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Research Article

Positive Solutions for a Second-Order p-Laplacian Boundary Value Problem with Impulsive Effects and Two Parameters

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The author considers an impulsive boundary value problem involving the one-dimensional p-Laplacian $-(\varphi_p(u'))' = \lambda\omega(t)f(t,u)$, $0<t<1$, $t \neq t_k$, $\Delta u|_{t=t_k} = \mu I_k(t_k, u(t_k))$, $\Delta u'|_{t=t_k} = 0$, $k = 1,2,\ldots,n$, $au(0)-bu'(0) = \int_0^1 g(t)u(t)dt, u'(1) = 0$, where $\lambda > 0$ and $\mu > 0$ are two parameters. Using fixed point theories, several new and more general existence and multiplicity results are derived in terms of different values of $\lambda > 0$ and $\mu > 0$. The exact upper and lower bounds for these positive solutions are also given. Moreover, the approach to deal with the impulsive term is different from earlier approaches. In this paper, our results cover equations without impulsive effects and are compared with some recent results by Ding and Wang.

1. Introduction

Impulsive differential equations, which provide a natural description of observed evolution processes, are regarded as important mathematical tools for the better understanding of several real-world problems in applied sciences, such as population dynamics, ecology, biological systems, biotechnology, industrial robotics, pharmacokinetics, and optimal control. Therefore, the study of this class of impulsive differential equations has gained prominence, and it is a rapidly growing field. For the general theory of impulsive differential equations, we refer the reader to [1–3], whereas the applications of impulsive differential equations can be found in [4–20]. In particular, we would like to mention some results of Lin and Jiang [8] and Feng and Xie [10]. In [8], Lin and Jiang investigated the following Dirichlet boundary value problem with impulse effects:

$$\begin{gathered}
-u''(t) = f(t,u(t)), \quad t \in J,\ t \neq t_k, \\
\Delta u'\big|_{t=t_k} = -I_k(u(t_k)), \quad k = 1,2,\ldots,m, \\
u(0) = u(1) = 0,
\end{gathered} \tag{1}$$

and, by virtue of the fixed point index theory in cones, the authors obtained some sufficient conditions for the existence of multiple positive solutions.

Recently, using fixed point theorems in a cone, Feng and Xie [10] considered the existence of positive solutions for the following problem:

$$\begin{gathered}
-u''(t) = f(t,u(t)), \quad t \in J,\ t \neq t_k, \\
-\Delta u'\big|_{t=t_k} = I_k(u(t_k)), \quad k = 1,2,\ldots,n, \\
u(0) = \sum_{i=1}^{m-2} a_i u(\xi_i), \qquad u(1) = \sum_{i=1}^{m-2} b_i u(\xi_i).
\end{gathered} \tag{2}$$

Moreover, differential equations with p-Laplacian arise naturally in the study of flow through porous media $p = 3/2$, nonlinear elasticity $p \geq 2$, glaciology $1 \leq p \leq 4/3$, and so forth. In recent years, many cases of the existence, multiplicity, and uniqueness of positive solution of differential equations with p-Laplacian have attracted considerable attention [21–46]. Here, it is worth mentioning the studies by Dai and Ma [25] and Kajikiya et al. [26]. In [25], Dai and Ma considered the following one-dimensional p-Laplacian problem:

$$\begin{gathered}
\left(\varphi_p\left(u'\right)\right)' + f(t,u) = 0, \quad t \in (0,1), \\
u(0) = u(1) = 0.
\end{gathered} \tag{3}$$

By using the global bifurcation theory, the authors showed the existence of nodal solutions.

In [26], Kajikiya et al. investigated the following one-dimensional p-Laplacian problem:

$$\left(\varphi_p\left(u'\right)\right)' + \lambda\omega(t) f(u) = 0, \quad t \in (0,1),$$
$$u(0) = u(1) = 0, \tag{4}$$

and, by virtue of the global bifurcation theory, they obtained the existence, nonexistence, uniqueness, and multiplicity of positive solutions as well as sign-changing solutions under suitable conditions imposed on the nonlinear term f.

At the same time, we notice that there has been a considerable attention on impulsive differential equations with one-dimensional p-Laplacian. For example, in [31], Ding and O'Regan studied the second-order p-Laplacian boundary value problems involving impulsive effects:

$$\begin{gathered}\left(\varphi_p\left(u'(t)\right)\right)' = -f(t,u(t)), \quad 0<t<1,\ t\neq t_k,\\ \Delta u|_{t=t_k} = I_k\left(u\left(t_k\right)\right),\\ \Delta u'\big|_{t=t_k} = 0, \quad k=1,2,\ldots,m,\\ u(0) = u'(1) = 0,\end{gathered} \tag{5}$$

and, via Jensens inequality, the first eigenvalue of a relevant linear operator, and the Krasnoselskii-Zabreiko fixed point theorem, they obtained the existence and multiplicity of positive solutions under suitable conditions imposed on the nonlinear term f and the impulsive terms I_k.

In [33], employing the classical fixed point index theorem for compact maps, Zhang and Ge obtained some sufficient conditions for the existence of multiple positive solutions of the following problem:

$$\begin{gathered}\left(\varphi_p\left(u'(t)\right)\right)' = -f(t,u(t)), \quad 0<t<1,\ t\neq t_k,\\ \Delta u|_{t=t_k} = -I_k\left(u\left(t_k\right)\right),\\ \Delta u'\big|_{t=t_k} = 0, \quad k=1,2,\ldots,m,\\ u(0) = \sum_{i=1}^{m-2} a_i u\left(\xi_i\right), \qquad u'(1) = 0.\end{gathered} \tag{6}$$

However, to the best of our knowledge, no paper has considered the second-order impulsive differential equations with one-dimensional p-Laplacian and two parameters till now; for example, see [4–20, 31, 32, 43–45] and the references therein. In this paper, we will use fixed point theorem to investigate the existence and multiplicity of positive solutions for a second-order impulsive differential equation involving one-dimensional p-Laplacian and two parameters.

Consider the following second-order impulsive differential equation with one-dimensional p-Laplacian:

$$\begin{gathered}-\left(\varphi_p\left(u'\right)\right)' = \lambda\omega(t) f(t,u), \quad 0<t<1,\ t\neq t_k,\\ \Delta u|_{t=t_k} = \mu I_k\left(t_k, u\left(t_k\right)\right),\\ \Delta u'\big|_{t=t_k} = 0, \quad k=1,2,\ldots,n,\\ au(0) - bu'(0) = \int_0^1 g(t)\,u(t)\,dt, \qquad u'(1) = 0,\end{gathered} \tag{7}$$

where $\lambda > 0$, $\mu > 0$ are two parameters, $\varphi_p(s) = |s|^{p-2}s$, $p > 1$, $(\varphi_p)^{-1} = \varphi_q$, $(1/p) + (1/q) = 1$, $a, b > 0$, ω may be singular at $t = 0$ and/or $t = 1$, t_k ($k = 1, 2, \ldots, n$, where n is fixed positive integer) are fixed points with $0 < t_1 < t_2 < \cdots < t_k < \cdots < t_n < 1$, and $\Delta u|_{t=t_k}$ denotes the jump of $u(t)$ at $t = t_k$; that is,

$$\Delta u|_{t=t_k} = u\left(t_k^+\right) - u\left(t_k^-\right), \tag{8}$$

where $u(t_k^+)$ and $u(t_k^-)$ represent the right-hand limit and left-hand limit of $u(t)$ at $t = t_k$, respectively. In addition, ω, f, I_k, and g satisfy the following:

(H_1) ω is a nonnegative measurable function on $(0, 1)$ and $\omega \not\equiv 0$ on any open subinterval in $(0, 1)$;

(H_2) $f \in C([0,1]\times[0,+\infty), [0,+\infty))$ with $f(t,u) > 0$ for all t and $u > 0$;

(H_3) $I_k \in C([0,1]\times[0,+\infty), [0,+\infty))$ with $I_k(t,u) > 0$ $(k = 1, 2, \ldots, n)$ for all t and $u > 0$;

(H_4) $g \in L^1[0,1]$ is nonnegative and $\sigma \in [0, a)$, where

$$\sigma = \int_0^1 g(t)\,dt. \tag{9}$$

Some special cases of (7) have been investigated. For example, Ding and Wang [14] considered problem (7) with $p = 2$, $\lambda = 1$, $\mu = 1$, and $\omega(t) \equiv 1$, $t \in [0,1]$. By using Krasnoselskii's fixed point theorem, they proved the existence results of positive solutions of problem (7). However, the authors only obtained that problem (7) has at least one positive solution.

Motivated by the papers mentioned above, we will extend the results of [11, 14, 23, 31, 33, 47, 48] to problem (7). We remark that on impulsive differential equations with a parameter only a few results have been obtained, not to mention impulsive differential equations with two parameters; see, for instance, [12, 18, 19, 45]. These results only dealt with the case that $p = 2$ and $\mu = 1$. Many difficulties occur when we study problem (7); for example, it is difficult to construct the cone and the operator because its state is discontinuous. It is also difficult to deal with λ and μ because of λ with one-dimensional p-Laplacian, and μ without one-dimensional p-Laplacian in the same equation (21). In this paper, we try to solve this kind of problem. Moreover, we will use a different approach to deal with the impulsive term to obtain the existence and multiplicity of positive solutions for problem (7); for details, see the proof of Theorem 1.

The rest of the paper is organized as follows: in Section 2, we state the main results of problem (7). In Section 3, we provide some preliminary results, and the proofs of the main results together with several technical lemmas are given in Section 4. The final section of the paper contains an example to illustrate the theoretical results.

2. Main Results

In this section, we state the main results, including existence and multiplicity results of positive solutions for problem (7).

For convenience, we introduce the following notations:

$$\begin{gathered} f^0=\limsup_{u\to 0^+}\max_{t\in J}\frac{f(t,u)}{\varphi_p(u)}, \qquad f^\infty=\limsup_{u\to\infty}\max_{t\in J}\frac{f(t,u)}{\varphi_p(u)}, \\ f_0=\liminf_{u\to 0^+}\min_{t\in J}\frac{f(t,u)}{\varphi_p(u)}, \qquad f_\infty=\liminf_{u\to\infty}\min_{t\in J}\frac{f(t,u)}{\varphi_p(u)}, \\ I^0(k)=\limsup_{u\to 0^+}\max_{t\in J}\frac{I_k(t,u)}{u}, \\ I^\infty(k)=\limsup_{u\to\infty}\max_{t\in J}\frac{I_k(t,u)}{u}, \\ I_0(k)=\liminf_{u\to 0^+}\min_{t\in J}\frac{I_k(t,u)}{u}, \\ I_\infty(k)=\liminf_{u\to\infty}\min_{t\in J}\frac{I_k(t,u)}{u}, \\ J=[0,1], \quad k=1,2,\dots,n. \end{gathered} \tag{10}$$

Moreover, we choose four numbers r, r_1, r_2, and R satisfying

$$0<r<r_1<\delta r_2<r_2<R<+\infty, \tag{11}$$

where δ is defined in (23).

Theorem 1. *Assume that (H_1)–(H_4) hold and f_∞, f^∞, $I_\infty(k)$, and $I^\infty(k)$ $(k=1,2,\dots,n)$ are positive constants. Then,*

(i) *there exist $\lambda_0>0$ and $\mu_0>0$ such that, for any $\lambda>\lambda_0$ and $\mu>\mu_0$, problem (7) has a positive solution u with*

$$\delta r\le u(t)\le\frac{1}{\delta}R, \quad t\in J; \tag{12}$$

(ii) *there exist $\overline{\lambda}_0>0$ and $\overline{\mu}_0>0$ such that, for any $0<\lambda<\overline{\lambda}_0$ and $0<\mu<\overline{\mu}_0$, problem (7) has a positive solution u with property (12).*

Theorem 2. *Assume that (H_1)–(H_4) hold and f_0, f^0, $I_0(k)$, and $I^0(k)$ $(k=1,2,\dots,n)$ are positive constants. Then,*

(i) *there exist $\lambda_0>0$ and $\mu_0>0$ such that, for any $\lambda>\lambda_0$ and $\mu>\mu_0$, problem (7) has a positive solution u with*

$$\delta r\le u(t)\le R, \quad t\in J; \tag{13}$$

(ii) *there exist $\overline{\lambda}_0>0$ and $\overline{\mu}_0>0$ such that, for any $0<\lambda<\overline{\lambda}_0$ and $0<\mu<\overline{\mu}_0$, problem (7) has a positive solution u with property (13).*

Remark 3. Some ideas of the proof of Theorems 1 and 2 are from Yan [47]. In [47], Yan studied a class of the periodic impulsive functional differential equations with two parameters and proved the following existence result by using a well-known fixed point index theorem due to Krasnoselskii.

Theorem 4 (see [47, Theorem 3.1]). *Assume that (A_1)–(A_6) hold and f^0, f_∞, I_∞, and I^0 are positive constants. If*

$$\begin{gathered} \beta\left(\lambda f^0P+\mu I^0Q\right)<1, \\ \alpha\sigma\left(\lambda f_\infty P+\mu I_\infty Q\right)>1, \end{gathered} \tag{14}$$

then problem (7) has a positive ω-periodic solution.

It is not difficult to see that the conditions of Theorem 4 are not the optimal conditions which guarantee the existence of at least one positive ω-periodic solution for the related problem. In fact, if

$$\beta\left(\lambda f^0P+\mu I^0Q\right)<1 \tag{15}$$

or

$$\alpha\sigma\left(\lambda f_\infty P+\mu I_\infty Q\right)>1, \tag{16}$$

we can prove that the problem studied in [47] has at least one positive ω-periodic solution, respectively.

Theorem 5. *Assume that (H_1)–(H_4) hold.*

(i) *If $f^\infty=0$ and $I^\infty(k)=0$, $k=1,2,\dots,n$, then there exist $\lambda_0>0$ and $\mu_0>0$ such that, for any $\lambda>\lambda_0$ and $\mu>\mu_0$, problem (7) has a positive solution u with property (12).*

(ii) *If $f^0=0$ and $I^0(k)=0$, $k=1,2,\dots,n$, then there exist $\lambda_0>0$ and $\mu_0>0$ such that, for any $\lambda>\lambda_0$ and $\mu>\mu_0$, problem (7) has a positive solution u with property (13).*

(iii) *If $f^0=f^\infty=I^\infty(k)=I^0(k)=0$, $k=1,2,\dots,n$, then there exist $\lambda_0>0$ and $\mu_0>0$ such that, for any $\lambda>\lambda_0$ and $\mu>\mu_0$, problem (7) has at least two positive solutions u_1 and u_2 with*

$$\delta r\le u_1(t)\le r_1<\delta r_2\le u_2(t)\le R, \quad t\in J. \tag{17}$$

Theorem 6. *Assume that (H_1)–(H_4) hold.*

(i) *If $f_\infty=\infty$ and $I_\infty(k)=\infty$, $k=1,2,\dots,n$, then there exist $\overline{\lambda}_0>0$ and $\overline{\mu}_0>0$ such that, for any $0<\lambda<\overline{\lambda}_0$ and $0<\mu<\overline{\mu}_0$, problem (7) has a positive solution u with property (12).*

(ii) *If $f_0=\infty$ and $I_0(k)=\infty$, $k=1,2,\dots,n$, then there exist $\overline{\lambda}_0>0$ and $\overline{\mu}_0>0$ such that, for any $0<\lambda<\overline{\lambda}_0$ and $0<\mu<\overline{\mu}_0$, problem (7) has a positive solution u with property (13).*

(iii) *If* $f_0 = f_\infty = I_\infty(k) = I_0(k) = +\infty$, $k = 1, 2, \ldots, n$, *then there exist* $\overline{\lambda}_0 > 0$ *and* $\overline{\mu}_0 > 0$ *such that, for any* $0 < \lambda < \overline{\lambda}_0$ *and* $0 < \mu < \overline{\mu}_0$, *problem* (7) *has at least two positive solutions* u_1 *and* u_2 *with*

$$\delta r \le u_1(t) \le r_1 < \delta r_2 \le u_2(t) \le \frac{1}{\delta} R, \quad t \in J. \tag{18}$$

Remark 7. Some ideas of the proof of Theorems 5 and 6 are from Graef et al. [48].

3. Preliminaries

Let $J' = J \setminus \{t_1, t_2, \ldots, t_n\}$ and let E be the Banach space:

$$\begin{aligned} E = \{ & u \mid u : J \longrightarrow \mathbb{R} \text{ is continuous at } t \neq t_k, \\ & u(t_k^-) = u(t_k), u(t_k^+) \text{ exist}, k = 1, 2, \ldots, n\} \end{aligned} \tag{19}$$

with $\|u\| = \max_{0 \le t \le 1} |u(t)|$. We denote

$$\Omega_r := \{u \in E : \|u\| < r\} \tag{20}$$

for all $r > 0$ in the sequel.

In our main results, we will make use of the following definitions and lemmas.

Definition 8 (see [49]). Let E be a real Banach space over $\mathbb{R}$. A nonempty closed set $P \subset E$ is said to be a cone provided that

(i) $au + bv \in P$ for all $u, v \in P$ and all $a \ge 0, b \ge 0$;

(ii) $u, -u \in P$ implies $u = 0$.

Every cone $P \subset E$ induces an ordering in E given by $x \le y$ if and only if $y - x \in P$.

Definition 9. A function $u \in E \cap C^1(0, 1)$ with $\varphi_p(u') \in C^1(0, 1)$ is called a solution of (7) if it satisfies (7). If $u(t) \ge 0$ and $u(t) \not\equiv 0$ on J, then u is called a positive solution of (7).

Lemma 10. *Assume that* (H_1)*–*(H_4) *hold. Then,* $u \in E \cap C^1(0, 1)$ *with* $\varphi_p(u') \in C^1(0, 1)$ *is a solution of problem* (7) *if and only if* $u \in E$ *is a solution of the following impulsive integral equation:*

$$\begin{aligned} u(t) = & \int_0^t \varphi_q\left(\lambda \int_s^1 \omega(r) f(r, u(r)) \, dr\right) ds \\ & + \mu \sum_{t_k < t} I_k(t_k, u(t_k)) + \frac{1}{a - \sigma} \\ & \times \left\{ \int_0^1 g(t) \left[\int_0^t \varphi_q\left(\lambda \int_s^1 \omega(r) f(r, u(r)) \, dr\right) ds \right] dt \right. \\ & + b\varphi_q\left(\lambda \int_0^1 \omega(s) f(s, u(s)) \, ds\right) \\ & \left. + \mu \int_0^1 g(t) \sum_{t_k < t} I_k(t_k, u(t_k)) \, dt \right\}. \end{aligned} \tag{21}$$

Moreover, if u *is a positive solution of problem* (7), *then*

$$\min_{t \in J} u(t) \ge \delta \|u\|, \tag{22}$$

where

$$\delta = \frac{\int_0^1 t g(t) \, dt}{a - \int_0^1 g(t) \, dt + \int_0^1 t g(t) \, dt}. \tag{23}$$

Proof. First, suppose that $u \in E$ is a solution of problem (7). It is easy to see by integration of (7) that

$$-\phi_p(u'(1)) + \phi_p(u'(t)) = \int_t^1 \lambda \omega(s) f(s, u(s)) \, ds. \tag{24}$$

By the boundary condition $u'(1) = 0$, we have

$$u'(t) = \phi_q\left(\int_t^1 \lambda \omega(s) f(s, u(s)) \, ds\right), \tag{25}$$

$$u'(0) = \phi_q\left(\int_0^1 \lambda \omega(s) f(s, u(s)) \, ds\right). \tag{26}$$

If $0 < t \le t_1$, integrating (25) from 0 to t we obtain

$$u(t) - u(0) = \int_0^t \phi_q\left(\int_s^1 \lambda \omega(r) f(r, u(r)) \, dr\right) ds. \tag{27}$$

If $t_1 < t \le t_2$, integrating (25) from 0 to t_1 we obtain

$$u(t_1) - u(0) = \int_0^{t_1} \phi_q\left(\int_s^1 \lambda \omega(r) f(r, u(r)) \, dr\right) ds, \tag{28}$$

and integrating (25) from t_1 to t we obtain

$$u(t) - u(t_1^+) = \int_{t_1}^t \phi_q\left(\int_s^1 \lambda \omega(r) f(r, u(r)) \, dr\right) ds. \tag{29}$$

It follows that

$$\begin{aligned} u(t) - u(0) = & \int_0^t \phi_q\left(\int_s^1 \lambda \omega(r) f(r, u(r)) \, dr\right) ds \\ & + \mu I_1(t_1, u(t_1)), \quad t_1 < t \le t_2. \end{aligned} \tag{30}$$

For $t_k < t \le t_{k+1}$, repeating the process we have

$$\begin{aligned} u(t) = u(0) + & \int_0^t \phi_q\left(\int_s^1 \lambda \omega(r) f(r, u(r)) \, dr\right) ds \\ & + \mu \sum_{t_k < t} I_k(t_k, u(t_k)). \end{aligned} \tag{31}$$

Combining this with the boundary condition, we have

$$\begin{aligned} u(0) = & \frac{1}{a - \int_0^1 g(t) \, dt} \\ & \times \left\{ \int_0^1 g(t) \left[\int_0^t \phi_q\left(\int_s^1 \lambda \omega(r) f(r, u(r)) \, dr\right) ds \right] dt \right. \\ & + b\phi_q\left(\int_0^1 \lambda \omega(s) f(s, u(s)) \, ds\right) \\ & \left. + \mu \int_0^1 g(t) \sum_{t_k < t} I_k(t_k, u(t_k)) \, dt \right\}. \end{aligned} \tag{32}$$

Then, the proof of sufficient is complete.

Conversely, if $u \in E$ is a solution of (21), then we have the following.

Direct differentiation of (21) implies

$$u'(t) = \phi_q\left(\int_t^1 \lambda\omega(s) f(s, u(s))\, ds\right), \quad t \in J. \tag{33}$$

Evidently,

$$\begin{gathered}\left(\phi_p\left(u'(t)\right)\right)' = -\lambda\omega(t) f(t, u(t)), \\ au(0) - bu'(0) = \int_0^1 g(t) u(t)\, dt, \quad u'(1) = 0.\end{gathered} \tag{34}$$

Finally, we show that (22) holds. It is clear that $u'(t) = \phi_q(\int_t^1 \lambda\omega(s) f(s, u(s)) ds) > 0$, which implies that

$$\|u\| = u(1), \qquad \min_{t\in J} u(t) = u(0). \tag{35}$$

As we assume that $f(t, u) \geq 0$ and $\omega(t) \geq 0$, we see that any nontrivial solution u of problem (7) is concave on J; that is, $u'' \leq 0$, and then we get that $u'(t)$ is nonincreasing on J.

So, for every $t \in (0, 1]$, we have

$$\frac{u(1) - u(0)}{1} \leq \frac{u(t) - u(0)}{t}; \tag{36}$$

that is, $u(t) - u(0) \geq tu(1) - tu(0)$. Therefore,

$$\begin{aligned}&\int_0^1 g(t) u(t)\, dt - \int_0^1 g(t)\, dt u(0) \\ &\quad \geq \int_0^1 tg(t)\, dt u(1) - \int_0^1 tg(t)\, dt u(0).\end{aligned} \tag{37}$$

Noticing that u is a positive solution of problem (7) and $au(0) - bu'(0) = \int_0^1 g(t) u(t) dt$, we have $\int_0^1 g(t) u(t) dt \leq au(0)$. Thus, we obtain

$$u(0) \geq \frac{\int_0^1 tg(t)\, dt}{a - \int_0^1 g(t)\, dt + \int_0^1 tg(t)\, dt} u(1). \tag{38}$$

The lemma is proved. □

Define a cone K in E by

$$K = \left\{u \in E : u \geq 0, \min_{t\in J} u(t) \geq \delta \|u\|\right\}, \tag{39}$$

where δ is defined in (23). It is easy to see that K is a closed convex cone of E.

Define $T_\lambda^\mu : K \to E$ by

$$\begin{aligned}&\left(T_\lambda^\mu u\right)(t) \\ &\quad = \int_0^t \varphi_q\left(\lambda \int_s^1 \omega(r) f(r, u(r))\, dr\right) ds \\ &\qquad + \mu \sum_{t<t_k} I_k\left(t_k, u\left(t_k\right)\right) + \frac{1}{a - \sigma} \\ &\qquad \times \left\{\int_0^1 g(t) \left[\int_0^t \varphi_q\left(\lambda \int_s^1 \omega(r) f(r, u(r))\, dr\right) ds\right] dt\right. \\ &\qquad\qquad + b\varphi_q\left(\lambda \int_0^1 \omega(s) f(s, u(s))\, ds\right) \\ &\qquad\qquad \left. + \mu \int_0^1 g(t) \sum_{t<t_k} I_k\left(t_k, u\left(t_k\right)\right) dt\right\}.\end{aligned} \tag{40}$$

From (40) and Lemma 10, it is easy to obtain the following result.

Lemma 11. *Assume that* (H_1)–(H_4) *hold. Problem* (7) *is equivalent to the fixed point problem of* T_λ^μ *in* K.

Lemma 12 (see [47, Lemmas 2.1 and 2.2]). *Assume that* (H_1)–(H_4) *hold. Then,* $T_\lambda^\mu : K \to K$ *is completely continuous.*

The following well-known result of the fixed point is crucial in our arguments.

Lemma 13 (see [49]). *Let* P *be a cone in a real Banach space* E. *Assume that* Ω_1, Ω_2 *are bounded open sets in* E *with* $0 \in \Omega_1$, $\overline{\Omega}_1 \subset \Omega_2$. *If*

$$A : P \cap \left(\overline{\Omega}_2 \setminus \Omega_1\right) \longrightarrow P \tag{41}$$

is completely continuous such that either

(a) $\|Ax\| \leq \|x\|$, $\forall x \in P \cap \partial\Omega_1$, *and* $\|Ax\| \leq \|x\|$, $\forall x \in P \cap \partial\Omega_2$, *or*

(b) $\|Ax\| \leq \|x\|$, $\forall x \in P \cap \partial\Omega_1$, *and* $\|Ax\| \leq \|x\|$, $\forall x \in P \cap \partial\Omega_2$,

then A *has at least one fixed point in* $P \cap (\overline{\Omega}_2 \setminus \Omega_1)$.

4. Proofs of the Main Results

For convenience, we introduce the following notation:

$$\gamma = \varphi_q\left(\int_0^1 \omega(s)\, ds\right), \qquad \sigma_1 = \int_{t_1}^1 g(s)\, ds. \tag{42}$$

Proof of Theorem 1. Part (i). Noticing that $f(t, u) > 0$, $I_k(t, u) > 0$ $(k = 1, 2, \ldots, n)$ for all t and $u > 0$, we can define

$$\begin{gathered}m_r = \min_{t\in J, \delta r \leq u \leq r} \{f(t, u)\} > 0, \\ m^* = \min\left\{m_k, k = 1, 2, \ldots, n\right\} > 0,\end{gathered} \tag{43}$$

where $r > 0$, $m_k = \min_{t\in J, \delta r\leq u\leq r}\{I_k(t, u)\}$, $k = 1, 2, \ldots, n$.

Let

$$\lambda_0 \geq \left(\frac{a-\sigma}{2b\delta\gamma} r\right)^{p-1} m_r^{-1}, \qquad \mu_0 \geq \frac{(a-\sigma) r}{2\sigma_1 m^*}. \tag{44}$$

Then, for $u \in K \cap \partial\Omega_r$ and $\lambda > \lambda_0$, $\mu > \mu_0$, we have

$$\begin{aligned}
&(T_\lambda^\mu u)(t) \\
&= \int_0^t \varphi_q\left(\lambda \int_s^1 \omega(r) f(r, u(r)) dr\right) ds \\
&\quad + \mu \sum_{t_k < t} I_k(t_k, u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\left\{\int_0^1 g(t)\right. \\
&\qquad \times \left[\int_0^t \varphi_q\left(\lambda \int_s^1 \omega(r) f(r, u(r)) dr\right) ds\right] dt \\
&\qquad + b\varphi_q\left(\lambda \int_0^1 \omega(s) f(s, u(s)) ds\right) \\
&\qquad \left. + \mu \int_0^1 g(t) \sum_{t_k < t} I_k(t_k, u(t_k)) dt\right\} \\
&\geq \frac{b}{a-\sigma}\varphi_q\left(\lambda \int_0^1 \omega(s) f(s, u(s)) ds\right) \\
&\quad + \frac{1}{a-\sigma}\mu \int_0^1 g(t) \sum_{t_k < t} I_k(t_k, u(t_k)) dt \\
&\geq \frac{b}{a-\sigma}\lambda^{q-1}\varphi_q\left(\int_0^1 \omega(s) m_r ds\right) \\
&\quad + \frac{\mu}{a-\sigma}\left[\int_{t_1}^{t_2} g(t) I_1(t_1, u(t_1)) dt\right. \\
&\qquad + \int_{t_2}^{t_3} g(t)\Big(I_1(t_1, u(t_1)) \\
&\qquad\qquad + I_2(t_2, u(t_2))\Big) dt + \cdots \\
&\qquad + \int_{t_n}^1 g(t)\Big(I_1(t_1, u(t_1)) + I_2(t_2, u(t_2)) \\
&\qquad\qquad \left. + \cdots + I_n(t_n, u(t_n))\Big) dt\right] \\
&= \frac{b}{a-\sigma}\lambda^{q-1}\varphi_q\left(\int_0^1 \omega(s) m_r ds\right) \\
&\quad + \frac{\mu}{a-\sigma}\left[\int_{t_1}^1 g(t)\Big(I_1(t_1, u(t_1)) dt\right. \\
&\qquad + \int_{t_2}^1 g(t) I_2(t_2, u(t_2)) dt \\
&\qquad \left. + \cdots + \int_{t_n}^1 g(t) I_n(t_n, u(t_n))\Big) dt\right] \\
&\geq \frac{b}{a-\sigma}\lambda^{q-1}\varphi_q\left(\int_0^1 \omega(s) m_r ds\right) \\
&\quad + \frac{\mu}{a-\sigma}\int_{t_1}^1 g(t) I_1(t_1, u(t_1)) dt \\
&\geq \frac{b}{a-\sigma}\lambda^{q-1} m_r^{q-1}\gamma + \frac{\sigma_1}{a-\sigma}\mu m^* \\
&> \frac{b}{a-\sigma}\lambda_0^{1-q} m_r^{q-1}\gamma + \frac{\sigma_1}{a-\sigma}\mu_0 m^* \\
&\geq \frac{1}{2}r + \frac{1}{2}r = r,
\end{aligned} \tag{45}$$

which implies that

$$\begin{gathered}
\left\|T_\lambda^\mu u\right\| > \|u\|, \quad \forall u \in K \cap \partial\Omega_r, \\
\lambda > \lambda_0, \quad \mu > \mu_0.
\end{gathered} \tag{46}$$

If $0 < f^\infty < +\infty$, $0 < I^\infty < +\infty$, then there exist $l_1 > 0$, and $l_2 > 0$ and $R > r > 0$ such that

$$\begin{gathered}
f(t, u) < l_1 \varphi_p(u), \qquad I_k(t, u) < l_2 u, \\
(\forall t \in J,\ u \geq R,\ k = 1, 2, \ldots, n),
\end{gathered} \tag{47}$$

where l_1 satisfies

$$\frac{2(a+b)}{a-\sigma}\varphi_q(\lambda)\varphi_q(l_1)\gamma \leq 1 \tag{48}$$

l_2 satisfies

$$\frac{2a}{a-\sigma}\mu n l_2 \leq 1. \tag{49}$$

Let $\nu = R/\delta$. Thus, when $u \in K \cap \partial\Omega_\nu$, we have

$$u(t) \geq \delta\|u\| = \delta\nu = R, \quad t \in J, \tag{50}$$

and then we get

$$\begin{aligned}
&(T_\lambda^\mu u)(t) \\
&= \int_0^t \varphi_q\left(\lambda \int_s^1 \omega(r) f(r, u(r)) dr\right) ds + \mu \sum_{t < t_k} I_k(t_k, u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\left\{\int_0^1 g(t)\right. \\
&\qquad \times \left[\int_0^t \varphi_q\left(\lambda \int_s^1 \omega(r) f(r, u(r)) dr\right) ds\right] dt \\
&\qquad + b\varphi_q\left(\lambda \int_0^1 \omega(s) f(s, u(s)) ds\right) \\
&\qquad \left. + \mu \int_0^1 g(t) \sum_{t < t_k} I_k(t_k, u(t_k)) dt\right\}
\end{aligned}$$

$$\begin{aligned}
&\le \int_0^1 \varphi_q\left(\lambda\int_s^1 \omega(r) f(r,u(r))\,dr\right) ds \\
&\quad + \frac{1}{a-\sigma}\left\{\int_0^1 g(t)\right. \\
&\qquad \times\left[\int_0^1 \varphi_q\left(\lambda\int_s^1 \omega(r) f(r,u(r))\,dr\right) ds\right] dt \\
&\qquad \left. + b\varphi_q\left(\lambda\int_0^1 \omega(s) f(s,u(s))\,ds\right)\right\} \\
&\quad + \mu\sum_{k=1}^n I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu\int_0^1 g(t)\sum_{k=1}^n I_k(t_k,u(t_k))\,dt \\
&= \int_0^1 \varphi_q\left(\lambda\int_s^1 \omega(r) f(r,u(r))\,dr\right) ds\left(1+\frac{\sigma}{a-\sigma}\right) \\
&\quad + \frac{b}{a-\sigma}\varphi_q\left(\lambda\int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\quad + \mu\sum_{k=1}^n I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu\int_0^1 g(t)\sum_{k=1}^n I_k(t_k,u(t_k))\,dt \\
&\le \frac{a}{a-\sigma}\int_0^1 \varphi_q\left(\lambda\int_0^1 \omega(r) f(r,u(r))\,dr\right) ds \\
&\quad + \frac{b}{a-\sigma}\varphi_q\left(\lambda\int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\quad + \mu\sum_{k=1}^n I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu\int_0^1 g(t)\sum_{k=1}^n I_k(t_k,u(t_k))\,dt \\
&= \frac{a+b}{a-\sigma}\varphi_q\left(\lambda\int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\quad + \mu\sum_{k=1}^n I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu\int_0^1 g(t)\sum_{k=1}^n I_k(t_k,u(t_k))\,dt \\
&\le \frac{a+b}{a-\sigma}\varphi_q\left(\lambda\int_0^1 \omega(s) l_1\varphi_p(u(s))\,ds\right) \\
&\quad + \mu\sum_{k=1}^n l_2 u(t_k) \\
&\quad + \frac{1}{a-\sigma}\mu\int_0^1 g(t)\sum_{k=1}^n l_2 u(t_k)\,dt \\
&\le \frac{a+b}{a-\sigma}\varphi_q(l_1)\|u\|\varphi_q\left(\lambda\int_0^1 \omega(s)\,ds\right) \\
&\quad + \mu n l_2\|u\| + \frac{1}{a-\sigma}\mu\sigma n l_2\|u\| \\
&= \frac{a+b}{a-\sigma}\varphi_q(\lambda)\varphi_q(l_1)\gamma\|u\| \\
&\quad + \frac{a}{a-\sigma}\mu n l_2\|u\| \\
&\le \frac{1}{2}\|u\| + \frac{1}{2}\|u\| = \|u\|.
\end{aligned} \tag{51}$$

This yields

$$\left\|T_\lambda^\mu u\right\| \le \|u\|, \quad \forall u \in K\cap\partial\Omega_\nu. \tag{52}$$

Applying (b) of Lemma 13 to (46) and (52) yields that T has a fixed point $u \in K\cap(\overline{\Omega}_\nu\setminus\Omega_r)$ with $r \le \|u\| \le \nu = (1/\delta)R$. Hence, since for $u \in K$ we have $u(t) \ge \delta\|u\|$ for $t \in J$, it follows that (12) holds. This gives the proof of Part (i).

Part (ii). Noticing that $f(t,u) > 0$, $I_k(t,u) > 0$ for all t and $u > 0$, we can define

$$\begin{gathered}
M_r = \max_{t\in J, 0\le u\le r}\{f(t,u)\} > 0, \\
M^* = \max\{M_k, k = 1,2,\ldots,n\} > 0,
\end{gathered} \tag{53}$$

where $r > 0$, $M_k = \max_{t\in J,0\le u\le r}\{I_k(t,u)\}$, $k = 1,2,\ldots,n$.

Let

$$\overline{\lambda}_0 \le \left(\frac{(a-\sigma)r}{2(a+b)\gamma}\right)^{p-1} M_r^{-1}, \qquad \overline{\mu}_0 \le \frac{(a-\sigma)r}{2anM^*}. \tag{54}$$

Then, for $u \in K\cap\partial\Omega_r$ and $\lambda < \overline{\lambda}_0$, $\mu < \overline{\mu}_0$, we have

$$\begin{aligned}
&(T_\lambda^\mu u)(t) \\
&\quad = \int_0^t \varphi_q\left(\lambda\int_s^1 \omega(r) f(r,u(r))\,dr\right) ds \\
&\qquad + \mu\sum_{t<t_k} I_k(t_k,u(t_k)) \\
&\qquad + \frac{1}{a-\sigma}\left\{\int_0^1 g(t)\right. \\
&\qquad\qquad \times\left[\int_0^t \varphi_q\left(\lambda\int_s^1 \omega(r) f(r,u(r))\,dr\right) ds\right] dt \\
&\qquad\qquad + b\varphi_q\left(\lambda\int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\qquad\qquad \left. + \mu\int_0^1 g(t)\sum_{t<t_k} I_k(t_k,u(t_k))\,dt\right\}
\end{aligned}$$

$$\begin{aligned}
&\le \int_0^1 \varphi_q\left(\lambda \int_s^1 \omega(r) f(r,u(r))\,dr\right)ds \\
&\quad + \frac{1}{a-\sigma}\left\{\int_0^1 g(t)\right. \\
&\qquad \times \left[\int_0^1 \varphi_q\left(\lambda \int_s^1 \omega(r) f(r,u(r))\,dr\right)ds\right]dt \\
&\qquad \left. + b\varphi_q\left(\lambda \int_0^1 \omega(s) f(s,u(s))\,ds\right)\right\} \\
&\quad + \mu \sum_{k=1}^{n} I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu \int_0^1 g(t) \sum_{k=1}^{n} I_k(t_k,u(t_k))\,dt \\
&= \int_0^1 \varphi_q\left(\lambda \int_s^1 \omega(r) f(r,u(r))\,dr\right)ds\left(1+\frac{\sigma}{a-\sigma}\right) \\
&\quad + \frac{b}{a-\sigma}\varphi_q\left(\lambda \int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\quad + \mu \sum_{k=1}^{n} I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu \int_0^1 g(t) \sum_{k=1}^{n} I_k(t_k,u(t_k))\,dt \\
&\le \frac{a}{a-\sigma}\int_0^1 \varphi_q\left(\lambda \int_0^1 \omega(r) f(r,u(r))\,dr\right)ds \\
&\quad + \frac{b}{a-\sigma}\varphi_q\left(\lambda \int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\quad + \mu \sum_{k=1}^{n} I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu \int_0^1 g(t) \sum_{k=1}^{n} I_k(t_k,u(t_k))\,dt \\
&= \frac{a+b}{a-\sigma}\varphi_q\left(\lambda \int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\quad + \mu \sum_{k=1}^{n} I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu \int_0^1 g(t) \sum_{k=1}^{n} I_k(t_k,u(t_k))\,dt \\
&\le \frac{a+b}{a-\sigma}\varphi_q\left(\lambda \int_0^1 \omega(s) M_r\,ds\right) \\
&\quad + \mu \sum_{k=1}^{n} M^* + \frac{1}{a-\sigma}\mu \int_0^1 g(t) \sum_{k=1}^{n} M^*\,dt \\
&\le \frac{a+b}{a-\sigma}\varphi_q(\lambda)\,\varphi_q(M_r)\,\gamma + \frac{a}{a-\sigma}\mu n M^* \\
&< \frac{a+b}{a-\sigma}\varphi_q\left(\overline{\lambda}_0\right)\varphi_q(M_r)\,\gamma + \frac{a}{a-\sigma}\overline{\mu}_0 n M^* \\
&\le \frac{1}{2}\|u\| + \frac{1}{2}\|u\| = \|u\|.
\end{aligned} \tag{55}$$

This implies that

$$\left\|T_\lambda^\mu u\right\| < \|u\|, \quad \forall u \in K \cap \partial\Omega_r. \tag{56}$$

If $0 < f_\infty < +\infty$, $0 < I_\infty < +\infty$, then there exist $l_3 > 0$, and $l_4 > 0$ and $R > r > 0$ such that

$$\begin{gathered}
f(t,u) > l_3 \varphi_p(u), \qquad I_k(t,u) > l_4 u, \\
(\forall t \in J,\ u \ge R,\ k = 1,2,\ldots,n),
\end{gathered} \tag{57}$$

where l_3 satisfies

$$\frac{2b}{a-\sigma}\lambda^{q-1} l_3^{q-1}\gamma\delta \ge 1 \tag{58}$$

l_4 satisfies

$$\frac{2\sigma_1}{a-\sigma}\mu l_4 \delta \ge 1. \tag{59}$$

Let $\nu = R/\delta$. Then, for $u \in K \cap \partial\Omega_\nu$, we obtain

$$u(t) \ge \delta R = \nu, \quad t \in J, \tag{60}$$

and it follows from (40) that

$$\begin{aligned}
&(T_\lambda^\mu u)(t) \\
&\quad = \int_0^t \varphi_q\left(\lambda \int_s^1 \omega(r) f(r,u_1(r))\,dr\right)ds \\
&\qquad + \mu \sum_{t<t_k} I_k(t_k,u(t_k)) \\
&\qquad + \frac{1}{a-\sigma}\left\{\int_0^1 g(t)\right. \\
&\qquad\quad \times \left[\int_0^t \varphi_q\left(\lambda \int_s^1 \omega(r) f(r,u_1(r))\,dr\right)ds\right]dt \\
&\qquad\quad + b\varphi_q\left(\lambda \int_0^1 \omega(s) f(s,u_1(s))\,ds\right) \\
&\qquad\quad \left. + \mu \int_0^1 g(t) \sum_{t<t_k} I_k(t_k,u(t_k))\,dt\right\} \\
&\quad \ge \frac{b}{a-\sigma}\varphi_q\left(\lambda \int_0^1 \omega(s) f(s,u_1(s))\,ds\right) \\
&\qquad + \frac{1}{a-\sigma}\mu \int_0^1 g(t) \sum_{t<t_k} I_k(t_k,u(t_k))\,dt
\end{aligned}$$

$$\begin{aligned}
&\geq \frac{b}{a-\sigma}\varphi_q\left(\lambda\int_0^1\omega(s)\,f(s,u_1(s))\,ds\right)\\
&\quad+\frac{1}{a-\sigma}\mu\int_{t_1}^1 g(t)\,I_1(t_1,u(t_1))\,dt\\
&\geq \frac{b}{a-\sigma}\lambda^{q-1}\varphi_q\left(\int_0^1\omega(s)\,l_3\varphi_p(u(s))\,ds\right)\\
&\quad+\frac{\mu}{a-\sigma}\int_{t_1}^1 g(t)\,l_4u(t_1)\,dt\\
&\geq \frac{b}{a-\sigma}\lambda^{q-1}l_3^{q-1}\gamma\delta\|u\|\\
&\quad+\frac{\sigma_1}{a-\sigma}\mu l_4\delta\|u\|\\
&\geq \frac{1}{2}\|u\|+\frac{1}{2}\|u\|=\|u\|,
\end{aligned}\tag{61}$$

which implies that

$$\|T_\lambda^\mu u\|>\|u\|,\quad \forall u\in K\cap\partial\Omega_\nu.\tag{62}$$

Applying (a) of Lemma 13 to (56) and (62) yields that T_λ^μ has a fixed point $u\in K\cap(\overline{\Omega}_\nu\setminus\Omega_r)$ with $r\leq\|u\|\leq\nu=(1/\delta)R$. Hence, since for $u\in K$ we have $u(t)\geq\delta\|u\|$ for $t\in J$, it follows that (13) holds. This finishes the proof of Part (ii). □

Proof of Theorem 2. Part (i). Noticing that $f(t,u)>0$, $I_k(t,u)>0$ $(k=1,2,\ldots,n)$ for all t and $u>0$, we can define

$$\begin{aligned}
m_R&=\min_{t\in J,\ \delta R\leq u\leq R}\{f(t,u)\}>0,\\
m_R^*&=\min\{m_{Rk},k=1,2,\ldots,n\}>0,
\end{aligned}\tag{63}$$

where $R>0$, $m_{Rk}=\min_{t\in J,\delta R\leq u\leq R}\{I_k(t,u)\}$, $k=1,2,\ldots,n$.

Let

$$\lambda_0\geq\left(\frac{a-\sigma}{2b\delta\gamma}R\right)^{p-1}m_R^{-1},\qquad \mu_0\geq\frac{(a-\sigma)R}{2\sigma_1 m_R^*}.\tag{64}$$

Then, for $u\in K\cap\partial\Omega_R$ and $\lambda>\lambda_0$, $\mu>\mu_0$, we have

$$\begin{aligned}
&(T_\lambda^\mu u)(t)\\
&=\int_0^t\varphi_q\left(\lambda\int_s^1\omega(r)\,f(r,u(r))\,dr\right)ds\\
&\quad+\mu\sum_{t<t_k}I_k(t_k,u(t_k))\\
&\quad+\frac{1}{a-\sigma}\left\{\int_0^1 g(t)\right.\\
&\qquad\times\left[\int_0^t\varphi_q\left(\lambda\int_s^1\omega(r)\,f(r,u(r))\,dr\right)ds\right]dt\\
&\qquad+b\varphi_q\left(\lambda\int_0^1\omega(s)\,f(s,u(s))\,ds\right)\\
&\qquad\left.+\mu\int_0^1 g(t)\sum_{t<t_k}I_k(t_k,u(t_k))\,dt\right\}\\
&\geq\frac{b}{a-\sigma}\varphi_q\left(\lambda\int_0^1\omega(s)\,f(s,u(s))\,ds\right)\\
&\quad+\frac{1}{a-\sigma}\mu\int_0^1 g(t)\sum_{t<t_k}I_k(t_k,u(t_k))\,dt\\
&\geq\frac{b}{a-\sigma}\varphi_q\left(\lambda\int_0^1\omega(s)\,f(s,u(s))\,ds\right)\\
&\quad+\frac{1}{a-\sigma}\mu\int_{t_1}^1 g(t)\,I_1(t_1,u(t_1))\,dt\\
&\geq\frac{b}{a-\sigma}\lambda^{q-1}\varphi_q\left(\int_0^1\omega(s)\,m_R\,ds\right)\\
&\quad+\frac{\mu}{a-\sigma}\int_{t_1}^1 g(t)\,m_R^*\,dt\\
&\geq\frac{b}{a-\sigma}\lambda^{q-1}m_R^{q-1}\gamma+\frac{\sigma_1}{a-\sigma}\mu m_R^*\\
&>\frac{b}{a-\sigma}\lambda_0^{1-q}m_R^{q-1}\gamma+\frac{\sigma_1}{a-\sigma}\mu_0 m_R^*\\
&\geq\frac{1}{2}R+\frac{1}{2}R=R,
\end{aligned}\tag{65}$$

which implies that

$$\begin{gathered}
\|T_\lambda^\mu u\|>\|u\|,\quad \forall u\in K\cap\partial\Omega_R,\\
\lambda>\lambda_0,\quad \mu>\mu_0.
\end{gathered}\tag{66}$$

If $0<f^0<+\infty$, $0<I^0<+\infty$, then there exist $l_1>0$, $l_2>0$ and $0<r<R$ such that

$$\begin{gathered}
f(t,u)<l_1\varphi_p(u),\qquad I_k(t,u)<l_2u,\\
(\forall t\in J,\ 0\leq u\leq r,\ 2k=1,2,\ldots,n),
\end{gathered}\tag{67}$$

where l_1 and l_2 satisfy (48) and (49), respectively.

Therefore, for $u\in K\cap\partial\Omega_r$, we have

$$\begin{aligned}
&(T_\lambda^\mu u)(t)\\
&=\int_0^t\varphi_q\left(\lambda\int_s^1\omega(r)\,f(r,u(r))\,dr\right)ds\\
&\quad+\mu\sum_{t<t_k}I_k(t_k,u(t_k))\\
&\quad+\frac{1}{a-\sigma}\left\{\int_0^1 g(t)\right.\\
&\qquad\times\left[\int_0^t\varphi_q\left(\lambda\int_s^1\omega(r)\,f(r,u(r))\,dr\right)ds\right]dt\\
&\qquad+b\varphi_q\left(\lambda\int_0^1\omega(s)\,f(s,u(s))\,ds\right)\\
&\qquad\left.+\mu\int_0^1 g(t)\sum_{t<t_k}I_k(t_k,u(t_k))\,dt\right\}
\end{aligned}$$

$$
\begin{aligned}
&\le \int_0^1 \varphi_q\left(\lambda\int_s^1 \omega(r) f(r,u(r))\,dr\right)ds \\
&\quad + \frac{1}{a-\sigma}\left\{\int_0^1 g(t)\right. \\
&\qquad \times\left[\int_0^1 \varphi_q\left(\lambda\int_s^1 \omega(r) f(r,u(r))\,dr\right)ds\right]dt \\
&\qquad \left. + b\varphi_q\left(\lambda\int_0^1 \omega(s) f(s,u(s))\,ds\right)\right\} \\
&\quad + \mu\sum_{k=1}^n I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu\int_0^1 g(t)\sum_{k=1}^n I_k(t_k,u(t_k))\,dt \\
&= \int_0^1 \varphi_q\left(\lambda\int_s^1 \omega(r) f(r,u(r))\,dr\right)ds\left(1+\frac{\sigma}{a-\sigma}\right) \\
&\quad + \frac{b}{a-\sigma}\varphi_q\left(\lambda\int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\quad + \mu\sum_{k=1}^n I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu\int_0^1 g(t)\sum_{k=1}^n I_k(t_k,u(t_k))\,dt \\
&\le \frac{a}{a-\sigma}\int_0^1 \varphi_q\left(\lambda\int_0^1 \omega(r) f(r,u(r))\,dr\right)ds \\
&\quad + \frac{b}{a-\sigma}\varphi_q\left(\lambda\int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\quad + \mu\sum_{k=1}^n I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu\int_0^1 g(t)\sum_{k=1}^n I_k(t_k,u(t_k))\,dt \\
&= \frac{a+b}{a-\sigma}\varphi_q\left(\lambda\int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\quad + \mu\sum_{k=1}^n I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu\int_0^1 g(t)\sum_{k=1}^n I_k(t_k,u(t_k))\,dt \\
&\le \frac{a+b}{a-\sigma}\varphi_q\left(\lambda\int_0^1 \omega(s) l_1\varphi_p(u(s))\,ds\right) \\
&\quad + \mu\sum_{k=1}^n l_2 u(t_k) \\
&\quad + \frac{1}{a-\sigma}\mu\int_0^1 g(t)\sum_{k=1}^n l_2 u(t_k)\,dt \\
&\le \frac{a+b}{a-\sigma}\varphi_q(l_1)\|u\|\varphi_q\left(\lambda\int_0^1 \omega(s)\,ds\right) \\
&\quad + \mu n l_2\|u\| + \frac{1}{a-\sigma}\mu\sigma n l_2\|u\| \\
&= \frac{a+b}{a-\sigma}\varphi_q(\lambda)\varphi_q(l_1)\gamma\|u\| \\
&\quad + \frac{a}{a-\sigma}\mu\sigma n l_2\|u\| \\
&\le \frac{1}{2}\|u\| + \frac{1}{2}\|u\| = \|u\|.
\end{aligned}
\tag{68}
$$

This yields

$$\left\|T_\lambda^\mu u\right\| \le \|u\|, \quad \forall u\in K\cap\partial\Omega_r. \tag{69}$$

Applying (a) of Lemma 13 to (66) and (69) yields that T_λ^μ has a fixed point $u\in K\cap(\overline{\Omega}_R\setminus\Omega_r)$ with $r\le\|u\|\le R$. Hence, since for $u\in K$ we have $u(t)\ge\delta\|u\|$ for $t\in J$, it follows that (13) holds. This gives the proof of Part (i).

Part (ii). Noticing that $f(t,u)>0$, $I_k(t,u)>0$ $(k=1,2,\ldots,n)$ for all t and $u>0$, we can define

$$
\begin{aligned}
M_R &= \max_{t\in J, 0\le u\le R}\{f(t,u)\} > 0, \\
M_R^* &= \max\{M_{Rk}, k=1,2,\ldots,n\} > 0,
\end{aligned}
\tag{70}
$$

where $R>0$, $M_{Rk}=\max_{t\in J,\,0\le u\le R}\{I_k(t,u)\}$, $k=1,2,\ldots,n$.

Let

$$\overline{\lambda}_0 \le \left(\frac{a-\sigma}{2(a+b)\gamma}R\right)^{p-1} M_R^{-1}, \qquad \overline{\mu}_0 \le \frac{(a-\sigma)R}{2\sigma n M_R^*}. \tag{71}$$

Then, for $u\in K\cap\partial\Omega_R$ and $\lambda<\overline{\lambda}_0$, $\mu<\overline{\mu}_0$, we have

$$
\begin{aligned}
&(T_\lambda^\mu u)(t) \\
&\quad = \int_0^t \varphi_q\left(\lambda\int_s^1 \omega(r) f(r,u(r))\,dr\right)ds \\
&\qquad + \mu\sum_{t<t_k} I_k(t_k,u(t_k)) \\
&\qquad + \frac{1}{a-\sigma}\left\{\int_0^1 g(t)\right. \\
&\qquad\quad \times\left[\int_0^t \varphi_q\left(\lambda\int_s^1 \omega(r) f(r,u(r))\,dr\right)ds\right]dt \\
&\qquad\quad + b\varphi_q\left(\lambda\int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\qquad\quad \left. + \mu\int_0^1 g(t)\sum_{t<t_k} I_k(t_k,u(t_k))\,dt\right\}
\end{aligned}
$$

$$\begin{aligned}
&\leq \int_0^1 \varphi_q\left(\lambda \int_s^1 \omega(r) f(r,u(r))\,dr\right) ds \\
&\quad + \frac{1}{a-\sigma}\left\{\int_0^1 g(t)\right. \\
&\qquad \times \left[\int_0^1 \varphi_q\left(\lambda \int_s^1 \omega(r) f(r,u(r))\,dr\right) ds\right] dt \\
&\qquad \left. + b\varphi_q\left(\lambda \int_0^1 \omega(s) f(s,u(s))\,ds\right)\right\} \\
&\quad + \mu \sum_{k=1}^{n} I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu \int_0^1 g(t) \sum_{k=1}^{n} I_k(t_k,u(t_k))\,dt \\
&= \int_0^1 \varphi_q\left(\lambda \int_s^1 \omega(r) f(r,u(r))\,dr\right) ds \left(1 + \frac{\sigma}{a-\sigma}\right) \\
&\quad + \frac{b}{a-\sigma}\varphi_q\left(\lambda \int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\quad + \mu \sum_{k=1}^{n} I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu \int_0^1 g(t) \sum_{k=1}^{n} I_k(t_k,u(t_k))\,dt \\
&\leq \frac{a}{a-\sigma} \int_0^1 \varphi_q\left(\lambda \int_0^1 \omega(r) f(r,u(r))\,dr\right) ds \\
&\quad + \frac{b}{a-\sigma}\varphi_q\left(\lambda \int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\quad + \mu \sum_{k=1}^{n} I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu \int_0^1 g(t) \sum_{k=1}^{n} I_k(t_k,u(t_k))\,dt \\
&= \frac{a+b}{a-\sigma}\varphi_q\left(\lambda \int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\quad + \mu \sum_{k=1}^{n} I_k(t_k,u(t_k)) \\
&\quad + \frac{1}{a-\sigma}\mu \int_0^1 g(t) \sum_{k=1}^{n} I_k(t_k,u(t_k))\,dt \\
&\leq \frac{a+b}{a-\sigma}\varphi_q\left(\lambda \int_0^1 \omega(s) M_R\,ds\right) \\
&\quad + \mu \sum_{k=1}^{n} M_R^* + \frac{1}{a-\sigma}\mu \int_0^1 g(t) \sum_{k=1}^{n} M_R^*\,dt \\
&\leq \frac{a+b}{a-\sigma}\varphi_q(\lambda)\varphi_q(M_R)\gamma + \frac{a}{a-\sigma}\mu n M_R^* \\
&< \frac{a+b}{a-\sigma}\varphi_q(\overline{\lambda}_0)\varphi_q(M_R)\gamma + \frac{a}{a-\sigma}\overline{\mu}_0 n M_R^* \\
&\leq \frac{1}{2}R + \frac{1}{2}R = R.
\end{aligned} \tag{72}$$

This implies that

$$\left\|T_\lambda^\mu u\right\| < \|u\|, \quad \forall u \in K \cap \partial\Omega_R. \tag{73}$$

If $0 < f_0 < +\infty$, $0 < I_0 < +\infty$, then there exist $l_3 > 0$, $l_4 > 0$ and $0 < r < R$ such that

$$\begin{gathered}
f(t,u) > l_3\varphi_p(u), \qquad I_k(t,u) > l_4 u \\
(\forall t \in J,\ 0 \leq u \leq r,\ k = 1,2,\ldots,n),
\end{gathered} \tag{74}$$

where l_3 and l_4 satisfy (58) and (59), respectively.

Therefore, for $u \in K \cap \partial\Omega_r$, we obtain

$$\begin{aligned}
&(T_\lambda^\mu u)(t) \\
&\quad = \int_0^t \varphi_q\left(\lambda \int_s^1 \omega(r) f(r,u(r))\,dr\right) ds \\
&\qquad + \mu \sum_{t<t_k} I_k(t_k,u(t_k)) \\
&\qquad + \frac{1}{a-\sigma}\left\{\int_0^1 g(t)\right. \\
&\qquad\quad \times \left[\int_0^t \varphi_q\left(\lambda \int_s^1 \omega(r) f(r,u(r))\,dr\right) ds\right] dt \\
&\qquad\quad + b\varphi_q\left(\lambda \int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\qquad\quad \left. + \mu \int_0^1 g(t) \sum_{t<t_k} I_k(t_k,u(t_k))\,dt\right\} \\
&\quad \geq \frac{b}{a-\sigma}\varphi_q\left(\lambda \int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\qquad + \frac{1}{a-\sigma}\mu \int_0^1 g(t) \sum_{t<t_k} I_k(t_k,u(t_k))\,dt \\
&\quad \geq \frac{b}{a-\sigma}\varphi_q\left(\lambda \int_0^1 \omega(s) f(s,u(s))\,ds\right) \\
&\qquad + \frac{1}{a-\sigma}\mu \int_{t_1}^1 g(t) I_1(t_1,u(t_1))\,dt \\
&\quad \geq \frac{b}{a-\sigma}\lambda^{q-1}\varphi_q\left(\int_0^1 \omega(s) l_3\varphi_p(u(s))\,ds\right) \\
&\qquad + \frac{\mu}{a-\sigma}\int_{t_1}^1 g(t) l_4 u(t_k)\,dt \\
&\quad \geq \frac{b}{a-\sigma}\lambda^{q-1} l_3^{q-1}\gamma\delta\|u\| \\
&\qquad + \frac{\sigma_1}{a-\sigma}\mu k l_4 \delta\|u\| \\
&\quad \geq \frac{1}{2}\|u\| + \frac{1}{2}\|u\| = \|u\|,
\end{aligned} \tag{75}$$

which implies that

$$\left\|T_\lambda^\mu u\right\| > \|u\|, \quad \forall u \in K \cap \partial\Omega_r. \tag{76}$$

Applying (a) of Lemma 13 to (73) and (76) yields that T_λ^μ has a fixed point $u \in K \cap (\overline{\Omega}_R \setminus \Omega_r)$ with $r \le \|u\| \le R$. Hence, since for $u \in K$ we have $u(t) \ge \delta\|u\|$ for $t \in J$, it follows that (13) holds. This finishes the proof of Part (ii). □

Proof of Theorem 5. Similar to the proof of Theorems 1(i) and 2(i), respectively, one can show that Theorems 5(i) and (ii) hold.

Considering Part (iii), choose two numbers r_1 and r_2 satisfying (11). By Theorems 1(i) and 2(i), there exist $\lambda_0 > 0$ and $\mu_0 > 0$ such that

$$\left\|T_\lambda^\mu u\right\| > \|u\|, \quad \forall u \in K \cap \partial\Omega_{r_i}, \quad i = 1, 2. \tag{77}$$

Since $f^0 = f^\infty = I^\infty = I^0 = 0$, from the proof of Theorem 1(i) and Theorem 2(i) and from (11), it follows that

$$\left\|T_\lambda^\mu u\right\| < \|u\|, \quad \forall u \in K \cap \partial\Omega_r, \tag{78}$$

$$\left\|T_\lambda^\mu u\right\| < \|u\|, \quad \forall u \in K \cap \partial\Omega_R. \tag{79}$$

Applying Lemma 12 to (77)–(79) yields that T_λ^μ has two fixed points u_1 and u_2 such that $u_1 \in K \cap (\overline{\Omega}_{r_1} \setminus \Omega_r)$ and $u_2 \in K \cap (\overline{\Omega}_R \setminus \Omega_{r_2})$. These are the desired distinct positive solutions of problem (7) for $\lambda_0 > 0$ and $\mu_0 > 0$ satisfying (17). Then, the result of Part (iii) follows. □

Proof of Theorem 6. Similar to the proof of Theorems 1(ii) and 2(ii), respectively, one can show that Theorems 6(i) and (ii) hold.

Now, considering Part (iii), choose two numbers r_1 and r_2 satisfying (11). By Theorems 1(ii) and 2(ii), there exist $\overline{\lambda}_0 > 0$ and $\overline{\mu}_0 > 0$ such that

$$\begin{gathered} \left\|T_\lambda^\mu u\right\| < \|u\|, \quad \forall 0 < \lambda < \overline{\lambda}_0, \\ 0 < \mu < \overline{\mu}_0, \qquad u \in K \cap \partial\Omega_{r_i}, \quad i = 1, 2. \end{gathered} \tag{80}$$

Since $f_0 = f_\infty = I_\infty = I_0 = \infty$, from the proof of Theorems 1(ii) and 2(ii) and from (11), it follows that

$$\left\|T_\lambda^\mu u\right\| > \|u\|, \quad \forall u \in K \cap \partial\Omega_r, \tag{81}$$

$$\left\|T_\lambda^\mu u\right\| > \|u\|, \quad \forall u \in K \cap \partial\Omega_R. \tag{82}$$

Applying Lemma 13 to (80)–(82) yields that T_λ^μ has two fixed points u_1 and u_2 such that $u_1 \in K \cap (\overline{\Omega}_{r_1} \setminus \Omega_r)$ and $u_2 \in K \cap (\overline{\Omega}_R \setminus \Omega_{r_2})$. These are the desired distinct positive solutions of problem (7) for $0 < \lambda < \overline{\lambda}_0$ and $0 < \mu < \overline{\mu}_0$ satisfying (18). Then, proof of Part (iii) is complete. □

5. An Example

To illustrate how our main results can be used in practice, we present an example.

Example 1. For $p = 3/2$, consider the following boundary value problem:

$$\begin{gathered} -\left(\varphi_p\left(u'\right)\right)' = \lambda(t(1-t))^{-1/2}\left(t^2+1\right)^{1/2} u^{p-1}, \quad t \in J, \\ \Delta u|_{t=1/2} = \mu I_1\left(\frac{1}{2}, u\left(\frac{1}{2}\right)\right), \\ \Delta u'\big|_{t=1/2} = 0, \\ u(0) - 2u'(0) = \int_0^1 \frac{1}{2} u(t)\, dt, \qquad u'(1) = 0. \end{gathered} \tag{83}$$

Evidently, $u(t) \equiv 0$ is the trivial solution of problem (83).

6. Conclusion

Problem (83) has at least one positive solution for any $\lambda > \sqrt{6}/4\pi$ and $\mu > 3$.

Proof. Problem (83) can be regarded as a problem of the form (7), where $a = 1$, $b = 2$, and

$$\begin{gathered} \omega(t) = (t(1-t))^{-1/2}, \qquad f(t,u) = \left(t^2+1\right)^{1/2} u^{p-1}, \\ I_1(t,u) = (1+t)\,u, \quad g(t) \equiv \frac{1}{2}, \quad \forall t \in J. \end{gathered} \tag{84}$$

It follows from the definition of ω, f, and g that (H_1)–(H_4) hold, and $\omega(t)$ is singular at $t = 0$ and $t = 1$. By calculating, we have

$$\begin{gathered} \int_0^1 \omega(t)\, dt = \pi, \quad 1 \le f_\infty \le f^\infty \le 2, \\ 1 \le I_\infty \le I^\infty \le 2, \\ q = 3, \qquad \delta = \frac{1}{3}, \qquad \sigma = \frac{1}{2}, \qquad \sigma_1 = \frac{1}{4}, \\ \gamma = \pi^2, \qquad m_r = \sqrt{\frac{1}{3}r}, \qquad m_r^* = \frac{1}{3}r, \\ \lambda_0 \ge \frac{\sqrt{6}}{4\pi}, \qquad \mu_0 \ge 3, \end{gathered} \tag{85}$$

where $r > 0$ is a constant.

Hence, by Theorem 1(i), the conclusion follows, and the proof is complete. □

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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