac{1}{\gamma_{\theta_n}}\dot{\hat{\theta}}_{g_n}\right) + \tilde{b}_{g_n}\left(z_n\rho_n(x)\tanh\left(\frac{z_n\rho_n(x)}{\omega}\right) - \frac{1}{\gamma_{b_n}}\dot{\hat{b}}_{g_n}\right)$$
$$+ \tilde{\bar{g}}_{g_n}\left(z_n\tanh\left(\frac{z_n}{\omega}\right) - \frac{1}{\gamma_{\bar{g}_n}}\dot{\hat{\bar{g}}}_{g_n}\right) - z_n z_{n-1}.$$

The actual control law u and the adaptive laws ζ, $\hat{\theta}_{g_n}$, $\hat{b}_{g_n}$, and $\hat{\bar{g}}_{g_n}$ are designed as

$$u = N(\zeta)\left[k_n z_n + z_{n-1} + \hat{\theta}_{g_n} f_n(x) + \hat{\bar{g}}_{g_n}\tanh\left(\frac{z_n}{\omega}\right) + \hat{b}_{g_n}\rho_n(x)\tanh\left(\frac{z_n\rho_n(x)}{\omega}\right) + \frac{e_{n-1}}{\tau_{n-1}}\right], \quad (42)$$

$$\dot{\zeta} = k_n z_n^2 + z_n z_{n-1} + z_n\hat{\theta}_{g_n} f_n(x) + \hat{\bar{g}}_{g_n} z_n\tanh\left(\frac{z_n}{\omega}\right) + z_n\hat{b}_{g_n}\rho_n(x)\tanh\left(\frac{z_n\rho_n(x)}{\omega}\right) + z_n\frac{e_{n-1}}{\tau_{n-1}},$$
$$\dot{\hat{\theta}}_{g_n} = \gamma_{\theta_n}\left(z_n f_n(x) - \varpi_n\hat{\theta}_{g_n}\right), \quad (43)$$
$$\dot{\hat{b}}_{g_n} = \gamma_{b_n}\left(z_n\rho_n(x)\tanh\left(\frac{z_n\rho_n(x)}{\omega}\right) - \mu_n\hat{b}_{g_n}\right),$$
$$\dot{\hat{\bar{g}}}_{g_n} = \gamma_{\bar{g}_n}\left(z_n\tanh\left(\frac{z_n}{\omega}\right) - \nu_n\hat{\bar{g}}_{g_n}\right),$$

where k_n, ϖ_n, μ_n, and ν_n are positive design parameters.

Similarly, the following inequalities will be utilized:

$$\varpi_n\tilde{\theta}_{g_n}\hat{\theta}_{g_n} \le \frac{\varpi_n}{2}\left(-\tilde{\theta}_{g_n}^2 + \theta_{g_n}^2\right),$$
$$\mu_n\tilde{b}_{g_n}\hat{b}_{g_n} \le \frac{\mu_n}{2}\left(-\tilde{b}_{g_n}^2 + b_{g_n}^2\right), \quad (44)$$
$$\nu_n\tilde{\bar{g}}_{g_n}\hat{\bar{g}}_{g_n} \le \frac{\nu_n}{2}\left(-\tilde{\bar{g}}_{g_n}^2 + \bar{g}_{g_n}^2\right);$$

then, we obtain

$$\dot{V}_n \le -\sum_{j=1}^{n} k_j z_j^2 - \sum_{j=1}^{n}\left(\frac{\varpi_j}{2}\tilde{\theta}_{g_j}^2 + \frac{\mu_j}{2}\tilde{b}_{g_j}^2 + \frac{\nu_j}{2}\tilde{\overline{g}}_{g_j}^2\right) + \left[g_n \dot{\lambda}(\vartheta) N(\zeta) + 1\right]\dot{\zeta} + 0.2785\omega\overline{g}_{g_n} + \sum_{j=1}^{n-1}\left(z_j e_j\right) + \sum_{j=1}^{n}\left(0.2785\omega b_{g_j} + \frac{\varpi_j}{2}\theta_{g_j}^2 + \frac{\mu_i}{2}b_{g_j}^2 + \frac{\nu_i}{2}\overline{g}_{g_j}^2\right). \tag{45}$$

4.2. Stability Analysis. In this subsection, the uniform ultimate boundedness of all signals in the closed-loop system will be proven.

From (27) and (31), we have

$$\begin{aligned}\dot{e}_i &= \dot{s}_i - \dot{\alpha}_i \\ &= -\frac{e_i}{\tau_i} + \left(\frac{\partial\alpha_i}{\partial z_i}\dot{z}_i + \frac{\partial\alpha_i}{\partial\hat{\theta}_{g_i}}\dot{\hat{\theta}}_{g_i} + \frac{\partial\alpha_i}{\partial\hat{b}_{g_i}}\dot{\hat{b}}_{g_i} + \frac{\partial\alpha_i}{\partial\hat{\overline{g}}_{g_i}}\dot{\hat{\overline{g}}}_{g_i}\right) \\ &= -\frac{e_i}{\tau_i} + B_i\left(z_1,\dots,z_i,e_1,\dots,e_{i-1},\hat{\theta}_{g_1},\dots,\hat{\theta}_{g_i},\hat{b}_{g_1},\dots,\hat{b}_{g_i},\hat{\overline{g}}_{g_1},\dots,\hat{g}_{g_i},y_d,\dot{y}_d,\ddot{y}_d\right),\end{aligned} \tag{46}$$

where $B_i(z_1,\dots,z_i,e_1,\dots,e_{i-1},\hat{\theta}_{g_1},\dots,\hat{\theta}_{g_i},\hat{b}_{g_1},\dots,\hat{b}_{g_i},\hat{\overline{g}}_{g_1},\dots,\hat{g}_{g_i},y_d,\dot{y}_d,\ddot{y}_d) = (\partial\alpha_i/\partial z_i)\dot{z}_i + (\partial\alpha_i/\partial\hat{\theta}_{g_i})\dot{\hat{\theta}}_{g_i} + (\partial\alpha_i/\partial\hat{b}_{g_i})\dot{\hat{b}}_{g_i} + (\partial\alpha_i/\partial\hat{\overline{g}}_{g_i})\dot{\hat{\overline{g}}}_{g_i}$ are the continuous functions, $i = 1,\dots,n-1$.

To establish the boundedness of the closed-loop system, the following Lyapunov function candidate is defined as

$$V = V_n + \frac{1}{2}\sum_{i=1}^{n-1} e_i^2. \tag{47}$$

The main results can be summarized as follows.

Theorem 9. *Consider the closed-loop system consisting of the plant* (1), *the controller* (42), *and adaptation laws* (43) *under Assumptions 1–7. If* $V(0) \le P_0$ *for any* $P_0 > 0$, *there exist the appropriate design parameters* k_i, τ_i, γ_{θ_i}, γ_{b_i}, $\gamma_{\overline{g}_i}$, ϖ_i, μ_i, ν_i, *and* ω, *such that all signals in the closed-loop system are semiglobally uniformly ultimately bounded.*

Proof. Define the set $\Omega_i := \{[z_1,\dots,z_i,e_1,\dots,e_{i-1},\hat{\theta}_{g_1},\dots,\hat{\theta}_{g_i},\hat{b}_{g_1},\dots,\hat{b}_{g_i},\hat{\overline{g}}_{g_1},\dots,\hat{g}_{g_i}] : V_i + \sum_{j=1}^{i-1} e_j^2 \le 2P_0\} \subset R^{5i-1}$. From Assumption 1 and $V(0) \le P_0$ for any $P_0 > 0$, the set Ω_d and Ω_i are compact in R^3 and R^{5i-1}. Thus, $B_i(z_1,\dots,z_i,e_1,\dots,e_{i-1},\hat{\theta}_{g_1},\dots,\hat{\theta}_{g_i},\hat{b}_{g_1},\dots,\hat{b}_{g_i},\hat{\overline{g}}_{g_1},\dots,\hat{g}_{g_i},y_d,\dot{y}_d,\ddot{y}_d)$ have a maximum value M_i, $i = 1,\dots,n-1$, on $\Omega_d \times \Omega_i$. Equation (46) can be further derived as

$$\begin{aligned}e_i\dot{e} &\le -\frac{e_i^2}{\tau_i} + \left|e_i B_i\left(z_1,\dots,z_i,e_1,\dots,e_{i-1},\hat{\theta}_{g_1},\dots,\hat{\theta}_{g_i},\hat{b}_{g_1},\dots,\hat{b}_{g_i},\hat{\overline{g}}_{g_1},\dots,\hat{g}_{g_i},y_d,\dot{y}_d,\ddot{y}_d\right)\right| \\ &\le -\frac{e_i^2}{\tau_i} + \frac{1}{2}e_i^2 + \frac{1}{2}M_i^2.\end{aligned} \tag{48}$$

The derivative of V with respect to t follows from (45), (46), and (48) that

$$\begin{aligned}\dot{V} \le& -\sum_{j=1}^{n} k_j z_j^2 - \sum_{j=1}^{n}\left(\frac{\varpi_j}{2}\tilde{\theta}_{g_j}^2 + \frac{\mu_j}{2}\tilde{b}_{g_j}^2 + \frac{\nu_j}{2}\tilde{\overline{g}}_{g_j}^2\right) + \left[g_n\dot{\lambda}(\vartheta)N(\zeta)+1\right]\dot{\zeta} + 0.2785\omega\overline{g}_{g_n} + \sum_{j=1}^{n-1}\left(z_je_j\right) \\ &+ \sum_{j=1}^{n}\left(0.2785\omega b_{g_j} + \frac{\varpi_j}{2}\theta_{g_j}^2 + \frac{\mu_i}{2}b_{g_j}^2 + \frac{\nu_i}{2}\overline{g}_{g_j}^2\right) + \sum_{i=1}^{n-1} e_i\dot{e}_i \\ \le& -\sum_{j=1}^{n} k_j z_j^2 - \sum_{j=1}^{n}\left(\frac{\varpi_j}{2}\tilde{\theta}_{g_j}^2 + \frac{\mu_j}{2}\tilde{b}_{g_j}^2 + \frac{\nu_j}{2}\tilde{\overline{g}}_{g_j}^2\right) + \left[g_n\dot{\lambda}(\vartheta)N(\zeta)+1\right]\dot{\zeta} + 0.2785\omega\overline{g}_{g_n} + \frac{1}{2}\sum_{j=1}^{n-1} {z_j}^2 \\ &+ \sum_{j=1}^{n}\left(0.2785\omega b_{g_j} + \frac{\varpi_j}{2}\theta_{g_j}^2 + \frac{\mu_i}{2}b_{g_j}^2 + \frac{\nu_i}{2}\overline{g}_{g_j}^2\right) - \frac{1}{2}\sum_{j=1}^{n-1}\left(\frac{1}{\tau_i} - \frac{3}{2}\right){e_j}^2 + \sum_{i=1}^{n-1}\frac{1}{2}M_i^2 \\ \le& -\xi V + \left[g_n\dot{\lambda}(\vartheta)N(\zeta)+1\right]\dot{\zeta} + \kappa,\end{aligned} \tag{49}$$

where

$$\begin{aligned}\xi &= \min\left\{2g_j\left(k_j - \frac{1}{2}\right), 2g_nk_n, \varpi_j\gamma_{\theta_j}, \varpi_n\gamma_{\theta_n}, \mu_j\gamma_{b_j}, \mu_n\gamma_{b_n}, \dots, \nu_j\gamma_{\overline{g}_j}, \nu_n\gamma_{\overline{g}_n}, \frac{1}{\tau_i} - \frac{3}{2}\right\}, \\ &\qquad j = 1,\dots,n-1, \\ \kappa &= 0.2785\omega\overline{g}_{g_n} + \sum_{j=1}^{n}\left(0.2785\omega b_{g_j} + \frac{\varpi_j}{2}\theta_{g_j}^2 + \frac{\mu_i}{2}b_{g_j}^2 + \frac{\nu_i}{2}\overline{g}_{g_j}^2\right) + \sum_{i=1}^{n-1}\frac{1}{2}M_i^2.\end{aligned} \tag{50}$$

Multiplying both sides of (49) by $e^{\xi t}$ yields

$$\frac{d}{dt}\left(Ve^{\xi t}\right) \le \kappa e^{\xi t} + \left[g_n \dot{\lambda}(\vartheta) N(\zeta) + 1\right]\dot{\zeta} e^{\xi t}. \tag{51}$$

Integrating (51) over $[0, t]$, it is deduced that

$$\begin{aligned} V &\le \frac{\kappa}{\xi} + \left[V(0) - \frac{\kappa}{\xi}\right] e^{\xi t} \\ &\quad + e^{-\xi t}\int_0^t \left[g_n \dot{\lambda}(\vartheta) N(\zeta) + 1\right]\dot{\zeta} e^{\xi \varrho} d\varrho \\ &\le \frac{\kappa}{\xi} + V(0) + e^{-\xi t}\int_0^t \left[g_n \dot{\lambda}(\vartheta) N(\zeta) + 1\right]\dot{\zeta} e^{\xi \varrho} d\varrho, \end{aligned} \tag{52}$$

where $c_1 = (\kappa/\xi) + V(0)$.

From Condition 1 of Duhem model and Assumption 7, it is easily concluded that $\dot{\lambda}(u) > 0$, and $\dot{\lambda}(u)$ is a bounded smooth even function of u, and hence $g_n\dot{\lambda}(\vartheta)$ is a nonzero bounded time-varying function. By using Lemma 8, it implies that $V(t)$, ζ, $\int_0^t [g_n\dot{\lambda}(\vartheta)N(\zeta) + 1]\dot{\zeta}e^{\xi\varrho}d\varrho$ are all bounded on $[0, t_f)$. From proposition 2 in [31], $t_f = \infty$, it can be concluded that all error signals $z_1, \dots, z_n, \tilde{\theta}_{g_1}, \dots, \tilde{\theta}_{g_n}$, $\tilde{b}_{g_1}, \dots, \tilde{b}_{g_n}, \tilde{\bar{g}}_{g_1}, \dots, \tilde{\bar{g}}_{g_n}, e_i, \dots, e_{n-1}$ in the closed-loop system are semiglobally uniformly ultimately bounded. □

5. Numerical Example

To demonstrate the effectiveness of the proposed control algorithm, in this section, one second-order nonlinear system with unknown Duhem hysteresis is considered:

$$\begin{aligned} \dot{x}_1 &= \theta_1 f_1(\bar{x}_1) + g_1 x_2 + d_1(x, t), \\ \dot{x}_2 &= \theta_2 f_2(x) + g_2 w(u) + d_2(x, t), \\ y &= x_1, \end{aligned} \tag{53}$$

where θ_1, θ_2, g_1, and g_2 are unknown parameters, $d_1(x,t)$ and $d_2(x,t)$ are unknown disturbances, and w is the output of the unknown hysteresis described by the Duhem model as (3). In the simulation, $\theta_1 = 0.1$, $\theta_2 = 1$, $g_1 = 1$, $g_2 = 1$, $f_1(x_1) = x_1^2$, $f_2(x) = -2x_1 - x_2$, $d_1(x,t) = 0.2\sin(x_2)\sin(t)$, and $d_2(x,t) = 0.1(x_1^2 + x_2^2)\sin(t)$. Correspondingly, $b_1 = 0.2$, $\rho_1(x_1) = 1$, $b_2 = 0.1$, and $\rho_2(x) = x_1^2 + x_2^2$. For the Duhem model, $\alpha = 1$, $\lambda(u) = 5\tanh 1.3u + 0.25u$, and $\psi(u) = \dot{\lambda}(u)(1 - e^{(-2.3|u|)})$. The objective is to make the output y of system (53) to track the desired trajectory $y_d = \sin(1.5t) - 0.3\cos(t)$.

In this simulation, the initial values of adaptive laws are selected as $\hat{\theta}_{g_1} = 0$, $\hat{\theta}_{g_2} = 0$, $\hat{b}_{g_1} = 0$, $\hat{b}_{g_2} = 0$, $\hat{\bar{g}}_{g_1} = 0$, and $\hat{\bar{g}}_{g_2} = 0$. In addition, the design parameters are chosen as $k_1 = 2$, $k_2 = 1$, $\gamma_{\theta_1} = \gamma_{\theta_2} = 2$, $\gamma_{b_1} = \gamma_{b_2} = 3$, $\gamma_{\bar{g}_1} = \gamma_{\bar{g}_2} = 4$, $\varpi_1 = \varpi_2 = \mu_1 = \mu_2 = \nu_1 = \nu_2 = 0.0001$, $\omega = 0.01$, and $\tau_1 = 0.1$. The Nussbaum function is chosen as $N(\zeta) = \zeta^2\cos(\zeta)$ with $\zeta(0) = 0$. The initial states of (53) are chosen as $x_1(0) = 0$, $x_2(0) = 0.5$.

The simulation results are shown in Figures 4, 5, 6, 7, 8, and 9. From Figure 4, it is observed that the good tracking performance is achieved under the proposed approach.

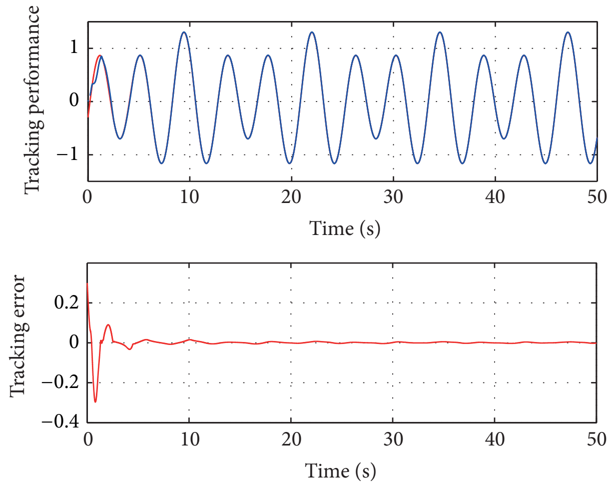

FIGURE 4: Tracking performance of the closed-loop system.

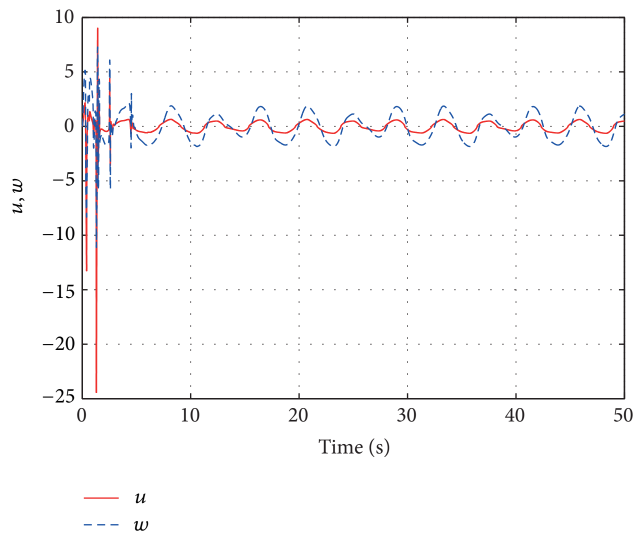

FIGURE 5: Control input u (real line) and hysteresis output w (dashed line).

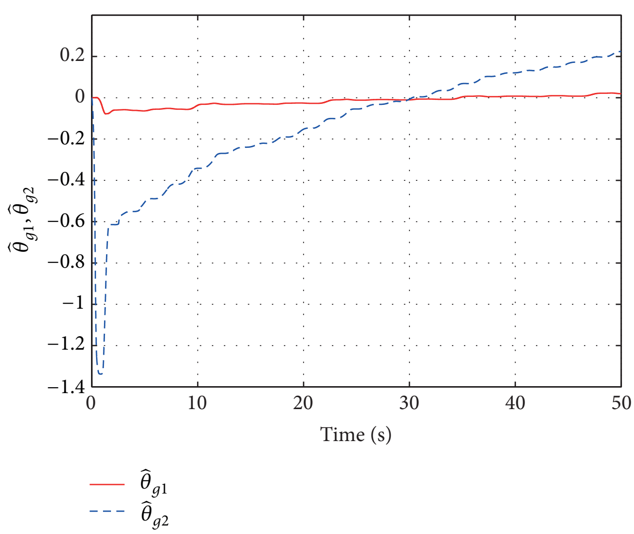

FIGURE 6: $\hat{\theta}_{g_1}$ (real line) and $\hat{\theta}_{g_2}$ (dashed line).

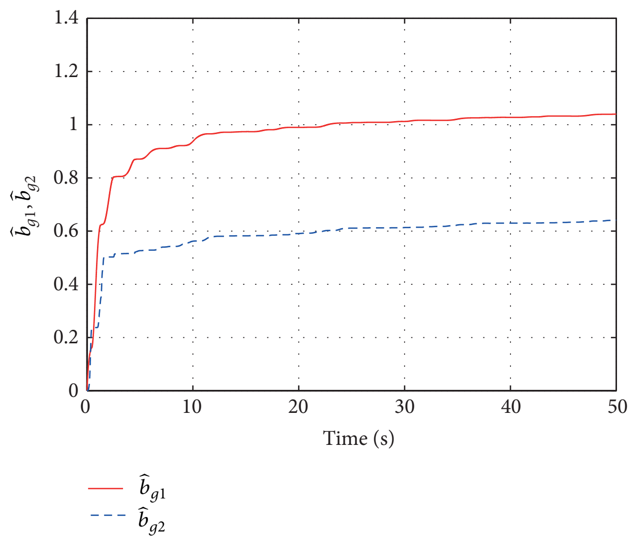

FIGURE 7: $\hat{b}_{g_1}$ (real line) and $\hat{b}_{g_2}$ (dashed line).

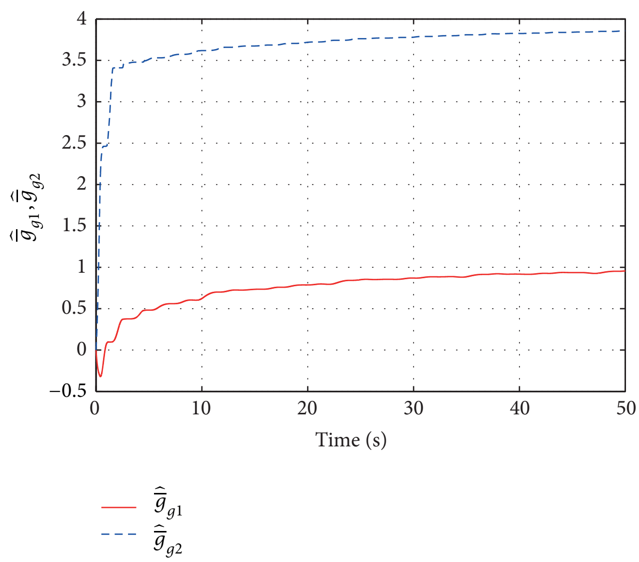

FIGURE 8: $\hat{\bar{g}}_{g_1}$ (real line) and $\hat{\bar{g}}_{g_2}$ (dashed line).

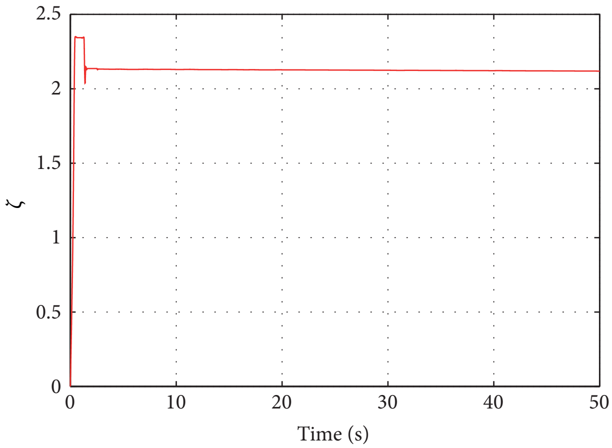

FIGURE 9: Variable ζ.

Figure 5 shows the control input u and the hysteresis output w. Figures 6, 7, 8, and 9 show the response curves of adaptive parameters $\hat{\theta}_{g_1}, \hat{\theta}_{g_2}, \hat{b}_{g_1}, \hat{b}_{g_2}, \hat{\bar{g}}_{g_1}, \hat{\bar{g}}_{g_2}$, and ζ. From these results, the proposed scheme can mitigate the detrimental effects of the unknown hysteresis and guarantee the boundedness of the closed-loop system.

6. Conclusion

In this paper, the adaptive DSC approach for a class of uncertain nonlinear systems in strict-feedback form with unknown Duhem hysteresis is discussed. How to utilize the properties of the hysteresis model and design the related control approach is the main task for this topic. To overcome the design difficulties of Duhem model, three conditions are used to get the analytical output expression of Duhem model. By using DSC technique, the "explosion complexity" in the standard backstepping design procedure is improved. For the last recursive step arising from the unknown hysteresis, the nonlinear smooth term of Duhem model is considered in the robust controller design by using mean value theorem and Nussbaum function lemma. Under the proposed approach, all the signals in the closed-loop system are uniformly semiglobally bounded, and a numerical example is shown to verify the effectiveness.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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