

Research Article

Oscillation Theorems for Second-Order Nonlinear Neutral Delay Differential Equations

Tongxing Li¹ and Yuriy V. Rogovchenko²

¹Department of Mathematics, Linyi University, Linyi, Shandong 276005, China

²Department of Mathematical Sciences, University of Agder, P.O. Box 422, 4604 Kristiansand, Norway

Correspondence should be addressed to Yuriy V. Rogovchenko; yuriy.rogovchenko@uia.no

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We analyze the oscillatory behavior of solutions to a class of second-order nonlinear neutral delay differential equations. Our theorems improve a number of related results reported in the literature.

1. Introduction

In this paper, we study the oscillatory behavior of a class of second-order nonlinear neutral delay differential equations

$$(r(t)|z'(t)|^{\alpha-1}z'(t))' + q(t)f(x(\sigma(t))) = 0, \quad (1)$$

where $t \in I := [t_0, \infty)$, $t_0 > 0$, $z(t) := x(t) + p(t)x(\tau(t))$, and $\alpha > 0$ is a constant. We assume that the following conditions hold:

- (A₁) $r, p, q \in C(I, \mathbb{R})$, $r(t) > 0$, $p(t) \geq 0$, $q(t) \geq 0$, and $q(t)$ is not identically zero for large t ;
- (A₂) $f \in C(\mathbb{R}, \mathbb{R})$, $uf(u) > 0$, for all $u \neq 0$, and there exists a positive constant k such that

$$\frac{f(u)}{|u|^{\alpha-1}u} \geq k, \quad \forall u \neq 0; \quad (2)$$

- (A₃) $\sigma \in C(I, \mathbb{R})$, $\sigma(t) \leq t$, and $\lim_{t \rightarrow \infty} \sigma(t) = \infty$;
- (A₄) $\tau \in C^1(I, \mathbb{R})$, $\tau(t) \leq t$, and $\lim_{t \rightarrow \infty} \tau(t) = \infty$;
- (A₅) $\tau'(t) \geq \tau_0 > 0$ and $\tau \circ \sigma = \sigma \circ \tau$.

By a solution of (1) we mean a function $x \in C([T_x, \infty), \mathbb{R})$, $T_x \geq t_0$, such that $r|z'|^{\alpha-1}z' \in C^1([T_x, \infty), \mathbb{R})$ and x satisfies (1) on $[T_x, \infty)$. We consider only those solutions of (1) which satisfy $\sup\{|x(t)| : t \geq T\} > 0$, for all $T \geq T_x$, and assume that (1) possesses such solutions. As

customary, we say that a solution of (1) is oscillatory if it has arbitrarily large zeros on the interval $[T_x, \infty)$; otherwise, it is called nonoscillatory. Equation (1) is termed oscillatory if all its solutions are oscillatory.

An increasing interest in oscillation of solutions to functional differential equations during the last few decades has been stimulated by applications arising in engineering and natural sciences; see Hale [1]. This resulted in publication of several monographs [2–5] and numerous research articles [6–20]; see also the references cited there. Prior to presenting our oscillation criteria, we briefly comment on a number of closely related results for (1) and its particular cases which motivated the present study. In the sequel, the following notation is frequently used:

$$f_+(t) := \max\{0, f(t)\}, \quad Q(t) := \min\{q(t), q(\tau(t))\}. \quad (3)$$

Grace and Lalli [10] studied a second-order nonlinear neutral delay differential equation

$$(r(t)(x(t) + p(t)x(t-\tau)))' + q(t)f(x(t-\sigma)) = 0 \quad (4)$$

under the assumptions that

$$0 \leq p(t) < 1, \quad \frac{f(u)}{u} \geq k > 0, \quad \forall u \neq 0, \quad (5)$$

$$\int_{t_0}^{\infty} r^{-1}(t) dt = \infty.$$

They proved that (4) is oscillatory if there exists a function $\rho \in C^1(I, (0, \infty))$ such that

$$\int_{t_0}^{\infty} \left[\rho(t) q(t) (1 - p(t - \sigma)) - \frac{(\rho'(t))^2 r(t - \sigma)}{4k\rho(t)} \right] dt = \infty. \tag{6}$$

Hasanbulli and Rogovchenko [11] obtained several oscillation criteria for a nonlinear neutral differential equation

$$(r(t)(x(t) + p(t)x(t - \tau)))' + q(t)f(x(t), x(\sigma(t))) = 0 \tag{7}$$

in the case where $0 \leq p(t) < 1$. Ye and Xu [18, Theorem 2.1] proved the following result for (1).

Theorem 1. *Suppose that $0 \leq p(t) < 1$, $\sigma \in C^1(I, \mathbb{R})$, and $\sigma'(t) > 0$. Assume also that conditions (A_1) – (A_4) are satisfied and*

$$\int_{t_0}^{\infty} r^{-1/\alpha}(t) dt = \infty. \tag{8}$$

If there exists a function $\rho \in C^1(I, (0, \infty))$ such that

$$\int_{t_0}^{\infty} \left[\rho(t) q(t) (1 - p(\sigma(t)))^\alpha - \frac{1}{k(\alpha + 1)^{\alpha+1}} \frac{(\rho'_+(t))^{\alpha+1} r(\sigma(t))}{(\rho(t)\sigma'(t))^\alpha} \right] dt = \infty, \tag{9}$$

then (1) is oscillatory.

In a special case $f(u) := |u|^{\alpha-1}u$, (1) reduces to a quasilinear neutral differential equation

$$(r(t)|z'(t)|^{\alpha-1}z'(t))' + q(t)|x(\sigma(t))|^{\alpha-1}x(\sigma(t)) = 0. \tag{10}$$

Equation (10) was studied by Sun et al. [17] and Zhong et al. [20] who established the following results.

Theorem 2 (see [17, Theorem 3.4]). *Suppose that $\alpha \geq 1$, $\sigma(t) \geq \tau(t)$, and $0 \leq p(t) \leq p_0 < \infty$. Assume also that conditions (A_1) , (A_3) – (A_5) , and (8) are satisfied. If there exists a function $\rho \in C^1(I, (0, \infty))$ such that*

$$\int_{t_0}^{\infty} \left[\frac{\rho(t)Q(t)}{2^{\alpha-1}} - \frac{1 + (p_0^\alpha/\tau_0)}{(\alpha + 1)^{\alpha+1}} \frac{(\rho'_+(t))^{\alpha+1} r(\tau(t))}{(\tau_0\rho(t))^\alpha} \right] dt = \infty, \tag{11}$$

then (10) is oscillatory.

Theorem 3 (see [20, Theorem 3.1]). *Assume that $r'(t) \geq 0$, $\tau(t) = t - \tau \leq t$, $0 \leq p(t) = p_0 < \infty$, $p_0 \neq 1$, $\sigma \in C^1(I, \mathbb{R})$, and $\sigma'(t) > 0$. Suppose also that conditions (A_1) , (A_3) , and*

(8) are satisfied. If there exist an $\varepsilon \in (0, 1)$ and a function $\rho \in C^1(I, (0, \infty))$ such that

$$\int_{t_0}^{\infty} \left[\frac{(1 - \varepsilon)^\alpha}{(1 + p_0(1 + \varepsilon))^\alpha} \rho(t) q(t) - \frac{1}{(\alpha + 1)^{\alpha+1}} \frac{(\rho'_+(t))^{\alpha+1} r(\sigma(t))}{(\rho(t)\sigma'(t))^\alpha} \right] dt = \infty, \tag{12}$$

then (10) is oscillatory.

The purpose of this note is to refine Theorems 1–3 in some cases. In what follows, all functional inequalities are assumed to hold for all t large enough. Without loss of generality, we can deal only with positive solutions of (1).

2. Main Results

For a more compact presentation of conditions in our results, we use the notation

$$R(l, t) := \left(\int_l^{\sigma(t)} r^{-1/\alpha}(s) ds \right) \left(\int_l^t r^{-1/\alpha}(s) ds \right)^{-1}. \tag{13}$$

Theorem 4. *Let $0 < \alpha \leq 1$ and $0 \leq p(t) \leq p_0 < \infty$. Assume also that conditions (A_1) – (A_5) and (8) are satisfied. If there exists a function $\rho \in C^1(I, (0, \infty))$ such that*

$$\int_{t_{**}}^{\infty} \left[\rho(t) Q(t) R^\alpha(t_*, t) - \frac{(\rho'_+(t))^{\alpha+1}}{k(\alpha + 1)^{\alpha+1} \rho^\alpha(t)} \left(r(t) + \frac{p_0^\alpha r(\tau(t))}{\tau_0^{\alpha+1}} \right) \right] dt = \infty, \tag{14}$$

for all sufficiently large t_* and for some $t_{**} \geq t_* \geq t_0$, then (1) is oscillatory.

Proof. Let $x(t)$ be a nonoscillatory solution of (1); we assume that it is eventually positive. Then there exists a $t_1 \geq t_0$ such that $x(t) > 0$, $x(\tau(t)) > 0$, and $x(\sigma(t)) > 0$, for all $t \geq t_1$. It follows from (1) that

$$(r(t)|z'(t)|^{\alpha-1}z'(t))' \leq -kq(t)x^\alpha(\sigma(t)) \leq 0, \tag{15}$$

$$\forall t \geq t_1.$$

Using condition (8), we conclude that there exists a $t_2 \geq t_1$ such that $z'(t) > 0$, for all $t \geq t_2$. Hence, for all $t \geq t_2$, inequality (15) reduces to

$$(r(t)(z'(t))^\alpha)' \leq -kq(t)x^\alpha(\sigma(t)) \leq 0, \tag{16}$$

and there exists a $t_3 \geq t_2$ such that, for all $t \geq t_3$,

$$p_0^\alpha \frac{(r(\tau(t))(z'(\tau(t)))^\alpha)'}{\tau'(t)} \leq -kp_0^\alpha q(\tau(t))x^\alpha(\sigma(\tau(t))). \tag{17}$$

Using the assumption $\tau'(t) \geq \tau_0 > 0$, we have, for all $t \geq t_3$,

$$\frac{p_0^\alpha}{\tau_0} (r(\tau(t)) (z'(\tau(t)))^\alpha)' \leq -k p_0^\alpha q(\tau(t)) x^\alpha(\sigma(\tau(t))). \tag{18}$$

Combining inequalities (16) and (18), using the condition $\tau \circ \sigma = \sigma \circ \tau$ and an auxiliary result due to Baculiková and Džurina [7, Lemma 2], we conclude that

$$\begin{aligned} & (r(t) (z'(t))^\alpha)' + \frac{p_0^\alpha}{\tau_0} (r(\tau(t)) (z'(\tau(t)))^\alpha)' \\ & \leq -k [q(t) x^\alpha(\sigma(t)) + p_0^\alpha q(\tau(t)) x^\alpha(\tau(\sigma(t)))] \\ & \leq -k \min\{q(t), q(\tau(t))\} [x^\alpha(\sigma(t)) + p_0^\alpha x^\alpha(\tau(\sigma(t)))] \\ & \leq -k Q(t) z^\alpha(\sigma(t)), \end{aligned} \tag{19}$$

for all $t \geq t_3$. Define a new function $\omega(t)$ by

$$\omega(t) := \rho(t) \frac{r(t) (z'(t))^\alpha}{z^\alpha(t)}. \tag{20}$$

Then $\omega(t) > 0$, for all $t \geq t_3$. Differentiation of (20) yields

$$\begin{aligned} \omega'(t) &= \rho'(t) \frac{r(t) (z'(t))^\alpha}{z^\alpha(t)} + \rho(t) \frac{(r(t) (z'(t))^\alpha)'}{z^\alpha(t)} \\ &\quad - \alpha \rho(t) \frac{r(t) (z'(t))^{\alpha+1}}{z^{\alpha+1}(t)} \leq \rho(t) \frac{(r(t) (z'(t))^\alpha)'}{z^\alpha(t)} \\ &\quad + \frac{\rho'_+(t)}{\rho(t)} \omega(t) - \frac{\alpha}{(\rho(t) r(t))^{1/\alpha}} \omega^{(\alpha+1)/\alpha}(t). \end{aligned} \tag{21}$$

Let

$$A := \frac{\rho'_+(t)}{\rho(t)}, \quad B := \frac{\alpha}{(\rho(t) r(t))^{1/\alpha}}, \quad u := \omega(t). \tag{22}$$

Using the inequality

$$Au - Bu^{(\alpha+1)/\alpha} \leq \frac{\alpha^\alpha}{(\alpha+1)^{\alpha+1}} \frac{A^{\alpha+1}}{B^\alpha}, \quad B > 0, \tag{23}$$

we deduce from (21) that

$$\omega'(t) \leq \rho(t) \frac{(r(t) (z'(t))^\alpha)'}{z^\alpha(t)} + \frac{1}{(\alpha+1)^{\alpha+1}} \frac{(\rho'_+(t))^{\alpha+1} r(t)}{\rho^\alpha(t)}. \tag{24}$$

Define another function $\nu(t)$ by

$$\nu(t) := \rho(t) \frac{r(\tau(t)) (z'(\tau(t)))^\alpha}{z^\alpha(\tau(t))}. \tag{25}$$

Observe that $\nu(t) > 0$, for all $t \geq t_3$. Differentiation of (25) yields

$$\begin{aligned} \nu'(t) &= \rho'(t) \frac{r(\tau(t)) (z'(\tau(t)))^\alpha}{z^\alpha(\tau(t))} \\ &\quad + \rho(t) \frac{(r(\tau(t)) (z'(\tau(t)))^\alpha)'}{z^\alpha(\tau(t))} \\ &\quad - \alpha \rho(t) \frac{r(\tau(t)) (z'(\tau(t)))^\alpha z'(\tau(t)) \tau'(t)}{z^{\alpha+1}(\tau(t))} \\ &\leq \rho(t) \frac{(r(\tau(t)) (z'(\tau(t)))^\alpha)'}{z^\alpha(\tau(t))} \\ &\quad + \frac{\rho'_+(t)}{\rho(t)} \nu(t) - \frac{\alpha \tau'(t)}{(\rho(t) r(\tau(t)))^{1/\alpha}} \nu^{(\alpha+1)/\alpha}(t). \end{aligned} \tag{26}$$

Let

$$A := \frac{\rho'_+(t)}{\rho(t)}, \quad B := \frac{\alpha \tau'(t)}{(\rho(t) r(\tau(t)))^{1/\alpha}}, \quad u := \nu(t). \tag{27}$$

Using the inequalities (23) and (26) along with the fact that $z'(t) > 0$, we have

$$\begin{aligned} \nu'(t) &\leq \rho(t) \frac{(r(\tau(t)) (z'(\tau(t)))^\alpha)'}{z^\alpha(\tau(t))} \\ &\quad + \frac{1}{(\alpha+1)^{\alpha+1}} \frac{(\rho'_+(t))^{\alpha+1} r(\tau(t))}{(\rho(t) \tau'(t))^\alpha}. \end{aligned} \tag{28}$$

Combining (24) and (28) and using the inequality (19), we obtain

$$\begin{aligned} & \omega'(t) + \frac{p_0^\alpha}{\tau_0} \nu'(t) \\ & \leq \rho(t) \frac{(r(t) (z'(t))^\alpha)'}{z^\alpha(t)} + \frac{p_0^\alpha}{\tau_0} \frac{(r(\tau(t)) (z'(\tau(t)))^\alpha)'}{z^\alpha(\tau(t))} \\ & \quad + \frac{(\rho'_+(t))^{\alpha+1}}{(\alpha+1)^{\alpha+1} \rho^\alpha(t)} \left(r(t) + \frac{p_0^\alpha r(\tau(t))}{\tau_0^{\alpha+1}} \right) \\ & \leq -k \rho(t) Q(t) \frac{z^\alpha(\sigma(t))}{z^\alpha(t)} \\ & \quad + \frac{(\rho'_+(t))^{\alpha+1}}{(\alpha+1)^{\alpha+1} \rho^\alpha(t)} \left(r(t) + \frac{p_0^\alpha r(\tau(t))}{\tau_0^{\alpha+1}} \right). \end{aligned} \tag{29}$$

Since $(r(t) (z'(t))^\alpha)' \leq 0$, we have

$$z(t) \geq r^{1/\alpha}(t) z'(t) \int_{t_2}^t r^{-1/\alpha}(s) ds, \tag{30}$$

and thus

$$\left(\frac{z(t)}{\int_{t_2}^t r^{-1/\alpha}(s)ds} \right)' \leq 0. \tag{31}$$

Consequently,

$$\frac{z^\alpha(\sigma(t))}{z^\alpha(t)} \geq R^\alpha(t_2, t). \tag{32}$$

Substitution of (32) in (29) yields

$$\begin{aligned} \omega'(t) + \frac{p_0^\alpha}{\tau_0} \nu'(t) &\leq -k\rho(t)Q(t)R^\alpha(t_2, t) \\ &+ \frac{(\rho'_+(t))^{\alpha+1}}{(\alpha+1)^{\alpha+1}\rho^\alpha(t)} \left(r(t) + \frac{p_0^\alpha r(\tau(t))}{\tau_0^{\alpha+1}} \right). \end{aligned} \tag{33}$$

Integrating (33) from t_3 to t , we have

$$\begin{aligned} &\int_{t_3}^t \left[\rho(s)Q(s)R^\alpha(t_2, s) \right. \\ &\quad \left. - \frac{(\rho'_+(s))^{\alpha+1}}{k(\alpha+1)^{\alpha+1}\rho^\alpha(s)} \left(r(s) + \frac{p_0^\alpha r(\tau(s))}{\tau_0^{\alpha+1}} \right) \right] ds \\ &\leq \frac{\omega(t_3)}{k} + \frac{p_0^\alpha}{k\tau_0} \nu(t_3). \end{aligned} \tag{34}$$

Passing in (34) to the limit as $t \rightarrow \infty$, we obtain contradiction with condition (14). Therefore, (1) is oscillatory. \square

Proceeding as in the proof of Theorem 4 and using another result by Baculiková and Džurina [7, Lemma 1], we obtain the following oscillation criterion for (1), for $\alpha \geq 1$.

Theorem 5. *Assume that $\alpha \geq 1$ and $0 \leq p(t) \leq p_0 < \infty$. Let conditions (A_1) – (A_5) and (8) be satisfied. If there exists a function $\rho \in C^1(I, (0, \infty))$ such that*

$$\begin{aligned} &\int_{t_{**}}^\infty \left[2^{1-\alpha} \rho(t)Q(t)R^\alpha(t_*, t) \right. \\ &\quad \left. - \frac{(\rho'_+(t))^{\alpha+1}}{k(\alpha+1)^{\alpha+1}\rho^\alpha(t)} \left(r(t) + \frac{p_0^\alpha r(\tau(t))}{\tau_0^{\alpha+1}} \right) \right] dt = \infty, \end{aligned} \tag{35}$$

for all sufficiently large t_* and for some $t_{**} \geq t_* \geq t_0$, then (1) is oscillatory.

3. Examples and Discussion

Example 1. For $t \geq 1$, consider a second-order neutral differential equation

$$\left(x(t) + \frac{1}{3}x(t-2) \right)'' + \frac{\gamma}{t^2}x(t) = 0, \tag{36}$$

where $\gamma > 0$ is a constant. We have $\alpha = 1$, $r(t) = 1$, $p(t) = p_0 = 1/3$, $\tau(t) = t - 2$, $q(t) = \gamma/t^2$, $\sigma(t) = t$, $f(u) = u$, and $k = 1$. Choose $\rho(t) = t$ and denote the left hand side of (14) by $\psi(t_{**})$. Then

$$\psi(t_{**}) = \left(\gamma - \frac{1}{3} \right) \int_{t_{**}}^\infty \frac{dt}{t} = \infty, \quad \text{provided that } \gamma > \frac{1}{3}. \tag{37}$$

Hence, (36) is oscillatory by Theorem 4 for any $\gamma > 1/3$. On the other hand, an application of Theorem 1 yields oscillation of (36) for $\gamma > 3/8$, whereas Theorem 3 implies that (36) is oscillatory if $\gamma > (4 + \varepsilon)/(3(4 - 4\varepsilon))$, for some $\varepsilon \in (0, 1)$. Therefore, we observe that our Theorem 4 improves Theorems 1 and 3.

Example 2. For $t \geq 1$, consider a second-order neutral differential equation

$$\left(x(t) + \frac{1}{3}x\left(\frac{t}{3}\right) \right)'' + \frac{\gamma}{t^2}x(t) = 0, \tag{38}$$

where $\gamma > 0$ is a constant. We have $\alpha = 1$, $r(t) = 1$, $p(t) = p_0 = 1/3$, $\tau(t) = t/3$, $q(t) = \gamma/t^2$, $\sigma(t) = t$, $f(u) = u$, and $k = 1$. Let $\rho(t) = t$ and let ψ be defined as in Example 1. Then

$$\psi(t_{**}) = (\gamma - 1) \int_{t_{**}}^\infty \frac{dt}{t} = \infty, \tag{39}$$

provided that $\gamma > 1$. Therefore, by Theorem 4, (38) is oscillatory for any $\gamma > 1$, whereas an application of Theorem 2 yields oscillation of (38) for all $\gamma > 3/2$. Hence, Theorem 4 improves Theorem 2.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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