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Research Article

On the Exponential Radon Transform and Its Extension to Certain Functions Spaces

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We investigate the exponential Radon transform on a certain function space of generalized functions. We establish certain space of generalized functions for the cited transform. The transform that is obtained is well defined. More properties of consistency, convolution, analyticity, continuity, and sufficient theorems have been established.

1. Introduction

The Radon transform of a sufficiently nice function f defined on $\mathbb{R}^n$ is given by

$$(\mathscr{R}_\vartheta f)(\eta) \equiv (\mathscr{R} f)(\vartheta, \eta) \equiv \int_{\vartheta^\perp} f(\eta\vartheta + u)\, \mathbf{d}u, \quad (1)$$

where $(\vartheta, \eta) \in \widetilde{\mathbb{R}}^n = \sum_{n-1} xp$, $\sum_{n-1}$ is the unit sphere in $\mathbb{R}^n$, and $\mathbf{d}u$ is the Euclidean measure on the subspace $\vartheta^\perp$ orthogonal to ϑ.

Applications of the Radon transform occur in a number of areas, such as seismic signal processing, remote sensing, and system identification from output data [1, 2]. The Radon transform is extended to various spaces of distributions, rapidly decreasing and integrable Boehmians [3, 4]. More about the Radon transform is given in [5–9].

The discrete Radon transform is defined by [10, 11]. The attenuated Radon transform is defined in Mikusiński et al. [12, 13]. For a uniform attenuation coefficient $\mu \in \mathbb{C}$, the exponential Radon transform of a compactly supported real valued function f, defined on $\mathbb{R}^2$, is given by Kurusa and Hertle [7, 8]:

$$\mathbf{T}^e_\mu f(\vartheta, t) = \int_{\mathbb{R}^2} f(\mathbf{x})\, \delta(\mathbf{x}\cdot\vartheta - t)\, e^{\mu \mathbf{x}\cdot\vartheta^\perp} \mathbf{dx}, \quad t \in \mathbb{R}, \quad (2)$$

where $\vartheta = (\cos\varphi, \sin\varphi)^\tau$ is a unit vector on $\mathscr{S}^1$, $\varphi \in [0, 2\pi)$, $\vartheta^\perp = (-\sin\varphi, \cos\varphi)$.

The exponential Radon transform constitutes a mathematical model for imaging modalities such as X-ray tomography for $\mu = 0$, single photon emission tomography for $\mu \in \mathbb{R}$, and optical polarization tomography of trass tensor field [14]. However, if in addition μ is unknown, then one first must find μ and then find f. This is the identification problem.

The exponential Radon transform, as a generalization of the Radon transform, is defined as a mapping of function spaces and is also represented in terms of Fourier transforms of its domain and range, and this leads to a characterization of the range of the transform. For more information about the exponential Radon transform, we refer to [15, 16].

2. General Construction of Boehmians

The minimal structure necessary for the construction of Boehmians consists of the following elements:

(i) a set $\mathbf{a}$ and a commutative semigroup $(\mathbf{g}, *)$;

(ii) an operation $\odot : \mathbf{a} \times \mathbf{g} \to \mathbf{a}$ such that for each $x \in \mathbf{a}$ and $\upsilon_1, \upsilon_2, \in \mathbf{g}$,

$$x \odot (\upsilon_1 * \upsilon_2) = (x \odot \upsilon_1) \odot \upsilon_2; \quad (3)$$

(iii) a collection $\Delta \subset \mathbf{g}^{\mathbb{N}}$ satisfying the following:

(1) if $x, y \in \mathbf{a}$, $(v_n) \in \Delta$, $x \odot v_n = y \odot v_n$ for all n, then $x = y$;

(2) if $(v_n), (\sigma_n) \in \Delta$, then $(v_n * \sigma_n) \in \Delta$, Δ being the set of all delta sequences.

Consider

$$\mathscr{A} = \{(x_n, v_n) : x_n \in \mathbf{a}, (v_n) \in \Delta, x_n \odot v_m = x_m \odot v_n, \forall m, n \in \mathbb{N}\}. \tag{4}$$

If $(x_n, v_n), (y_n, \sigma_n) \in \mathscr{A}$, $x_n \odot \sigma_m = y_m \odot v_n$, $\forall m, n \in \mathbb{N}$, then we say $(x_n, v_n) \sim (y_n, \sigma_n)$. The relation $\sim$ is an equivalence relation in $\mathscr{A}$. The space of equivalence classes in $\mathscr{A}$ is denoted by $\boldsymbol{\delta}(\mathbf{a}, (\mathbf{g}, *), \odot, \Delta)$. Elements of $\boldsymbol{\delta}(\mathbf{a}, (\mathbf{g}, *), \odot, \Delta)$ are called Boehmians.

Between $\mathbf{a}$ and $\boldsymbol{\delta}(\mathbf{a}, (\mathbf{g}, *), \odot, \Delta)$ there is a canonical embedding expressed as

$$x \longrightarrow \frac{x \odot s_n}{s_n} \quad \text{as } n \longrightarrow \infty. \tag{5}$$

The operation $\odot$ can be extended to $\boldsymbol{\delta}(\mathbf{a}, (\mathbf{g}, *), \odot, \Delta) \times \mathbf{a}$ by

$$\frac{x_n}{v_n} \odot t = \frac{x_n \odot t}{v_n}. \tag{6}$$

In $\boldsymbol{\delta}(\mathbf{a}, (\mathbf{g}, *), \Delta)$, two types of convergence exist:

(1) a sequence (h_n) in $\boldsymbol{\delta}(\mathbf{a}, (\mathbf{g}, *), \odot, \Delta)$ is said to be δ-convergent to h in $\boldsymbol{\delta}(\mathbf{a}, (\mathbf{g}, *), \odot, \Delta)$, denoted by $h_n \xrightarrow{\delta} h$ as $n \to \infty$, if there exists a delta sequence (v_n) such that $(h_n \odot v_n), (h \odot v_n) \in \mathbf{a}$, $\forall k, n \in \mathbb{N}$, and $(h_n \odot v_k) \to (h \odot v_k)$ as $n \to \infty$, in $\mathbf{a}$, for every $k \in \mathbb{N}$;

(2) a sequence (h_n) in $\boldsymbol{\delta}(\mathbf{a}, (\mathbf{g}, *), \odot, \Delta)$ is said to be Δ-convergent to h in $\boldsymbol{\delta}(\mathbf{a}, (\mathbf{g}, *), \odot, \Delta)$, denoted by $h_n \xrightarrow{\Delta} h$ as $n \to \infty$, if there exists a $(v_n) \in \Delta$ such that $(h_n - h) \odot v_n \in \mathbf{a}$, $\forall n \in \mathbb{N}$, and $(h_n - h) \odot v_n \to 0$ as $n \to \infty$ in $\mathbf{a}$.

The following theorem is equivalent to the statement of δ-convergence.

Theorem 1. *$h_n \xrightarrow{\delta} h$ $(n \to \infty)$ in $\boldsymbol{\delta}(\mathbf{a}, (\mathbf{g}, *), \odot, \Delta)$ if and only if there is $f_{n,k}, f_k \in \mathbf{a}$ and $v_k \in \Delta$ such that $h_n = [f_{n,k}/v_k]$, $h = [f_k/v_k]$ and for each $k \in \mathbb{N}$, $f_{n,k} \to f_k$ as $n \to \infty$ in $\mathbf{a}$.*

For further discussion see [3, 17–21].

3. Necessary and Sufficient Conditions

Denote by $\mathbf{l}^1(\mathscr{S}^1 \times \mathbb{R})$ the space of Lebesgue complex-valued measurable functions of bounded support defined on $\mathscr{S}^1 \times \mathbb{R}$ and satisfying

$$\||\psi((\vartheta, t))\|| = \int_{\mathscr{S}^1} \int_{\mathbb{R}} |\psi((\vartheta, t))|^2 e^{\mu \mathbf{y} \cdot \vartheta^{\perp}} \mathbf{d}\vartheta \, \mathbf{d}t < \infty \tag{7}$$

then $\mathbf{T}^e_\mu \psi \in \mathbf{l}^1(\mathscr{S}^1 \times \mathbb{R})$, $\mu \in \mathbb{R}$, $\mathbf{y} \in \mathbb{R}^2$ being arbitrary but fixed.

By $\boldsymbol{\kappa}(\mathbb{R}^2)$ denote the space of test functions of bounded support defined on $\mathbb{R}^2$.

Let Δ be the set of sequences $(\mu_n(\mathbf{x})) \in \boldsymbol{\kappa}(\mathbb{R}^2)$ such that [3, (2.6)–(2.8)]

$$\begin{gathered} \int_{\mathbb{R}^2} \mu n(\mathbf{x}) \, \mathbf{dx} = 1, \\ \int_{\mathbb{R}^2} |\mu n(\mathbf{x})| \, \mathbf{dx} < M, \quad 0 < M \in \mathbb{R}, \\ \operatorname{supp}(\mu_n(\mathbf{x})) \longrightarrow 0 \quad \text{as } n \longrightarrow \infty. \end{gathered} \tag{8}$$

The convolution product between two functions is defined by the integral equation

$$(f * g)(\mathbf{x}) = \int_{\mathbb{R}^2} f(\mathbf{x} - \mathbf{y}) \, g(\mathbf{y}) \, \mathbf{dy}, \tag{9}$$

where $\mathbf{x} \in \mathbb{R}^2$.

Now we construct the space $\boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), \times, \Delta)$ of Boehmians.

We have the following definition.

Definition 2. Let $\psi \in \mathbf{l}^1(\mathscr{S}^1 \times \mathbb{R})$ and $\xi \in \boldsymbol{\kappa}(\mathbb{R}^2)$; then we define the mapping $\times$ as

$$(\psi \times \xi)(\vartheta, t) = \int_{\mathbb{R}^2} \psi(\vartheta, t - \mathbf{y} \cdot \vartheta) \, \xi(\mathbf{y}) \, e^{\mu \mathbf{y} \cdot \vartheta^{\perp}} \mathbf{dy} \qquad (\vartheta, t) \in \mathscr{S}^1 \times \mathbb{R}. \tag{10}$$

Theorem 3. *Let $\psi \in \mathbf{l}^1(\mathscr{S}^1 \times \mathbb{R})$ and $\xi \in \boldsymbol{\kappa}(\mathbb{R}^2)$; then $\psi \times \xi \in \mathbf{l}^1(\mathscr{S}^1 \times \mathbb{R})$.*

Proof. Let $\psi \in \mathbf{l}^1(\mathscr{S}^1 \times \mathbb{R})$ and $\xi \in \boldsymbol{\kappa}(\mathbb{R}^2)$. By using (10), Fubini's theorem, and Jensen's inequality we get

$$\begin{aligned} &\||(\psi \times \xi)(\vartheta, t)\||^2 \\ &\quad = \int_{\mathscr{S}^1} \int_{\mathbb{R}} |(\psi \times \xi)(\vartheta, t)|^2 \mathbf{d}\vartheta \, \mathbf{d}t \\ &\quad = \int_{\mathscr{S}^1} \int_{\mathbb{R}} \left| \int_{\mathbb{R}^2} \psi(\vartheta, t - \mathbf{y} \cdot \theta) \, \xi(\mathbf{y}) \, e^{\mu \mathbf{y} \cdot \vartheta^{\perp}} \mathbf{dy} \right|^2 \mathbf{d}\vartheta \, \mathbf{d}t \\ &\quad = \int_{\mathbb{R}^2} |\xi(\mathbf{y})| \int_{\mathscr{S}^1} \int_{\mathbb{R}} |\psi(\vartheta, t - \mathbf{y} \cdot \vartheta)|^2 e^{\mu \mathbf{y} \cdot \vartheta^{\perp}} \mathbf{d}\vartheta \, \mathbf{d}t \, \mathbf{dy} \\ &\quad < M \||\psi\||^2, \end{aligned} \tag{11}$$

where M is a positive constant.

The proof is therefore completed. □

Theorem 4. *Let $\psi_n \to \psi$ in $\mathbf{l}^1(\mathscr{S}^1 \times \mathbb{R})$ and $\xi \in \boldsymbol{\kappa}(\mathbb{R}^2)$; then $\psi_n \times \xi \to \psi \times \xi$ as $n \to \infty$.*

Proof of this theorem follows from Theorem 3.

Theorem 5. *Let $\psi \in \mathbf{l}^1(\mathscr{S}^1 \times \mathbb{R})$ and $\xi_1, \xi_2 \in \boldsymbol{\kappa}(\mathbb{R}^2)$; then one has*

$$\psi \times (\xi_1 * \xi_2) = (\psi \times \xi_1) \times \xi_2. \tag{12}$$

Proof. Let $(\vartheta, t) \in \mathcal{S}^1 \times \mathbb{R}$. Using (10) and (9) we write

$$\begin{aligned}
&(\psi \times (\xi_1 * \xi_2))(\vartheta, t) \\
&= \int_{\mathbb{R}^2} \psi(\vartheta, t - \mathbf{y}\cdot\vartheta)(\xi_1 * \xi_2)(\mathbf{y}) e^{\mu\mathbf{y}\cdot\vartheta^\perp} \mathbf{dy} \\
&= \int_{\mathbb{R}^2} \psi(\vartheta, t - \mathbf{y}\cdot\vartheta)\left(\int_{\mathbb{R}^2} \xi_1(\mathbf{y}-\mathbf{x})\xi_2(\mathbf{x})\mathbf{dx}\right) e^{\mu\mathbf{y}\cdot\vartheta^\perp} \mathbf{dy} \\
&= \int_{\mathbb{R}^2}\left(\int_{\mathbb{R}^2} \psi(\vartheta, t - \mathbf{y}\cdot\vartheta)\xi_1(\mathbf{y}-\mathbf{x}) e^{\mu\mathbf{y}\cdot\vartheta^\perp}\mathbf{dy}\right)\xi_2(\mathbf{x})\mathbf{dx}.
\end{aligned} \tag{13}$$

The substitution $\mathbf{y} - \mathbf{x} = \mathbf{z}$, $\mathbf{y}, \mathbf{x}, \mathbf{z} \in \mathbb{R}^2$, implies

$$\begin{aligned}
&(\psi \times (\xi_1 * \xi_2))(\vartheta, t) \\
&= \int_{\mathbb{R}^2}\left(\int_{\mathbb{R}^2} \psi(\vartheta, t - (\mathbf{z}+\mathbf{x})\cdot\vartheta)\xi_1(\mathbf{z}) e^{\mu(\mathbf{z}+\mathbf{x})\vartheta^\perp}\mathbf{dz}\right) \\
&\quad \times \xi_2(\mathbf{x})\mathbf{dx} \\
&= \int_{\mathbb{R}^2}\left(\int_{\mathbb{R}^2} \psi(\vartheta, (t - \mathbf{x}\cdot\vartheta) - \mathbf{z}\cdot\vartheta)\xi_1(\mathbf{z}) e^{\mu\mathbf{x}\cdot\vartheta^\perp}\mathbf{dz}\right) \\
&\quad \times \xi_2(\mathbf{x}) e^{\mu\mathbf{x}\cdot\vartheta^\perp}\mathbf{dx} \\
&= \int_{\mathbb{R}^2} (\psi \times \xi_1)(\vartheta, t - \mathbf{x}\cdot\vartheta)\xi_2(\mathbf{x}) e^{\mu\mathbf{x}\cdot\vartheta^\perp}\mathbf{dx} \\
&= ((\psi \times \xi_1) \times \xi_2)(\vartheta, t).
\end{aligned} \tag{14}$$

This completes the proof of the theorem. □

Theorem 6. *Let* $\psi_1, \psi_2 \in \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$ *and* $\xi \in \boldsymbol{\kappa}(\mathbb{R}^2)$; *then*

$$(\psi_1 + \psi_2) \times \xi = \psi_1 \times \xi + \psi_2 \times \xi_2. \tag{15}$$

Theorem 7. *Let* $\alpha \in \mathbb{C}$, $\psi \in \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$ *and* $\xi \in \boldsymbol{\kappa}(\mathbb{R}^2)$; *then*

$$\alpha(\psi \times \xi) = (\alpha\psi) \times \xi. \tag{16}$$

Proof of Theorems 6 and 7 follows from simple integration. Detailed proof is thus avoided.

Theorem 8. *Let* $\psi \in \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$ *and* $(\mu_n) \in \Delta$; *then* $\psi \times \mu_n \to \psi$ *as* $n \to \infty$.

Proof. Let $\psi \in \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$. Since $\boldsymbol{\kappa}(\mathbb{R} \times \mathbb{R})$ is dense in $\mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$, we can choose $\alpha \in \boldsymbol{\kappa}(\mathbb{R} \times \mathbb{R})$ such that

$$\||\psi - \alpha\|| < \epsilon, \quad \epsilon > 0. \tag{17}$$

From the analysis applied for proving Theorem 3 and by applying (17) we get

$$\||(\psi - \alpha) \times \mu_n\|| \le M \||\psi - \alpha\|| < M\epsilon. \tag{18}$$

Also, for each fixed $(\vartheta, t) \in \mathcal{S}^1 \times \mathbb{R}$ define

$$g(\mathbf{y}) = \alpha(\vartheta, t - \mathbf{y}\cdot\vartheta) e^{\mu\mathbf{y}\cdot\vartheta^\perp}, \quad \mathbf{y} \in \mathbb{R}^2; \tag{19}$$

then $g(\mathbf{y}) \in \boldsymbol{\kappa}(\mathbb{R}^2)$ and hence $g(\mathbf{y})$ uniformly continuous on $\mathbb{R}^2$. Thus, there is $\delta > 0$ such that $|g(\mathbf{y}) - g(\mathbf{x})| < \epsilon$ whenever $|\mathbf{y} - \mathbf{x}| \le \delta$.

Moreover $\operatorname{supp}\alpha(\vartheta, t) \subseteq [a, b] \times \Bbbk$, $\Bbbk \subseteq \mathbb{R} - \{0\}$ implies $\alpha(\vartheta, t) = 0$, $\forall(\vartheta, t) \notin [a - \delta, b + \delta] \times \Bbbk$.

Hence, by (8) and the fact that $|(g(y) - g(0))| < \epsilon$, by Jensen's inequality, we have

$$\begin{aligned}
&\||(\alpha \times \mu_n - \alpha)(\vartheta, t)\||^2 \\
&= \int_{\mathcal{S}^1}\int_{\mathbb{R}} |(\alpha \times \mu_n - \alpha)(\vartheta, t)|^2 \mathbf{d}\vartheta\,\mathbf{d}t \\
&= \int_{\mathcal{S}^1}\int_{\mathbb{R}} \left|\int_{\mathbb{R}^2}\left(\alpha(\vartheta, t - \mathbf{y}\cdot\vartheta)\mu_n(\mathbf{y}) e^{\mu\mathbf{y}\cdot\vartheta^\perp}\right.\right. \\
&\qquad\left.\left. - \alpha(\vartheta, t)\mu_n(\mathbf{y})\right)\mathbf{dy}\right|^2 \mathbf{d}\vartheta\,\mathbf{d}t \\
&= \int_{\mathcal{S}^1}\int_{\mathbb{R}} \left|\int_{\mathbb{R}^2} (g(\mathbf{y}) - g(0))\mu_n(\mathbf{y})\mathbf{dy}\right|^2 \mathbf{d}\vartheta\,\mathbf{d}t \\
&\le \int_{\Bbbk}\int_{a-\delta}^{b+\delta}\int_{\mathbb{R}^2} |(g(\mathbf{y}) - g(0))|^2 |\mu_n(\mathbf{y})|\mathbf{dy}\,\mathbf{d}\vartheta\,\mathbf{d}t \\
&\le \epsilon^2 \int_{\Bbbk}\int_{a-\delta}^{b+\delta}\int_{\mathbb{R}^2} |\mu_n(\mathbf{y})|\mathbf{dy}\,\mathbf{d}\vartheta\,\mathbf{d}t \\
&= \epsilon^2 Mm(\Bbbk)(b - a + 2\delta),
\end{aligned} \tag{20}$$

where $m(\Bbbk)$ is the Lebesgue measure of $\Bbbk$.

Hence, using (17), (18), and (20) we, for large values of n, get

$$\begin{aligned}
\||\psi \times \mu_n - \psi\|| &\le \||(\psi - \alpha) \times \mu_n\|| \\
&\quad + \||\alpha \times \mu_n - \alpha\|| + \||\alpha - \psi\|| \\
&< M\epsilon + \epsilon^2 Mm(\Bbbk)(b - a + 2\delta) + \epsilon \\
&= \left(M + \epsilon^2 Mm(\Bbbk)(b - a + 2\delta) + 1\right)\epsilon.
\end{aligned} \tag{21}$$

Hence $\psi \times \mu_n \to \psi$ as $n \to \infty$.

The Boehmian space $\boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), \times, \Delta)$ is constructed.

The sum and multiplication by a scalar of two Boehmians are naturally defined in the respective ways:

$$\begin{aligned}
&\left[\frac{(f_n)}{(\phi_n)}\right] + \left[\frac{(g_n)}{(\tau_n)}\right] = \left[\frac{(f_n \times \tau_n) + (g_n \times \phi_n)}{(\phi_n * \tau_n)}\right], \\
&\eta\left[\frac{(f_n)}{(\phi_n)}\right] = \left[\frac{(\eta f_n)}{(\phi_n)}\right],
\end{aligned} \tag{22}$$

η being complex number.

The operations $\times$ and the derivative are defined by

$$\begin{aligned}
&\left[\frac{(f_n)}{(\phi_n)}\right] \times \left[\frac{(g_n)}{(\tau_n)}\right] = \left[\frac{(f_n \times g_n)}{(\epsilon_n * \tau_n)}\right], \\
&\mathscr{D}^\alpha\left[\frac{(f_n)}{(\phi_n)}\right] = \left[\frac{(\mathscr{D}^\alpha f_n)}{(\phi_n)}\right].
\end{aligned} \tag{23}$$

Between $\mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$ and $\boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), \times, \Delta)$ the canonical embedding admits

$$f \longrightarrow \left[\frac{(f \times \phi_n)}{(\phi_n)}\right] \quad \text{as } n \longrightarrow \infty. \tag{24}$$

The operation $\times$ can be extended to $\boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), \times, \Delta) \times \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$ by

$$\left[\frac{(f_n)}{(\phi_n)}\right] \times f = \left[\frac{(f_n \times f)}{(\phi_n)}\right]. \tag{25}$$

By $\boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), *, \Delta)$ denote the corresponding Boehmian space obtained from $\mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$, $\boldsymbol{\kappa}(\mathbb{R}^2)$, Δ and the product $*$. □

Theorem 9. *Let $\psi \in \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$ and $\xi \in \boldsymbol{\kappa}(\mathbb{R}^2)$; then*

$$\mathbf{T}^e_\mu(\psi * \xi)(\vartheta, t) = \left(\mathbf{T}^e_\mu \psi \times \xi\right)(\vartheta, t), (\vartheta, t) \in \mathcal{S}^1 \times \mathbb{R}. \tag{26}$$

Proof. Let $(\vartheta, t) \in (\mathcal{S}^1 \times \mathbb{R})$. By employing (2) for (9) we get

$$\begin{aligned} &\mathbf{T}^e_\mu(\psi * \xi)(\vartheta, t) \\ &\quad = \int_{\mathbb{R}^2} (\psi * \xi)(\mathbf{x})\, \delta(\mathbf{x}\cdot\vartheta - t)\, e^{\mu \mathbf{x}\cdot\vartheta^\perp} \mathbf{dx} \\ &\quad = \int_{\mathbb{R}^2} \left(\int_{\mathbb{R}^2} \psi(\mathbf{x}-\mathbf{y})\, \xi(\mathbf{y})\, \mathbf{dy}\right) \delta(\mathbf{x}\cdot\vartheta - t)\, e^{\mu \mathbf{x}\cdot\vartheta^\perp} \mathbf{dx} \\ &\quad = \int_{\mathbb{R}^2} \left(\int_{\mathbb{R}^2} \psi(\mathbf{x}-\mathbf{y})\, \delta(\mathbf{x}\cdot\vartheta - t)\, e^{\mu \mathbf{x}\cdot\vartheta^\perp} \mathbf{dx}\right) \xi(\mathbf{y})\, \mathbf{dy}. \end{aligned} \tag{27}$$

The substitution $\mathbf{x}-\mathbf{y} = \mathbf{z}$ implies $\mathbf{x} = \mathbf{y}+\mathbf{z}$ and $\mathbf{dx} = \mathbf{dz}$. Thus we get

$$\begin{aligned} &\mathbf{T}^e_\mu(\psi * \xi)(\vartheta, t) \\ &\quad = \int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \psi(\mathbf{z})\, \delta(\mathbf{z}\cdot\vartheta - (t - \mathbf{y}\vartheta))\, e^{\mu \mathbf{z}\cdot\vartheta^\perp} e^{\mu \mathbf{y}\cdot\vartheta^\perp} \mathbf{dz}\xi(\mathbf{y})\, \mathbf{dy} \\ &\quad = \int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \mathbf{T}^e_\mu \psi(t - \mathbf{y}\cdot\vartheta)\, e^{\mu \mathbf{y}\cdot\vartheta^\perp} \xi(\mathbf{y})\, \mathbf{dy} \\ &\quad = \left(\mathbf{T}^e_\mu \psi \times \xi\right)(\vartheta, t). \end{aligned} \tag{28}$$

This completes the proof of the theorem. □

4. The Exponential Radon Transform of Boehmians

Definition 10. Let $\beta_n = [\psi_n/\mu_n] \in \boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), *, \Delta)$; then we define its exponential Radon transform as the mapping

$$\overrightarrow{\mathbf{T}^e_\mu}\left[\frac{\psi_n}{\mu_n}\right] = \left[\frac{\mathbf{T}^e_\mu \psi_n}{\mu_n}\right] \tag{29}$$

in the space $\boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), \times, \Delta)$.

Definition 10 is well defined by Theorem 9.

To show that (29) is well defined, let $[\psi_n/\mu_n], [\xi_n/\epsilon_n] \in \boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), *, \Delta)$ and $[\psi_n/\mu_n] = [\xi_n/\epsilon_n]$; then

$$\psi_n * \epsilon_m = \xi_m * \mu_n. \tag{30}$$

Employing $\mathbf{T}^e_\mu$ for (30) and using Theorem 9 imply that

$$\mathbf{T}^e_\mu \psi_n \times \epsilon_m = \mathbf{T}^e_\mu \xi_m \times \mu_n. \tag{31}$$

From (31) we see that $\mathbf{T}^e_\mu \psi_n/\mu_n \sim \mathbf{T}^e_\mu \xi_n/\epsilon_n$ in the sense of $\boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), \times, \Delta)$.

This completes the proof of the theorem.

Theorem 11. *Let $\beta_1, \beta_2 \in \boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), *, \Delta)$; then $\overrightarrow{\mathbf{T}^e_\mu}(\beta_1 * \beta_1) = \overrightarrow{\mathbf{T}^e_\mu}\beta_1 \times \beta_2$.*

Proof. Assume the requirements of the theorem are satisfied for some $\beta_1, \beta_2 \in \boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), *, \Delta)$; then there are $(f_n), (\kappa_n) \in \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$ and $(\varphi_n), (\phi_n) \in \Delta$ such that $\beta_1 = [(f_n)/(\varphi_n)]$ and $\beta_2 = [(\kappa_n)/(\phi_n)]$. Therefore, we write

$$\begin{aligned} &\overrightarrow{\mathbf{T}^e_\mu}(\beta_1 * \beta_2) \\ &\quad = \overrightarrow{\mathbf{T}^e_\mu}\left(\left[\frac{(f_n) * (\kappa_n)}{(\varphi_n) * (\phi_n)}\right]\right) = \left[\frac{\mathbf{T}^e_\mu((f_n) * (\kappa_n))}{(\varphi_n) * (\phi_n)}\right] \\ &\quad = \left[\frac{\left(\mathbf{T}^e_\mu f_n\right) \times (\kappa_n)}{(\varphi_n) \times (\phi_n)}\right] = \left[\frac{\left(\mathbf{T}^e_\mu f_n\right)}{(\varphi_n)}\right] \times \left[\frac{(\kappa_n)}{(\phi_n)}\right]. \end{aligned} \tag{32}$$

Thus we get that $\overrightarrow{\mathbf{T}^e_\mu}(\beta_1 * \beta_2) = \overrightarrow{\mathbf{T}^e_\mu}(\beta_1) \times \beta_2$.

This completes the proof. □

Theorem 12. *$\overrightarrow{\mathbf{T}^e_\mu}$ defines a linear mapping from $\boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), *, \Delta)$ into $\boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), \times, \Delta)$.*

The proof is straightforward.

Definition 13. Let $\beta \in \boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), \times, \Delta)$ be such that $\beta = [(\mathbf{T}^e_\mu f_n)/(\phi_n)]$. Then we define the inverse transform of $\mathcal{T}^e_\mu$ as

$$\left(\overrightarrow{\mathbf{T}^e_\mu}\right)^{-1}\left[\frac{\left(\mathbf{T}^e_\mu f_n\right)}{(\phi_n)}\right] = \left[\frac{(f_n)}{(\phi_n)}\right] \tag{33}$$

for each $(\phi_n) \in \Delta$.

Theorem 14. *$\overrightarrow{\mathbf{T}^e_\mu}$ defines an isomorphism from $\boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), *, \Delta)$ onto $\boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), \times, \Delta)$.*

Proof. Assume that $[(\mathbf{T}^e_\mu f_n)/(\phi_n)] = [(\mathbf{T}^e_\mu g_n)/(\psi_n)]$ in $\boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), \times, \Delta)$. Using (29) and Theorem 9 we get $\mathbf{T}^e_\mu f_n \times \psi_m = \mathbf{T}^e_\mu g_m \times \phi_n$. Once again, Theorem 9 implies $\mathbf{T}^e_\mu(f_n * \psi_m) = \mathbf{T}^e_\mu(g_m * \phi_n)$. Hence $f_n * \psi_m = g_m * \phi_n$. Therefore, $[(f_n)/(\phi_n)] = [(g_n)/(\psi_n)]$.

Now, let $[(\mathbf{T}^e_\mu f_n)/(\phi_n)] \in \boldsymbol{\delta}(\mathbf{l}^1,(\boldsymbol{\kappa},*),\times,\Delta)$; then $\mathbf{T}^e_\mu f_n \times \phi_m = \mathbf{T}^e_\mu f_m \times \phi_n$, $\forall m,n \in \mathbb{N}$. Theorem 9 leads to $\mathbf{T}^e_\mu(f_n * \phi_m) = \mathbf{T}^e_\mu(f_m * \phi_n)$. Hence $[(f_n)/(\phi_n)] \in \boldsymbol{\delta}(\mathbf{l}^1,(\boldsymbol{\kappa},*),*,\Delta)$ is the Boehmian that satisfies $\overrightarrow{\mathbf{T}^e_\mu}[(f_n)/(\phi_n)] = [(\mathbf{T}^e_\mu f_n)/(\phi_n)]$.

This completes the proof of the theorem. □

Theorem 15. *Let* $[(f_n)/(\phi_n)] \in \boldsymbol{\delta}(\mathbf{l}^1,(\boldsymbol{\kappa},*),*,\Delta)$ *and* $\phi \in \boldsymbol{\kappa}(\mathbb{R}^2)$*; then*

$$\overrightarrow{\mathbf{T}^e_\mu}\left(\left[\frac{(f_n)}{(\phi_n)}\right] * \phi\right) = \left[\frac{(\mathbf{T}^e_\mu f_n)}{(\phi_n)}\right] \times \phi. \quad (34)$$

Proof. Applying Definition 10 for each $[(f_n)/(\phi_n)] \in \boldsymbol{\delta}(\mathbf{l}^1,(\boldsymbol{\kappa},*),\times,\Delta)$ and $\phi \in \boldsymbol{\kappa}(\mathbb{R}^2)$ yields

$$\left(\overrightarrow{\mathbf{T}^e_\mu}\right)\left(\left[\frac{(f_n)}{(\phi_n)}\right] * \phi\right) = \left[\frac{(\mathbf{T}^e_\mu)((f_n) * \phi)}{(\phi_n)}\right]. \quad (35)$$

By Theorem 9 we get

$$\left(\overrightarrow{\mathbf{T}^e_\mu}\right)\left(\left[\frac{(f_n)}{(\phi_n)}\right] * \phi\right) = \left[\frac{(f_n)\times\phi}{(\phi_n)}\right] = \left[\frac{(f_n)}{(\phi_n)}\right] \times \phi. \quad (36)$$

This completes the proof of the theorem. □

Theorem 16. *The mappings* $\overrightarrow{\mathbf{T}^e_\mu}$ *and* $(\overrightarrow{\mathbf{T}^e_\mu})^{-1}$ *are continuous with respect to* δ *and* Δ *convergence.*

Proof. First of all, we show that $\overrightarrow{\mathbf{T}^e_\mu}$ and $(\overrightarrow{\mathbf{T}^e_\mu})^{-1}$ are continuous with respect to δ convergence.

Let $\beta_n \xrightarrow{\delta} \beta$ in $\boldsymbol{\delta}(\mathbf{l}^1,(\boldsymbol{\kappa},*),*,\Delta)$ as $n \to \infty$; then we show that $\overrightarrow{\mathbf{T}^e_\mu}\beta_n \to \overrightarrow{\mathbf{T}^e_\mu}\beta$ as $n \to \infty$. By virtue of Theorem 1 we can find $f_{n,k}$ and f_k in $\mathbf{l}^1(\mathbb{R}^2)$ such that $\beta_n = [f_{n,k}/\phi_k]$ and $\beta = [f_k/\phi_k]$ such that $f_{n,k} \to f_k$ as $n \to \infty$ for every $k \in \mathbb{N}$.

Employing the continuity condition of $\mathbf{T}^e_\mu$ transform implies $\mathbf{T}^e_\mu f_{n,k} \to \mathbf{T}^e_\mu f_k$ as $n \to \infty$ in the space $\mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$.

Thus,

$$\left[\frac{\mathbf{T}^e_\mu f_{n,k}}{\phi_k}\right] \longrightarrow \left[\frac{\mathbf{T}^e_\mu f_k}{\phi_k}\right] \quad \text{as } n \longrightarrow \infty \quad (37)$$

in $\boldsymbol{\delta}(\mathbf{l}^1,(\boldsymbol{\kappa},*),\times,\Delta)$.

To prove the second part, let $g_n \xrightarrow{\delta} g$ in $\boldsymbol{\delta}(\mathbf{l}^1,(\boldsymbol{\kappa},*),\times,\Delta)$ as $n \to \infty$. Then, once again, by Theorem 1, $g_n = [\mathbf{T}^e_\mu f_{n,k}/\phi_k]$ and $g = [\mathbf{T}^e_\mu f_k/\phi_k]$ and $\mathbf{T}^e_\mu f_{n,k} \to \mathbf{T}^e_\mu f_k$ as $n \to \infty$. Hence $f_{n,k} \to f_k$ in $\boldsymbol{\delta}(\mathbf{l}^1,(\boldsymbol{\kappa},*),*,\Delta)$ as $n \to \infty$. That is, $[f_{n,k}/\phi_k] \to [f_k/\phi_k]$ as $n \to \infty$. Using (33) we get

$$\left(\overrightarrow{\mathbf{T}^e_\mu}\right)^{-1}\left[\frac{\mathbf{T}^e_\mu f_{n,k}}{\phi_k}\right] \longrightarrow \left(\overrightarrow{\mathbf{T}^e_\mu}\right)^{-1}\left[\frac{\mathbf{T}^e_\mu f_k}{\phi_k}\right] \quad \text{as } n \longrightarrow \infty. \quad (38)$$

Now, we establish continuity of $\overrightarrow{\mathbf{T}^e_\mu}$ and $(\overrightarrow{\mathbf{T}^e_\mu})^{-1}$ with respect to Δ convergence.

Let $\beta_n, \beta \in \boldsymbol{\delta}(\mathbf{l}^1,(\boldsymbol{\kappa},*),*,\Delta)$ be such that $\beta_n \xrightarrow{\Delta} \beta$ as $n \to \infty$. Then, by Theorem 1 we can find that $f_n \in \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$ and $(\phi_n) \in \Delta$ such that $(\beta_n - \beta) * \phi_n = [((f_n) * \phi_k)/\phi_k]$ and $f_n \to 0$ as $n \to \infty$. Employing (29) we get

$$\overrightarrow{\mathbf{T}^e_\mu}((\beta_n - \beta) * \phi_n) = \left[\frac{\mathbf{T}^e_\mu((f_n) * \phi_k)}{\phi_k}\right]. \quad (39)$$

Hence, we have $\overrightarrow{\mathbf{T}^e_\mu}((\beta_n - \beta) * \phi_n) = [((\mathbf{T}^e_\mu f_n) \times \phi_k)/\phi_k] = \mathbf{T}^e_\mu f_n \to 0$ as $n \to \infty$ in $\mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$.

Therefore

$$\overrightarrow{\mathbf{T}^e_\mu}((\beta_n - \beta) * \phi_n) = \left(\overrightarrow{\mathbf{T}^e_\mu}\beta_n - \overrightarrow{\mathbf{T}^e_\mu}\beta\right) \times \phi_n \Longrightarrow \text{as } n \longrightarrow \infty. \quad (40)$$

Hence, $\overrightarrow{\mathbf{T}^e_\mu}\beta_n \xrightarrow{\Delta} \overrightarrow{\mathbf{T}^e_\mu}\beta$ as $n \to \infty$.

Finally, let $g_n \xrightarrow{\Delta} g$ in $\boldsymbol{\delta}(\mathbf{l}^1,(\boldsymbol{\kappa},*),*,\Delta)$ as $n \to \infty$; then we find $\mathbf{T}^e_\mu f_k \in \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$ such that $(g_n - g) \times \phi_n = [(\mathbf{T}^e_\mu f_k \times \phi_k)/\phi_k]$ and $\mathbf{T}^e_\mu f_k \to 0$ as $n \to \infty$ for some $(\phi_n) \in \Delta$.

Now, using (33), we obtain

$$\left(\overrightarrow{\mathbf{T}^e_\mu}\right)^{-1}((g_n - g) \times \phi_n) = \left[\frac{(\mathbf{T}^e_\mu)^{-1}(\mathbf{T}^e_\mu f_k \times \phi_k)}{\phi_k}\right]. \quad (41)$$

Theorem 9 implies

$$\left(\overrightarrow{\mathbf{T}^e_\mu}\right)^{-1}((g_n - g) \times \phi_n) = \left[\frac{(f_n) * \phi_k}{\phi_k}\right] = f_n \longrightarrow 0 \in \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R}) \quad \text{as } n \longrightarrow \infty. \quad (42)$$

Thus

$$\left(\overrightarrow{\mathbf{T}^e_\mu}\right)^{-1}((g_n - g) \times \phi_n) = \left(\left(\overrightarrow{\mathbf{T}^e_\mu}\right)^{-1} g_n - \left(\overrightarrow{\mathbf{T}^e_\mu}\right)^{-1} g\right) * \phi_n \to 0 \in \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R}) \quad \text{as } n \longrightarrow \infty. \quad (43)$$

From this we find that $(\overrightarrow{\mathbf{T}^e_\mu})^{-1} g_n \xrightarrow{\Delta} (\overrightarrow{\mathbf{T}^e_\mu})^{-1} g \in \boldsymbol{\delta}(\mathbf{l}^1,(\boldsymbol{\kappa},*),*,\Delta)$ as $n \to \infty$.

This completes the proof of the theorem. □

Theorem 17. *The transform* $\overrightarrow{\mathbf{T}^e_\mu}$ *is consistent with* $\overrightarrow{\mathbf{T}^e_\mu} : \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R}) \to \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$.

Proof. For every $f \in \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$, let $\beta \in \boldsymbol{\delta}(\mathbf{l}^1,(\boldsymbol{\kappa},*),*,\Delta)$ be its representative; then $\forall n \in \mathbb{N}$, $(\varphi_n) \in \Delta$, $\beta = [(f * (\varphi_n))/(\varphi_n)]$. For all $n \in \mathbb{N}$ it is clear that (φ_n) is independent of the representative.

We have

$$\overrightarrow{\mathbf{T}_{\mu}^{e}}(\beta) = \overrightarrow{\mathbf{T}_{\mu}^{e}}\left(\left[\frac{f * (\varphi_n)}{(\varphi_n)}\right]\right) = \left[\frac{\mathbf{T}_{\mu}^{e}(f * (\varphi_n))}{(\varphi_n)}\right] = \left[\frac{\mathbf{T}_{\mu}^{e} f \times (\varphi_n)}{(\varphi_n)}\right] \quad (44)$$

which is the representative of $\mathbf{T}_{\mu}^{e} f$ in the space $\mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$.

Hence the proof is completed. □

Theorem 18. *The necessary and sufficient condition for* $[(g_n)/(\psi_n)] \in \boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), \times, \Delta)$ *to be in the range of* $\overrightarrow{\mathbf{T}_{\mu}^{e}}$ *is that* g_n *belongs to range of* $\mathbf{T}_{\mu}^{e}$ *for every* $n \in \mathbb{N}$.

Proof. Let $[(g_n)/(\psi_n)]$ be in the range of $\overrightarrow{\mathbf{T}_{\mu}^{e}}$; then of course g_n belongs to the range of $\mathbf{T}_{\mu}^{e}$, $\forall n \in \mathbb{N}$.

To establish the converse, let g_n be in the range of $\mathbf{T}_{\mu}^{e}$, $\forall n \in \mathbb{N}$. Then there is $f_n \in \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$ such that $\mathbf{T}_{\mu}^{e} f_n = g_n, n \in \mathbb{N}$.

Since $[(g_n)/(\psi_n)] \in \boldsymbol{\delta}(\mathbf{l}^1, (\boldsymbol{\kappa}, *), \times, \Delta)$ we get $g_n \times \psi_m = g_m \times \psi_n$, $\forall m, n \in \mathbb{N}$.

Therefore, Theorem 9 yields

$$\mathbf{T}_{\mu}^{e}(f_n * \varphi_n) = \mathbf{T}_{\mu}^{e}(f_m * \varphi_n), \quad \forall m, n \in \mathbb{N}, \quad (45)$$

where $f_n \in \mathbf{l}^1(\mathcal{S}^1 \times \mathbb{R})$ and $\varphi_n \in \Delta$, $\forall n \in \mathbb{N}$.

Thus $f_n * \varphi_m = f_m * \varphi_n$, $m, n \in \mathbb{N}$. Hence,

$$\left[\frac{(f_n)}{(\varphi_n)}\right] \in \boldsymbol{\delta}\left(\mathbf{l}^1, (\boldsymbol{\kappa}, *), *, \Delta\right), \qquad \overrightarrow{\mathbf{T}_{\mu}^{e}}\left(\left[\frac{(f_n)}{(\varphi_n)}\right]\right) = \left[\frac{(g_n)}{(\psi_n)}\right]. \quad (46)$$

The theorem is therefore completely proved. □

Conflict of Interests

The authors declare that they have no conflict of interests regarding publication of this paper.

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