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Research Article

Exponential Stability of BAM Fuzzy Cellular Neural Networks with Time-Varying Delays in Leakage Terms and Impulses

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BAM fuzzy cellular neural networks with time-varying delays in leakage terms and impulses are considered. Some sufficient conditions for the exponential stability of the networks are established by using differential inequality techniques. The results of this paper are completely new and complementary to the previously known results. Finally, an example is given to demonstrate the effectiveness and conservativeness of our theoretical results.

1. Introduction

The bidirectional associative memory (BAM) neural networks were first introduced by Kosko [1–3]. It is a special class of recurrent neural networks that can store bipolar rector pairs. The BAM neural networks are composed of neurons arranged in two layers, the X-layer and Y-layer. Recently, many researchers have studied the dynamics of BAM neural networks with or without delays [4–15]. However, in mathematical modeling of real world problems, uncertainty or vagueness is unavoidable. In order to take vagueness into consideration, fuzzy theory is considered as a suitable method. In [16, 17], the authors first combined those operations with cellular neural networks (FCNNs). Some results have been reported on stability and periodicity of FCNNs. More recently, state estimation problem for the fuzzy BAM neural networks has been obtained in the paper [18, 19] and passivity criteria for the fuzzy BAM neural networks have been studied in the papers [20, 21].

Very recently, a leakage delay, which is the time delay in leakage term of the systems and a considerable factor affecting dynamics for the worse in the systems, is being put to use in the problem of stability for neural networks [22, 23]. However, so far, very little attention has been paid to neural networks with time delay in the leakage (or "forgetting") term [24–30]. Such time delays in leakage terms are difficult to handle but have great impact on the dynamical behavior of neural networks.

In [31], the authors studied the following BAM fuzzy cellular network with time delay in leakage terms and discrete and unbounded distributed delays:

$$\begin{aligned} x_i'(t) &= -d_i x_i(t-\sigma_1) + \sum_{j=1}^{m} a_{ij}\tilde{f}_j(y_j(t)) \\ &\quad + \sum_{j=1}^{m} b_{ij}\tilde{f}_j(y_j(t-\tau(t))) + \sum_{j=1}^{m} c_{ij}\omega_j \\ &\quad + \bigwedge_{j=1}^{m}\alpha_{ij}\int_{-\infty}^{t} k_j(t-s)\tilde{f}_j(y_j(s))\,ds \\ &\quad + \bigvee_{j=1}^{m}\beta_{ij}\int_{-\infty}^{t} k_j(t-s)\tilde{f}_j(y_j(s))\,ds \\ &\quad + \bigwedge_{j=1}^{m} T_{ij}\omega_j + \bigvee_{j=1}^{m} H_{ij}\omega_j + I_i, \\ &\qquad t \geq 0, \quad i = 1, 2, \ldots, n, \\ y_j'(t) &= -\tilde{d}_j y_j(t-\sigma_2) + \sum_{i=1}^{n}\tilde{a}_{ji}\tilde{g}_i(x_i(t)) \\ &\quad + \sum_{i=1}^{n}\tilde{b}_{ji}\tilde{g}_i(x_i(t-\rho(t))) + \sum_{i=1}^{n}\tilde{c}_{ji}\tilde{\omega}_i \end{aligned}$$

$$+\bigwedge_{i=1}^{n}\widetilde{\alpha}_{ji}\int_{-\infty}^{t}k_i(t-s)\,\widetilde{g}_i(x_i(s))\,ds$$
$$+\bigvee_{i=n}^{m}\widetilde{\beta}_{ji}\int_{-\infty}^{t}k_i(t-s)\,\widetilde{g}_i(x_i(s))\,ds$$
$$+\bigwedge_{i=1}^{n}\widetilde{T}_{ji}\widetilde{\omega}_i+\bigvee_{i=1}^{n}\widetilde{H}_{ji}\widetilde{\omega}_i+J_j,$$
$$t\ge 0,\quad j=1,2,\ldots,m. \tag{1}$$

However, time-varying delays in the leakage terms inevitably occur in electronic neural networks owing to the unavoidable finite switching speed of amplifiers. It is desirable to study the fuzzy BAM neural networks with time-varying delays in leakage terms. In [32], by using a fixed point theorem and differential inequality techniques, the authors studied the existence and exponential stability of equilibrium point for the following BAM neural network with time-varying delays in leakage terms on time scales:

$$\begin{aligned}
x_i^{\Delta}(t)&=-a_ix_i(t-\delta_i(t))+\sum_{j=1}^{m}c_{ji}f_j\left(y_j\left(t-\tau_{ji}(t)\right)\right)\\
&\quad+\bigwedge_{j=1}^{m}\alpha_{ji}f_j\left(y_j\left(t-\tau_{ji}(t)\right)\right)\\
&\quad+\bigwedge_{j=1}^{m}T_{ji}\mu_j+\bigvee_{j=1}^{m}\beta_{ji}f_j\left(y_j\left(t-\tau_{ji}(t)\right)\right)\\
&\quad+\bigvee_{j=1}^{m}H_{ji}\mu_j+I_i,\quad t\in\mathbb{T},\ i=1,2,\ldots,n,\\
y_j^{\Delta}(t)&=-b_jy_j\left(t-\eta_j(t)\right)+\sum_{i=1}^{n}d_{ij}g_i\left(x_i\left(t-\sigma_{ij}(t)\right)\right)\\
&\quad+\bigwedge_{i=1}^{n}p_{ij}g_i\left(x_i\left(t-\sigma_{ij}(t)\right)\right)\\
&\quad+\bigwedge_{i=1}^{n}F_{ij}\nu_i+\bigvee_{i=1}^{n}q_{ij}g_i\left(x_i\left(t-\sigma_{ij}(t)\right)\right)\\
&\quad+\bigvee_{i=1}^{n}G_{ij}\nu_i+J_j,\quad t\in\mathbb{T},\ j=1,2,\ldots,m,
\end{aligned}\tag{2}$$

where $\mathbb{T}$ is a time scale. Though the nonimpulsive systems have been well studied in theory and in practice, the theory of impulsive differential equations is now being recognized to be richer than the corresponding theory of differential equations without impulses (see [33–35]). What is more, very few results are available on exponential stability of equilibrium point for fuzzy BAM neural networks with time-varying delays in leakage terms and impulses.

Motivated by the above discussion, in this paper, we consider the following model:

$$\begin{aligned}
x_i'(t)&=-a_i(t)\,x_i(t-\alpha_i(t))+\sum_{j=1}^{m}a_{ij}(t)\,f_j\left(y_j(t)\right)\\
&\quad+\sum_{j=1}^{m}b_{ij}(t)\,f_j\left(y_j(t-\tau(t))\right)+\sum_{j=1}^{m}c_{ij}(t)\,\omega_j\\
&\quad+\bigwedge_{j=1}^{m}\alpha_{ij}(t)\int_{-\infty}^{t}k_j(t-s)\,f_j\left(y_j(s)\right)ds\\
&\quad+\bigvee_{j=1}^{m}\beta_{ij}(t)\int_{-\infty}^{t}k_j(t-s)\,f_j\left(y_j(s)\right)ds\\
&\quad+\bigwedge_{j=1}^{m}T_{ij}\omega_j+\bigvee_{j=1}^{m}H_{ij}\omega_j+A_i(t),\\
&\qquad t\ge 0,\quad t\ne t_k,\quad i=1,2,\ldots,n,\\
\Delta x_i(t_k)&=I_k\left(x_i(t_k)\right),\quad i=1,2,\ldots,n,\ k=1,2,\ldots,\\
y_j'(t)&=-b_j(t)\,y_j\left(t-\beta_j(t)\right)+\sum_{i=1}^{n}d_{ji}(t)\,g_i(x_i(t))\\
&\quad+\sum_{i=1}^{n}p_{ji}(t)\,g_i(x_i(t-\rho(t)))+\sum_{i=1}^{n}q_{ji}(t)\,\mu_i\\
&\quad+\bigwedge_{i=1}^{n}\gamma_{ji}(t)\int_{-\infty}^{t}k_i(t-s)\,g_i(x_i(s))\,ds\\
&\quad+\bigvee_{i=1}^{n}\eta_{ji}(t)\int_{-\infty}^{t}k_i(t-s)\,g_i(x_i(s))\,ds\\
&\quad+\bigwedge_{i=1}^{n}R_{ji}\mu_i+\bigvee_{i=1}^{n}S_{ji}\mu_i+B_j(t),\\
&\qquad t\ge 0,\quad t\ne t_k,\quad j=1,2,\ldots,m,\\
\Delta y_j(t_k)&=J_k\left(y_j(t_k)\right),\quad j=1,2,\ldots,m,\ k=1,2,\ldots,
\end{aligned}\tag{3}$$

where $x_i(t)$ and $y_j(t)$ are the states of the ith neuron and the jth neuron at time t, $g_i(t)$ and $f_j(t)$ denote the activation functions of the ith neuron and the jth neuron at time t, μ_i and ω_j denote the inputs of the ith neuron and the jth neuron, $A_i(t)$ and $B_j(t)$ denote the bias of the ith neuron and the jth neuron at time t, $a_i(t)$ and $b_j(t)$ represent the rates with which the ith neuron and the jth neuron at time t will reset their potential to the resting state in isolation when disconnected from the networks and external inputs, $a_{ij}(t)$, $b_{ij}(t)$, $d_{ji}(t)$, and $p_{ji}(t)$ denote the connection weights of the feedback template at time t and $c_{ij}(t)$, $q_{ji}(t)$ denote the connection weights of the feedforward template at time t, $\alpha_{ij}(t)$, $\gamma_{ji}(t)$ and $\beta_{ij}(t)$, $\eta_{ji}(t)$ denote the connection weights of the delays fuzzy feedback MIN template at time t and the delays fuzzy feedback MAX template at time t, T_{ij}, R_{ji} and H_{ij}, S_{ji} are the elements of the fuzzy feedforward MIN template and fuzzy feedforward MAX template, $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operators, $0 \le \tau(t) \le \tau$ and $0 \le \rho(t) \le \rho$ denote the transmission delays at time t, $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k)$, $\Delta y_j(t_k) = y_j(t_k^+) - y_j(t_k)$ are the impulses at moments t_k, and $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k\to\infty} t_k = +\infty$, $k_j(s) \ge 0$, and $k_i(s) \ge 0$ are the feedback kernels and satisfy $\int_0^{+\infty} k_j(s)ds = 1$, $\int_0^{+\infty} k_i(s)ds = 1$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

The initial conditions are given by

$$\begin{aligned} x_i(s) &= \varphi_i(s), \quad s \in (-\infty, 0], \\ y_j(s) &= \psi_j(s), \quad s \in (-\infty, 0], \end{aligned} \tag{4}$$

where $\varphi_i, \psi_j \in C((-\infty, 0], R)$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

For convenience, for a continuous function $f : R \to R$, we denote $f^+ = \sup_{t\in R} |f(t)|$, $f^- = \inf_{t\in R} |f(t)|$.

Throughout this paper, we make the following assumptions.

(H_1) The neuron activation functions $f_j(t)$ and $g_i(t)$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, are continuous on R and there exist some real constants L_j, M_i such that

$$\begin{aligned} \left|f_j(u) - f_j(v)\right| &\le L_j |u - v|, \\ \left|g_i(u) - g_i(v)\right| &\le M_i |u - v| \end{aligned} \tag{5}$$

for all $u, v \in R$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

(H_2) The leakage delays satisfy $\alpha_i(t) \ge 0$, $\beta_j(t) \ge 0$ and $a_i(t) > 0$, $b_j(t) > 0$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

Definition 1. Let $z^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t), y_1^*(t), y_2^*(t), \ldots, y_m^*(t))^T$ be a solution of system (3) with the initial condition $\phi^*(t) = (\varphi_1^*(t), \varphi_2^*(t), \ldots, \varphi_n^*(t), \psi_1^*(t), \psi_2^*(t), \ldots, \psi_m^*(t))^T$ and let $z(t) = (x_1(t), x_2(t), \ldots, x_n(t), y_1(t), y_2(t), \ldots, y_m(t))^T$ be any solution of system (3) with the initial condition $\phi(t) = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t), \psi_1(t), \psi_2(t), \ldots, \psi_m(t))^T$; if these two solutions satisfy

$$x_i(t) - x_i^*(t) = O\left(e^{-\lambda t}\right), \qquad y_j(t) - y_j^*(t) = O\left(e^{-\lambda t}\right), \qquad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m, \tag{6}$$

then, we say that system (3) is exponentially stable.

Lemma 2 (see [36]). *Let z, z' be two states of system (3); for $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, one has*

$$\begin{aligned} &\left|\bigwedge_{j=1}^{m}\alpha_{ij}(t) f_j(z) - \bigwedge_{j=1}^{m}\alpha_{ij}(t) f_j\left(z'\right)\right| \le \sum_{j=1}^{m}\left|\alpha_{ij}(t)\right|\left|f_j(z) - f_j\left(z'\right)\right|, \\ &\left|\bigvee_{j=1}^{m}\beta_{ij}(t) f_j(z) - \bigvee_{j=1}^{m}\beta_{ij}(t) f_j\left(z'\right)\right| \le \sum_{j=1}^{m}\left|\beta_{ij}(t)\right|\left|f_j(z) - f_j\left(z'\right)\right|, \\ &\left|\bigwedge_{i=1}^{n}\gamma_{ji}(t) g_i(z) - \bigwedge_{i=1}^{n}\gamma_{ji}(t) g_i\left(z'\right)\right| \le \sum_{i=1}^{n}\left|\gamma_{ji}(t)\right|\left|g_i(z) - g_i\left(z'\right)\right|, \\ &\left|\bigvee_{j=1}^{n}\eta_{ji}(t) g_i(z) - \bigvee_{i=1}^{n}\eta_{ji}(t) g_i\left(z'\right)\right| \le \sum_{i=1}^{n}\left|\eta_{ji}(t)\right|\left|g_i(z) - g_i\left(z'\right)\right|. \end{aligned} \tag{7}$$

Our main purpose of this paper is by using differential inequality techniques to study the exponential stability of (3). The results of this paper are completely new and complementary to the previously known results and the methods used in this paper are different from those used in [31, 32].

2. Exponential Stability

In this section, we will give some sufficient conditions to guarantee the exponential stability of system (3).

Theorem 3. *Suppose that (H_1) and (H_2) hold. Let $z^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t), y_1^*(t), y_2^*(t), \ldots, y_m^*(t))^T$ be a solution of system (3) with the initial condition $\phi^*(t) = (\varphi_1^*(t), \varphi_2^*(t), \ldots, \varphi_n^*(t), \psi_1^*(t), \psi_2^*(t), \ldots, \psi_m^*(t))^T$. Furthermore, assume that*

(H_3)

$$- \left[a_i(t) - a_i(t)\alpha_i(t) a_i^+\right] + \left(1 + a_i(t)\alpha_i(t)\right) \times \sum_{j=1}^{m}\left(a_{ij}^+ + b_{ij}^+ + \alpha_{ij}^+ + \beta_{ij}^+\right) L_j < 0, \qquad i = 1, 2, \ldots, n,$$

$$-\left[b_j(t)-b_j(t)\beta_j(t)b_j^+\right]+\left(1+b_j(t)\beta_j(t)\right)\times\sum_{i=1}^{n}\left(d_{ji}^++p_{ji}^++\gamma_{ji}^++\eta_{ji}^+\right)M_i<0,$$
$$j=1,2,\ldots,m. \quad (8)$$

(H$_4$) *There exist constants θ_{ik}, $\overline{\theta}_{jk}$ such that*

$$\begin{aligned}I_k(x_i(t_k))&=-\theta_{ik}x_i(t_k),\\ 0\le\theta_{ik}\le 2,\quad i&=1,2,\ldots,n,\quad k=1,2,\ldots,\\ J_k\left(y_j(t_k)\right)&=-\overline{\theta}_{jk}y_j(t_k),\\ 0\le\overline{\theta}_{jk}\le 2,\quad j&=1,2,\ldots,m,\quad k=1,2,\ldots.\end{aligned}\quad (9)$$

Then system (3) *is exponentially stable.*

Proof. Let $z(t)=(x_1(t),x_2(t),\ldots,x_n(t),y_1(t),y_2(t),\ldots,y_m(t))^T$ be an arbitrary solution of system (3) with the initial condition $\phi(t)=(\varphi_1(t),\varphi_2(t),\ldots,\varphi_n(t),\psi_1(t),\psi_2(t),\ldots,\psi_m(t))^T$. Set

$$\begin{aligned}\overline{x}_i(t)&=x_i(t)-x_i^*(t),\quad i=1,2,\ldots,n,\\ \overline{y}_j(t)&=y_j(t)-y_j^*(t),\quad j=1,2,\ldots,m,\end{aligned}\quad (10)$$

$$\begin{aligned}\widetilde{f}_j\left(\overline{y}_j(t)\right)&=f_j\left(\overline{y}_j(t)+y_j^*(t)\right)-f_j\left(y_j^*(t)\right),\\ &\qquad j=1,2,\ldots,m,\\ \widetilde{g}_i\left(\overline{x}_i(t)\right)&=g_i\left(\overline{x}_i(t)+x_i^*(t)\right)-g_i\left(x_i^*(t)\right),\\ &\qquad i=1,2,\ldots,n.\end{aligned}\quad (11)$$

From (3) and (11), for $t>0$, $t\neq t_k$, $k=1,2,\ldots$, we have

$$\begin{aligned}\overline{x}_i'(t)=&-a_i(t)\overline{x}_i(t-\alpha_i(t))+\sum_{j=1}^{m}a_{ij}(t)\widetilde{f}_j\left(\overline{y}_j(t)\right)\\ &+\sum_{j=1}^{m}b_{ij}(t)\widetilde{f}_j\left(\overline{y}_j(t-\tau(t))\right)\\ &+\bigwedge_{j=1}^{m}\alpha_{ij}(t)\int_{-\infty}^{t}k_j(t-s)\widetilde{f}_j\left(\overline{y}_j(s)\right)ds\\ &+\bigvee_{j=1}^{m}\beta_{ij}(t)\int_{-\infty}^{t}k_j(t-s)\widetilde{f}_j\left(\overline{y}_j(s)\right)ds,\\ &\qquad i=1,2,\ldots,n,\end{aligned}\quad (12)$$

$$\begin{aligned}\overline{y}_j'(t)=&-b_j(t)\overline{y}_j\left(t-\beta_j(t)\right)+\sum_{i=1}^{n}d_{ji}(t)\widetilde{g}_i\left(\overline{x}_i(t)\right)\\ &+\sum_{i=1}^{n}p_{ji}(t)\widetilde{g}_i\left(\overline{x}_i(t-\rho(t))\right)\\ &+\bigwedge_{i=1}^{n}\gamma_{ji}(t)\int_{-\infty}^{t}k_i(t-s)\widetilde{g}_i\left(\overline{x}_i(s)\right)ds\\ &+\bigvee_{i=1}^{n}\eta_{ji}(t)\int_{-\infty}^{t}k_i(t-s)\widetilde{g}_i\left(\overline{x}_i(s)\right)ds,\\ &\qquad j=1,2,\ldots,m.\end{aligned}\quad (13)$$

According to (H$_4$), we get

$$\begin{aligned}x_i&(t_k^+)-x_i^*(t_k^+)\\ &=x_i(t_k)+I_k(x_i(t_k))-x_i^*(t_k)-I_k(x_i^*(t_k))\\ &=(1-\theta_{ik})(x_i(t_k)-x_i^*(t_k)),\\ &\quad i=1,2,\ldots,n,\quad k=1,2,\ldots,\\ y_j&(t_k^+)-y_j^*(t_k^+)\\ &=y_j(t_k)+J_k\left(y_j(t_k)\right)-y_j^*(t_k)-J_k\left(y_j^*(t_k)\right)\\ &=\left(1-\overline{\theta}_{jk}\right)\left(y_j(t_k)-y_j^*(t_k)\right),\\ &\quad j=1,2,\ldots,m,\quad k=1,2,\ldots.\end{aligned}\quad (14)$$

So,

$$\begin{aligned}&\left|x_i(t_k^+)-x_i^*(t_k^+)\right|\\ &=\left|1-\theta_{ik}\right|\left|x_i(t_k)-x_i^*(t_k)\right|\le\left|x_i(t_k)-x_i^*(t_k)\right|,\\ &\qquad i=1,2,\ldots,n,\quad k=1,2,\ldots,\\ &\left|y_j(t_k^+)-y_j^*(t_k^+)\right|\\ &\quad=\left|1-\overline{\theta}_{jk}\right|\left|y_j(t_k)-y_j^*(t_k)\right|\le\left|y_j(t_k)-y_j^*(t_k)\right|,\\ &\qquad j=1,2,\ldots,m,\quad k=1,2,\ldots,\end{aligned}\quad (15)$$

which implies that

$$\begin{aligned} \left|\overline{x}_i\left(t_k^+\right)\right| &= \left|x_i\left(t_k^+\right) - x_i^*\left(t_k^+\right)\right| \\ &\le \left|x_i\left(t_k\right) - x_i^*\left(t_k\right)\right| = \left|\overline{x}_i\left(t_k^-\right)\right|, \\ & i = 1,2,\ldots,n, \quad k = 1,2,\ldots, \\ \left|\overline{y}_j\left(t_k^+\right)\right| &= \left|y_j\left(t_k^+\right) - y_j^*\left(t_k^+\right)\right| \\ &\le \left|y_j\left(t_k\right) - y_j^*\left(t_k\right)\right| = \left|\overline{y}_j\left(t_k^-\right)\right|, \\ & j = 1,2,\ldots,m, \quad k = 1,2,\ldots. \end{aligned} \tag{16}$$

Define continuous functions $\Gamma_i(\omega)$ and $\Gamma_j(\omega)$ by setting

$$\begin{aligned} &\Gamma_i(\omega) \\ &= -\left[a_i(t)e^{\omega\alpha_i(t)} - \omega - a_i(t)e^{\omega\alpha_i(t)}\alpha_i(t) \right. \\ &\quad \left. \times\left(\omega + a_i^+ e^{\omega\alpha_i^+}\right)\right] + \left[1 + a_i(t)e^{\omega\alpha_i(t)}\alpha_i(t)\right] \\ &\quad \times \sum_{j=1}^{m}\left[a_{ij}^+ + b_{ij}^+ e^{\omega\tau} + \left(\alpha_{ij}^+ + \beta_{ij}^+\right)\int_0^{+\infty} k_j(u)e^{\omega u}du\right] L_j, \\ &\qquad i = 1,2,\ldots,n, \\ &\Gamma_j(\omega) \\ &= -\left[b_j(t)e^{\omega\beta_j(t)} - \omega - b_j(t)e^{\omega\beta_j(t)}\beta_j(t) \right. \\ &\quad \left. \times\left(\omega + b_j^+ e^{\omega\beta_j^+}\right)\right] + \left[1 + b_j(t)e^{\omega\beta_j(t)}\beta_j(t)\right] \\ &\quad \times \sum_{i=1}^{n}\left[d_{ji}^+ + p_{ji}^+ e^{\omega\rho} + \left(\gamma_{ji}^+ + \eta_{ji}^+\right)\int_0^{+\infty} k_i(u)e^{\omega u}du\right] M_i, \\ &\qquad j = 1,2,\ldots,m. \end{aligned} \tag{17}$$

Then, for $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$, we have

$$\begin{aligned} \Gamma_i(0) &= -\left[a_i(t) - a_i(t)\alpha_i(t)a_i^+\right] + \left(1 + a_i(t)\alpha_i(t)\right) \\ &\quad \times \sum_{j=1}^{m}\left(a_{ij}^+ + b_{ij}^+ + \alpha_{ij}^+ + \beta_{ij}^+\right)L_j < 0, \\ \Gamma_j(0) &= -\left[b_j(t) - b_j(t)\beta_j(t)b_j^+\right] + \left(1 + b_j(t)\beta_j(t)\right) \\ &\quad \times \sum_{i=1}^{n}\left(d_{ji}^+ + p_{ji}^+ + \gamma_{ji}^+ + \eta_{ji}^+\right)M_i < 0. \end{aligned} \tag{18}$$

The continuity of $\Gamma_i(\omega)$ and $\Gamma_j(\omega)$ implies that there exists $\lambda > 0$ such that

$$\begin{aligned} \Gamma_i(\lambda) &= -\left[a_i(t)e^{\lambda\alpha_i(t)} - \lambda - a_i(t)e^{\lambda\alpha_i(t)}\alpha_i(t) \right. \\ &\quad \left. \times\left(\lambda + a_i^+ e^{\lambda\alpha_i^+}\right)\right] + \left[1 + a_i(t)e^{\lambda\alpha_i(t)}\alpha_i(t)\right] \\ &\quad \times \sum_{j=1}^{m}\left[a_{ij}^+ + b_{ij}^+ e^{\lambda\tau} + \left(\alpha_{ij}^+ + \beta_{ij}^+\right)\int_0^{+\infty} k_j(u)e^{\lambda u}du\right] \\ &\quad \times L_j < 0, \quad i = 1,2,\ldots,n, \\ \Gamma_j(\lambda) &= -\left[b_j(t)e^{\lambda\beta_j(t)} - \lambda - b_j(t)e^{\lambda\beta_j(t)}\beta_j(t) \right. \\ &\quad \left. \times\left(\lambda + b_j^+ e^{\lambda\beta_j^+}\right)\right] + \left[1 + b_j(t)e^{\lambda\beta_j(t)}\beta_j(t)\right] \\ &\quad \times \sum_{i=1}^{n}\left[d_{ji}^+ + p_{ji}^+ e^{\lambda\rho} + \left(\gamma_{ji}^+ + \eta_{ji}^+\right)\int_0^{+\infty} k_i(u)e^{\lambda u}du\right] \\ &\quad \times M_i < 0, \quad j = 1,2,\ldots,m. \end{aligned} \tag{19}$$

Let

$$\begin{gathered} X_i(t) = \overline{x}_i(t)e^{\lambda t}, \qquad Y_j(t) = \overline{y}_j(t)e^{\lambda t}, \\ i = 1,2,\ldots,n, \quad j = 1,2,\ldots,m. \end{gathered} \tag{20}$$

From (12) and (20), we obtain

$$\begin{aligned} X_i'(t) &= \lambda e^{\lambda t}\overline{x}_i(t) + e^{\lambda t}\overline{x}_i'(t) \\ &= \lambda X_i(t) + e^{\lambda t} \\ &\quad \times \left[-a_i(t)\overline{x}_i\left(t - \alpha_i(t)\right) + \sum_{j=1}^{m} a_{ij}(t)\widetilde{f}_j\left(\overline{y}_j(t)\right) \right. \\ &\quad + \sum_{j=1}^{m} b_{ij}(t)\widetilde{f}_j\left(\overline{y}_j\left(t - \tau(t)\right)\right) \\ &\quad + \bigwedge_{j=1}^{m}\alpha_{ij}(t)\int_{-\infty}^{t} k_j(t-s)\widetilde{f}_j\left(\overline{y}_j(s)\right)ds \\ &\quad \left. + \bigvee_{j=1}^{m}\beta_{ij}(t)\int_{-\infty}^{t} k_j(t-s)\widetilde{f}_j\left(\overline{y}_j(s)\right)ds\right] \end{aligned}$$

$$
\begin{aligned}
&= \lambda X_i(t) - a_i(t) e^{\lambda\alpha_i(t)} X_i(t-\alpha_i(t)) \\
&\quad + \sum_{j=1}^{m} a_{ij}(t) e^{\lambda t} \tilde{f}_j(\overline{y}_j(t)) \\
&\quad + \sum_{j=1}^{m} b_{ij}(t) e^{\lambda t} \tilde{f}_j(\overline{y}_j(t-\tau(t))) \\
&\quad + \bigwedge_{j=1}^{m} \alpha_{ij}(t) e^{\lambda t} \int_{-\infty}^{t} k_j(t-s) \tilde{f}_j(\overline{y}_j(s))\, ds \\
&\quad + \bigvee_{j=1}^{m} \beta_{ij}(t) e^{\lambda t} \int_{-\infty}^{t} k_j(t-s) \tilde{f}_j(\overline{y}_j(s))\, ds, \\
&\qquad i = 1, 2, \ldots, n.
\end{aligned} \tag{21}
$$

Similarly,

$$
\begin{aligned}
Y_j'(t) &= \lambda Y_j(t) - b_j(t) e^{\lambda\beta_j(t)} Y_j(t-\beta_j(t)) \\
&\quad + \sum_{i=1}^{n} d_{ji}(t) e^{\lambda t} \tilde{g}_i(\overline{x}_i(t)) \\
&\quad + \sum_{i=1}^{n} p_{ji}(t) e^{\lambda t} \tilde{g}_i(\overline{x}_i(t-\rho(t))) \\
&\quad + \bigwedge_{i=1}^{n} \gamma_{ji}(t) e^{\lambda t} \int_{-\infty}^{t} k_i(t-s) \tilde{g}_i(\overline{x}_i(s))\, ds \\
&\quad + \bigvee_{i=1}^{n} \eta_{ji}(t) e^{\lambda t} \int_{-\infty}^{t} k_i(t-s) \tilde{g}_i(\overline{x}_i(s))\, ds, \\
&\qquad j = 1, 2, \ldots, m.
\end{aligned} \tag{22}
$$

We rewrite (21) and (22) as follows:

$$
\begin{aligned}
&X_i'(t) \\
&= \lambda X_i(t) - a_i(t) e^{\lambda\alpha_i(t)} X_i(t) + a_i(t) e^{\lambda\alpha_i(t)} \\
&\quad \times \int_{t-\alpha_i(t)}^{t} \Big\{ \lambda X_{\mathrm{i}}(s) - a_i(s) e^{\lambda\alpha_i(s)} \times X_i(s-\alpha_i(s)) \\
&\qquad + \sum_{j=1}^{m} a_{ij}(s) e^{\lambda s} \tilde{f}_j(\overline{y}_j(s)) \\
&\qquad + \sum_{j=1}^{m} b_{ij}(s) e^{\lambda s} \tilde{f}_j(\overline{y}_j(s-\tau(s))) \\
&\qquad + \bigwedge_{j=1}^{m} \alpha_{ij}(s) e^{\lambda s} \int_{-\infty}^{s} k_j(s-u) \tilde{f}_j(\overline{y}_j(u))\, du \\
&\qquad + \bigvee_{j=1}^{m} \beta_{ij}(s) e^{\lambda s} \int_{-\infty}^{s} k_j(s-u) \tilde{f}_j(\overline{y}_j(u))\, du \Big\}\, ds \\
&\quad + \sum_{j=1}^{m} a_{ij}(t) e^{\lambda t} \tilde{f}_j(\overline{y}_j(t)) + \sum_{j=1}^{m} b_{ij}(t) e^{\lambda t} \tilde{f}_j(\overline{y}_j(t-\tau(t))) \\
&\quad + \bigwedge_{j=1}^{m} \alpha_{ij}(t) e^{\lambda t} \int_{-\infty}^{t} k_j(t-s) \tilde{f}_j(\overline{y}_j(s))\, ds \\
&\quad + \bigvee_{j=1}^{m} \beta_{ij}(t) e^{\lambda t} \int_{-\infty}^{t} k_j(t-s) \tilde{f}_j(\overline{y}_j(s))\, ds, \\
&\qquad i = 1, 2, \ldots, n,
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
&Y_j'(t) \\
&= \lambda Y_j(t) - b_j(t) e^{\lambda\beta_j(t)} Y_j(t) + b_j(t) e^{\lambda\beta_j(t)} \\
&\quad \times \int_{t-\beta_j(t)}^{t} \Big\{ \lambda Y_j(s) - b_j(s) e^{\lambda\beta_j(s)} \\
&\qquad \times Y_j(s-\beta_j(s)) + \sum_{i=1}^{n} d_{ji}(s) e^{\lambda s} \tilde{g}_i(\overline{x}_i(s)) \\
&\qquad + \sum_{i=1}^{n} p_{ji}(s) e^{\lambda s} \tilde{g}_i(\overline{x}_i(s-\rho(s))) \\
&\qquad + \bigwedge_{i=1}^{n} \gamma_{ji}(s) e^{\lambda s} \int_{-\infty}^{s} k_i(s-u) \tilde{g}_i(\overline{x}_i(u))\, du \\
&\qquad + \bigvee_{i=1}^{n} \eta_{ji}(s) e^{\lambda s} \int_{-\infty}^{s} k_i(s-u) \tilde{g}_i(\overline{x}_i(u))\, du \Big\}\, ds \\
&\quad + \sum_{i=1}^{n} d_{ji}(t) e^{\lambda t} \tilde{g}_i(\overline{x}_i(t)) \\
&\quad + \sum_{i=1}^{n} p_{ji}(t) e^{\lambda t} \tilde{g}_i(\overline{x}_i(t-\rho(t))) \\
&\quad + \bigwedge_{i=1}^{n} \gamma_{ji}(t) e^{\lambda t} \int_{-\infty}^{t} k_i(t-s) \tilde{g}_i(\overline{x}_i(s))\, ds \\
&\quad + \bigvee_{i=1}^{n} \eta_{ji}(t) e^{\lambda t} \int_{-\infty}^{t} k_i(t-s) \tilde{g}_i(\overline{x}_i(s))\, ds, \\
&\qquad j = 1, 2, \ldots, m.
\end{aligned} \tag{24}
$$

We define a positive number such that

$$M=\max_{\substack{1\le i\le n,\\ 1\le j\le m}}\left\{\sup_{s\in[-\tau,0]}\left|X_i(s)\right|,\ \sup_{s\in[-\rho,0]}\left|Y_j(s)\right|\right\}. \tag{25}$$

It follows that

$$\begin{aligned}&\left|X_i(t)\right|<M,\quad \forall t\in[-\tau,0],\ i=1,2,\dots,n,\\ &\left|Y_j(t)\right|<M,\quad \forall t\in[-\rho,0],\ j=1,2,\dots,m.\end{aligned} \tag{26}$$

We claim that

$$\begin{gathered}\left|X_i(t)\right|<M,\qquad \left|Y_j(t)\right|<M,\\ \forall t>0,\quad t\neq t_k,\quad i=1,2,\dots,n,\quad j=1,2,\dots,m.\end{gathered} \tag{27}$$

If (27) is not valid, then there exist some $i\in\{1,2,\dots,n\}$, some $j\in\{1,2,\dots,m\}$, and a first $T_1>0$ such that one of the following four cases must occur:

(1) $X_i(T_1)=M$, $X_i'(T_1)\ge 0$, $|X_i(t)|<M$, $|Y_j(t)|<M$, for $t<T_1$;

(2) $X_i(T_1)=-M$, $X_i'(T_1)\le 0$, $|X_i(t)|<M$, $|Y_j(t)|<M$, for $t<T_1$;

(3) $Y_j(T_1)=M$, $Y_j'(T_1)\ge 0$, $|X_i(t)|<M$, $|Y_j(t)|<M$, for $t<T_1$;

(4) $Y_j(T_1)=-M$, $Y_j'(T_1)\le 0$, $|X_i(t)|<M$, $|Y_j(t)|<M$, for $t<T_1$.

If (1) holds from (19), (23), and (H_3), we have

$$\begin{aligned}
&X_i'(T_1)\\
&=\lambda X_i(T_1)-a_i(T_1)e^{\lambda\alpha_i(T_1)}X_i(T_1)+a_i(T_1)e^{\lambda\alpha_i(T_1)}\\
&\quad\times\int_{T_1-\alpha_i(T_1)}^{T_1}\Big\{\lambda X_i(s)-a_i(s)\,e^{\lambda\alpha_i(s)}X_i(s-\alpha_i(s))\\
&\qquad+\sum_{j=1}^{m}a_{ij}(s)e^{\lambda s}\tilde f_j(\bar y_j(s))\\
&\qquad+\sum_{j=1}^{m}b_{ij}(s)e^{\lambda s}\tilde f_j(\bar y_j(s-\tau(s)))\\
&\qquad+\bigwedge_{j=1}^{m}\alpha_{ij}(s)e^{\lambda s}\int_{-\infty}^{s}k_j(s-u)\tilde f_j(\bar y_j(u))\,du\\
&\qquad+\bigvee_{j=1}^{m}\beta_{ij}(s)e^{\lambda s}\\
&\qquad\times\int_{-\infty}^{s}k_j(s-u)\tilde f_j(\bar y_j(u))\,du\Big\}\,ds\\
&\quad+\sum_{j=1}^{m}a_{ij}(T_1)e^{\lambda T_1}\tilde f_j(\bar y_j(T_1))\\
&\quad+\sum_{j=1}^{m}b_{ij}(T_1)e^{\lambda T_1}\tilde f_j(\bar y_j(t-\tau(t)))\\
&\quad+\bigwedge_{j=1}^{m}\alpha_{ij}(T_1)e^{\lambda t}\int_{-\infty}^{T_1}k_j(T_1-s)\tilde f_j(\bar y_j(s))\,ds\\
&\quad+\bigvee_{j=1}^{m}\beta_{ij}(T_1)e^{\lambda T_1}\int_{-\infty}^{T_1}k_j(T_1-s)\tilde f_j(\bar y_j(s))\,ds\\
&\le\lambda X_i(T_1)-a_i(T_1)e^{\lambda\alpha_i(T_1)}X_i(T_1)+a_i(T_1)e^{\lambda\alpha_i(T_1)}\\
&\quad\times\int_{T_1-\alpha_i(T_1)}^{T_1}\Big\{\lambda X_i(T_1)+a_i^+e^{\lambda\alpha_i^+}X_i(T_1)+\sum_{j=1}^{m}a_{ij}^+L_j\left|Y_j(s)\right|\\
&\qquad+\sum_{j=1}^{m}b_{ij}^+e^{\lambda\tau}L_j\left|Y_j(s-\tau(s))\right|\\
&\qquad+\sum_{j=1}^{m}\alpha_{ij}^+\int_{-\infty}^{s}k_j(s-u)e^{\lambda(s-u)}L_j\left|Y_j(u)\right|du\\
&\qquad+\sum_{j=1}^{m}\beta_{ij}^+\int_{-\infty}^{s}k_j(s-u)e^{\lambda(s-u)}\\
&\qquad\times L_j\left|Y_j(u)\right|du\Big\}\,ds\\
&\quad+\sum_{j=1}^{m}a_{ij}^+L_j\left|Y_j(T_1)\right|\\
&\quad+\sum_{j=1}^{m}b_{ij}^+L_je^{\lambda\tau}\left|Y_j(T_1-\tau(T_1))\right|\\
&\quad+\sum_{j=1}^{m}\alpha_{ij}^+\int_{-\infty}^{T_1}k_j(T_1-s)e^{\lambda(T_1-s)}L_j\left|Y_j(s)\right|ds\\
&\quad+\sum_{j=1}^{m}\beta_{ij}^+\int_{-\infty}^{T_1}k_j(T_1-s)e^{\lambda(T_1-s)}L_j\left|Y_j(s)\right|ds\\
&\le-\Big[a_i(T_1)e^{\lambda\alpha_i(T_1)}-\lambda-a_i(T_1)e^{\lambda\alpha_i(T_1)}\\
&\quad\times\alpha_i(T_1)\left(\lambda+a_i^+e^{\lambda\alpha_i^+}\right)\Big]X_i(T_1)+a_i(T_1)e^{\lambda\alpha_i(T_1)}
\end{aligned}$$

$$\times \int_{T_1-\alpha_i(T_1)}^{T_1} \left[\sum_{j=1}^{m} a_{ij}^{+} L_j M + \sum_{j=1}^{m} b_{ij}^{+} e^{\lambda\tau} L_j M + \sum_{j=1}^{m} \alpha_{ij}^{+} L_j M \times \int_{-\infty}^{s} k_j(s-u) e^{\lambda(s-u)} du + \sum_{j=1}^{m} \beta_{ij}^{+} L_j M \int_{-\infty}^{s} k_j(s-u) e^{\lambda(s-u)} du \right] ds$$

$$+ \sum_{j=1}^{m} a_{ij}^{+} L_j M + \sum_{j=1}^{m} b_{ij}^{+} e^{\lambda\tau} L_j M + \sum_{j=1}^{m} \alpha_{ij}^{+} L_j M \times \int_{-\infty}^{T_1} k_j(T_1-s) e^{\lambda(T_1-s)} du + \sum_{j=1}^{m} \beta_{ij}^{+} L_j M \int_{-\infty}^{T_1} k_j(T_1-s) e^{\lambda(T_1-s)} du$$

$$\leq -\Big[a_i(T_1) e^{\lambda\alpha_i(T_1)} - \lambda - a_i(T_1) e^{\lambda\alpha_i(T_1)} \times \alpha_i(T_1) \left(\lambda + a_i^{+} e^{\lambda\alpha_i^{+}} \right) \Big] X_i(T_1) + \left(1 + a_i(T_1) e^{\lambda\alpha_i(T_1)} \alpha_i(T_1)\right) \times \sum_{j=1}^{m} \left[a_{ij}^{+} + b_{ij}^{+} e^{\lambda\tau} + \left(\alpha_{ij}^{+} + \beta_{ij}^{+}\right) \times \int_{0}^{+\infty} k_j(u) e^{\lambda u} du \right] L_j M$$

$$= \Big\{ -\Big[a_i(T_1) e^{\lambda\alpha_i(T_1)} - \lambda - a_i(T_1) e^{\lambda\alpha_i(T_1)} \times \alpha_i(T_1) \left(\lambda + a_i^{+} e^{\lambda\alpha_i^{+}} \right) \Big] + \left(1 + a_i(T_1) e^{\lambda\alpha_i(T_1)} \alpha_i(T_1)\right) \times \sum_{j=1}^{m} \Big[a_{ij}^{+} + b_{ij}^{+} e^{\lambda\tau} + \left(\alpha_{ij}^{+} + \beta_{ij}^{+}\right) \times \int_{0}^{+\infty} k_j(u) e^{\lambda u} du \Big] L_j \Big\} M$$

$$< 0, \tag{28}$$

and this is a contradiction. Hence, (1) does not hold.

If (2) holds, then, from (H_3), (19), and (24), we have

$$X_i'(T_1) = \lambda X_i(T_1) - a_i(T_1) e^{\lambda\alpha_i(T_1)} X_i(T_1) + a_i(T_1) e^{\lambda\alpha_i(T_1)} \times \int_{T_1-\alpha_i(T_1)}^{T_1} \Big\{ \lambda X_i(s) - a_i(s) \times e^{\lambda\alpha_i(s)} X_i(s-\alpha_i(s)) + \sum_{j=1}^{m} a_{ij}(s) e^{\lambda s} \tilde{f}_j\left(\overline{y}_j(s)\right) + \sum_{j=1}^{m} b_{ij}(s) e^{\lambda s} \tilde{f}_j\left(\overline{y}_j(s-\tau(s))\right) + \bigwedge_{j=1}^{m} \alpha_{ij}(s) e^{\lambda s} \int_{-\infty}^{s} k_j(s-u) \tilde{f}_j\left(\overline{y}_j(u)\right) du + \bigvee_{j=1}^{m} \beta_{ij}(s) e^{\lambda s} \times \int_{-\infty}^{s} k_j(s-u) \tilde{f}_j\left(\overline{y}_j(u)\right) du \Big\} ds$$

$$+ \sum_{j=1}^{m} a_{ij}(T_1) e^{\lambda T_1} \tilde{f}_j\left(\overline{y}_j(T_1)\right) + \sum_{j=1}^{m} b_{ij}(T_1) e^{\lambda T_1} \tilde{f}_j\left(\overline{y}_j(t-\tau(t))\right) + \bigwedge_{j=1}^{m} \alpha_{ij}(T_1) e^{\lambda t} \int_{-\infty}^{T_1} k_j(T_1-s) \tilde{f}_j\left(\overline{y}_j(s)\right) ds + \bigvee_{j=1}^{m} \beta_{ij}(T_1) e^{\lambda T_1} \int_{-\infty}^{T_1} k_j(T_1-s) \tilde{f}_j\left(\overline{y}_j(s)\right) ds$$

$$\geq \lambda X_i(T_1) - a_i(T_1) e^{\lambda\alpha_i(T_1)} X_i(T_1) + a_i(T_1) e^{\lambda\alpha_i(T_1)} \times \int_{T_1-\alpha_i(T_1)}^{T_1} \Big\{ \lambda X_i(T_1) + a_i^{+} e^{\lambda\alpha_i^{+}} X_i(T_1) - \sum_{j=1}^{m} a_{ij}^{+} L_j \left|Y_j(s)\right| - \sum_{j=1}^{m} b_{ij}^{+} e^{\lambda\tau} L_j \left|Y_j(s-\tau(s))\right| - \sum_{j=1}^{m} \alpha_{ij}^{+} \int_{-\infty}^{s} k_j(s-u) e^{\lambda(s-u)} L_j \left|Y_j(u)\right| du - \sum_{j=1}^{m} \beta_{ij}^{+} \int_{-\infty}^{s} k_j(s-u) e^{\lambda(s-u)} \times L_j \left|Y_j(u)\right| du \Big\} ds$$

$$- \sum_{j=1}^{m} a_{ij}^{+} L_j \left|Y_j(T_1)\right| - \sum_{j=1}^{m} b_{ij}^{+} L_j e^{\lambda\tau} \left|Y_j(T_1-\tau(T_1))\right| - \sum_{j=1}^{m} \alpha_{ij}^{+} \int_{-\infty}^{T_1} k_j(T_1-s) e^{\lambda(T_1-s)} L_j \left|Y_j(s)\right| ds - \sum_{j=1}^{m} \beta_{ij}^{+} \int_{-\infty}^{T_1} k_j(T_1-s) e^{\lambda(T_1-s)} L_j \left|Y_j(s)\right| ds$$

$$\geq -\Big[a_i(T_1) e^{\lambda\alpha_i(T_1)} - \lambda - a_i(T_1) e^{\lambda\alpha_i(T_1)} \times \alpha_i(T_1) \left(\lambda + a_i^{+} e^{\lambda\alpha_i^{+}} \right) \Big] X_i(T_1) + a_i(T_1) e^{\lambda\alpha_i(T_1)}$$

$$\times \int_{T_1-\alpha_i(T_1)}^{T_1} \Bigg[-\sum_{j=1}^{m} a_{ij}^{+} L_j M - \sum_{j=1}^{m} b_{ij}^{+} e^{\lambda\tau} L_j M - \sum_{j=1}^{m} \alpha_{ij}^{+} L_j M \times \int_{-\infty}^{s} k_j (s-u) e^{\lambda(s-u)} du - \sum_{j=1}^{m} \beta_{ij}^{+} L_j M \int_{-\infty}^{s} k_j (s-u) e^{\lambda(s-u)} du \Bigg] ds$$

$$- \sum_{j=1}^{m} a_{ij}^{+} L_j M - \sum_{j=1}^{m} b_{ij}^{+} e^{\lambda\tau} L_j M - \sum_{j=1}^{m} \alpha_{ij}^{+} L_j M \times \int_{-\infty}^{T_1} k_j (T_1 - s) e^{\lambda(T_1-s)} du - \sum_{j=1}^{m} \beta_{ij}^{+} L_j M \int_{-\infty}^{T_1} k_j (T_1 - s) e^{\lambda(T_1-s)} du$$

$$\geq \Bigg\{ -\left[a_i (T_1) e^{\lambda\alpha_i(T_1)} - \lambda - a_i (T_1) e^{\lambda\alpha_i(T_1)} \times \alpha_i (T_1) \left(\lambda + a_i^{+} e^{\lambda\alpha_i^{+}} \right) \right] X_i (T_1) + \left(1 + a_i (T_1) e^{\lambda\alpha_i(T_1)} \alpha_i (T_1) \right) \times \sum_{j=1}^{m} \left[a_{ij}^{+} + b_{ij}^{+} e^{\lambda\tau} + \left(\alpha_{ij}^{+} + \beta_{ij}^{+} \right) \times \int_{0}^{+\infty} k_j (u) e^{\lambda u} du \right] L_j \Bigg\} (-M) > 0. \tag{29}$$

This is also a contradiction.

Similarly, if (3) (or (4)) holds, we can derive a contradiction. Therefore, (27) holds.

Furthermore, together with (16) and (17), we have

$$\begin{aligned}
\left|\overline{x}_i (t_k^{+})\right| &= \left|x_i (t_k^{+}) - x_i^{*} (t_k^{+})\right| \\
&\leq \left|x_i (t_k) - x_i^{*} (t_k)\right| = \left|\overline{x}_i (t_k^{-})\right| \\
&= \left|X_i (t_k^{-})\right| e^{-\lambda t_k} \leq M e^{-\lambda t_k}, \\
\left|\overline{y}_j (t_k^{+})\right| &= \left|y_j (t_k^{+}) - y_j^{*} (t_k^{+})\right| \\
&\leq \left|y_j (t_k) - y_j^{*} (t_k)\right| = \left|\overline{y}_j (t_k^{-})\right| \\
&= \left|Y_j (t_k^{-})\right| e^{-\lambda t_k} \leq M e^{-\lambda t_k},
\end{aligned} \tag{30}$$

where $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, and $k = 1, 2, \ldots$.

From (27) and (30), we get

$$\begin{aligned}
x_i (t) - x_i^{*} (t) &= O\left(e^{-\lambda t}\right), \\
y_j (t) - y_j^{*} (t) &= O\left(e^{-\lambda t}\right)
\end{aligned} \tag{31}$$

for all $t > 0$, $i = 1, 2, \ldots, n$, and $j = 1, 2, \ldots, m$. Therefore, system (3) is exponentially stable. This completes the proof. □

3. An Example

In this section, we present an example to illustrate the feasibility of our results obtained in previous sections.

Example 4. Consider the following fuzzy BAM neural networks with distributed delays and impulses:

$$\begin{aligned}
x_i' (t) &= -a_i (t) x_i (t - \alpha_i (t)) + \sum_{j=1}^{2} a_{ij} (t) f_j \left(y_j (t) \right) + \sum_{j=1}^{2} b_{ij} (t) f_j \left(y_j (t - \tau (t)) \right) + \sum_{j=1}^{2} c_{ij} (t) \omega_j \\
&\quad + \bigwedge_{j=1}^{2} \alpha_{ij} (t) \int_{-\infty}^{t} k_j (t-s) f_j \left(y_j (s) \right) ds + \bigvee_{j=1}^{2} \beta_{ij} (t) \int_{-\infty}^{t} k_j (t-s) f_j \left(y_j (s) \right) ds \\
&\quad + \bigwedge_{j=1}^{2} T_{ij} \omega_j + \bigvee_{j=1}^{m} H_{ij} \omega_j + A_i (t), \quad t \geq 0,\ t \neq t_k, \\
\Delta x_i (t_k) &= -\theta_{ik} x_i (t_k), \quad k = 1, 2, \ldots, \\
y_j' (t) &= -b_j (t) y_j \left(t - \beta_j (t) \right) + \sum_{i=1}^{2} d_{ji} (t) g_i (x_i (t)) + \sum_{i=1}^{2} p_{ji} (t) g_i (x_i (t - \rho (t))) \\
&\quad + \sum_{i=1}^{2} q_{ji} (t) \mu_i + \bigwedge_{i=1}^{n} \gamma_{ji} (t) \int_{-\infty}^{t} k_i (t-s) g_i (x_i (s)) ds + \bigvee_{i=1}^{2} \eta_{ji} (t) \int_{-\infty}^{t} k_i (t-s) g_i (x_i (s)) ds \\
&\quad + \bigwedge_{i=1}^{2} R_{ji} \mu_i + \bigvee_{i=1}^{n} S_{ji} \mu_i + B_j (t), \quad t \geq 0,\ t \neq t_k, \\
\Delta y_j (t_k) &= -\overline{\theta}_{jk} y_j (t_k), \quad k = 1, 2, \ldots,
\end{aligned} \tag{32}$$

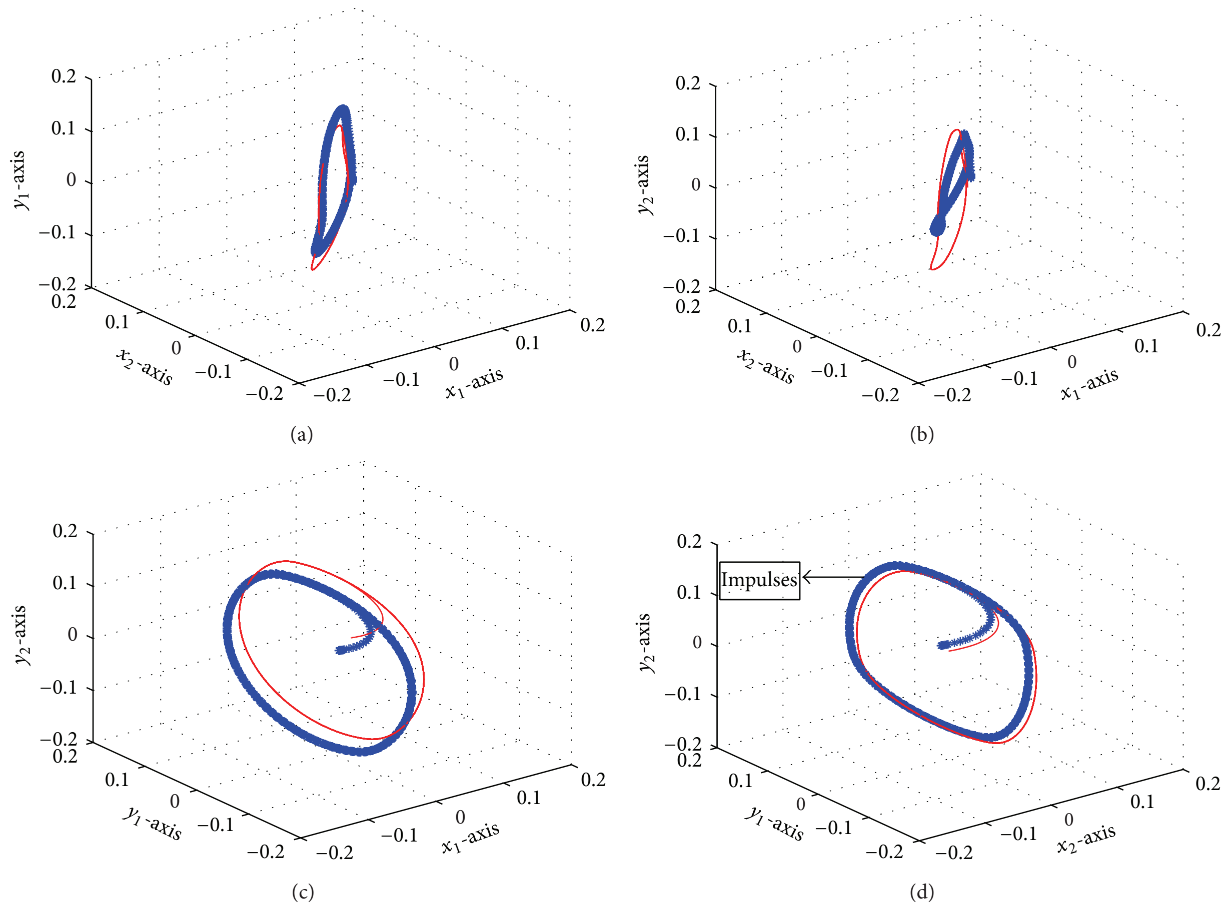

Figure 1: Phase responses of states x_1, x_2, y_1, and y_2.

where $i, j = 1, 2$, $f_j(x) = g_i(x) = (1/8)(|x+1| - |x-1|)$, and $t_1 < t_2 < \cdots$ is strictly increasing sequences such that $\lim_{k\to\infty} t_k = +\infty$; the coefficients are as follows:

$$a_1(t) = \frac{1}{2}(9 + \sin t), \qquad a_2(t) = 4 + |\sin t|,$$
$$\alpha_1(t) = \alpha_2(t) = \frac{1}{20}(1 + \sin t),$$
$$b_1(t) = \frac{1}{2}(9 + \cos t), \qquad b_2(t) = 4 + |\cos t|,$$
$$\beta_1(t) = \beta_2(t) = \frac{1}{20}(1 + \cos t),$$
$$a_{11}(t) = a_{12}(t) = a_{21}(t) = a_{22}(t) = \frac{1}{9}(1 + 2\sin t),$$
$$b_{11}(t) = b_{12}(t) = b_{21}(t) = b_{22}(t) = \frac{1}{9}(1 + 2\cos t),$$
$$d_{11}(t) = d_{12}(t) = d_{21}(t) = d_{22}(t) = \frac{1}{6}(1 + \sin t),$$
$$p_{11}(t) = p_{12}(t) = p_{21}(t) = p_{22}(t) = \frac{1}{6}(1 + \cos t),$$
$$\alpha_{11}(t) = \alpha_{12}(t) = \alpha_{21}(t) = \alpha_{22}(t) = \frac{1}{3}\sin t,$$
$$\beta_{11}(t) = \beta_{12}(t) = \beta_{21}(t) = \beta_{22}(t) = \frac{1}{3}\cos t,$$
$$\gamma_{11}(t) = \gamma_{12}(t) = \gamma_{21}(t) = \gamma_{22}(t) = \frac{1}{12}(1 + 3\sin t),$$
$$\eta_{11}(t) = \eta_{12}(t) = \eta_{21}(t) = \eta_{22}(t) = \frac{1}{12}(1 + 3\cos t),$$
$$T_{ij} = H_{ij} = R_{ij} = S_{ij} = 1,$$
$$\omega_j = \mu_i = 1 \quad (i, j = 1, 2),$$
$$A_1(t) = A_2(t) = \frac{1}{2}\cos t, \qquad B_1(t) = B_2(t) = \frac{1}{2}\sin t,$$
$$\theta_{ik} = 1 - \frac{1}{3}\sin(2 + k),$$
$$\bar{\theta}_{jk} = 1 + \frac{2}{3}\cos(3k), \quad (i, j = 1, 2). \tag{33}$$

By calculating, we have

$$a_1^+ = a_2^+ = b_1^+ = b_2^+ = 5,$$
$$a_1^- = a_2^- = b_1^- = b_2^- = 4,$$
$$\alpha_1^+ = \alpha_2^+ = \beta_1^+ = \beta_2^+ = \frac{1}{10},$$

$$a_{ij}^{+} = b_{ij}^{+} = d_{ji}^{+} = p_{ji}^{+} = \alpha_{ij}^{+} = \beta_{ij}^{+} = \gamma_{ji}^{+} = \eta_{ji}^{+} = \frac{1}{3} \quad (i, j = 1, 2),$$
$$L_j = M_i = \frac{1}{4} \quad (i, j = 1, 2). \tag{34}$$

We can see that system (32) satisfies the conditions (H_1), (H_2), and (H_4); for $i = 1, 2$, $j = 1, 2$, we have

$$\begin{aligned} &-a_i^{-}\left(1 - a_i^{+}\alpha_i^{+}\right) + \left(1 + a_i^{+}\alpha_i^{+}\right) \times \sum_{j=1}^{2}\left(a_{ij}^{+} + b_{ij}^{+} + \alpha_{ij}^{+} + \beta_{ij}^{+}\right) L_j < 0, \\ &-b_j^{-}\left(1 - b_j^{+}\beta_j^{+}\right) + \left(1 + b_j^{+}\beta_j^{+}\right) \times \sum_{i=1}^{2}\left(d_{ji}^{+} + p_{ji}^{+} + \gamma_{ji}^{+} + \eta_{ji}^{+}\right) M_i < 0, \end{aligned} \tag{35}$$

which implies that (H_3) holds. Therefore, from Theorem 3, system (32) is exponentially stable (Figure 1 illustrates our plausible results).

4. Conclusion

In this paper, we consider a class of BAM fuzzy cellular neural networks with time-varying delays in leakage terms and impulses. By using differential inequality techniques, we obtain sufficient conditions for the exponential stability of this class of networks. Our results are completely new and complementary to the previously known results. Finally, an example is given to demonstrate the effectiveness and conservativeness of our theoretical results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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