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Research Article

Stability of Infinite Dimensional Interconnected Systems with Impulsive and Stochastic Disturbances

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Some research on the stability with mode constraint for a class of infinite dimensional look-ahead interconnected systems with impulsive and stochastic disturbances is studied by using the vector Lyapunov function approach. Intuitively, the stability with mode constraint is the property of damping disturbance propagation. Firstly, we derive a set of sufficient conditions to assure the stability with mode constraint for a class of general infinite dimensional look-ahead interconnected systems with impulsive and stochastic disturbances. The obtained conditions are less conservative than the existing ones. Secondly, the controller for a class of look-ahead vehicle following systems with the above uncertainties is constructed by the sliding mode control method. Based on the obtained new stability conditions, the domain of the control parameters of the systems is proposed. Finally, a numerical example with simulations is given to show the effectiveness and correctness of the obtained results.

1. Introduction

In the real industries, the control problem of many complex dynamic systems can be translated into the stability analysis. At present, there have been lots of research results about the stability analysis for the finite dimensional interconnected systems; see [1–11]. Nevertheless, considering that the connections or disconnections between the subsystems of the real interconnected systems are uncertain, which means that the dimension of the interconnected systems is uncertain, the interconnected systems can be described by infinite dimensional equations. On the other hand, there are some unavoidable disturbances in the real systems, such as stochastic disturbance and impulsive disturbance. Therefore some researchers have given stability analysis for some finite dimensional complex dynamic systems with impulsive and stochastic disturbances; see [6–11].

The applied methods presented in [1–11] are based on the scalar Lyapunov function approach or the LMI tool. In fact, the LMI method is essentially a kind of the method using the scalar Lyapunov function method. It should be noted that until now there is no general constructive method for building the Lyapunov functions for nonlinear systems. In comparison with the vector Lyapunov function method, the scalar method or the LMI method needs to discuss the convergence of the scalar Lyapunov function when analyzing the stability of infinite dimensional systems. Hence the vector Lyapunov function method is more efficient. The research team led by Professor Zhang has studied the stability of some infinite dimensional nonlinear interconnected systems with stochastic disturbances based on the vector Lyapunov function approach and obtained some important stability results; see [12–14].

The obtained stability results in [2–12, 14] are focused on the stability of the steady state of the systems without considering the size relationship of the state variables when the systems converge to steady-state process. For example, considering interconnected system $\dot{x} = f(t, x)$ (here $x = \mathrm{col}(x_1, x_2, \ldots, x_n)$), the states are needed to be satisfied that $\sup \|x_1(t)\|_{t\geq 0} \geq \sup \|x_2(t)\|_{t\geq 0} \geq, \ldots, \geq \sup \|x_n(t)\|_{t\geq 0}$ or $\sup \|x_1(t)\|_{t\geq 0} \geq \max_{i=2,3,\ldots}\{\sup \|x_i(t)\|_{t\geq 0}\}$. The stability with the above constraint condition is named the stability with mode constraint. The Lyapunov stability in the general sense cannot describe the stability with mode constraint condition, and the existing Lyapunov function methods cannot be used to analyze the stability with mode constraint for the systems directly. The motivation for the stability problem with mode constraint comes from the analysis and the design of controllers for automated highway system [13], multirobot operation system [15], formation flying of unmanned aerial vehicles [16], and so on. In a formation one wants controllers to be designed so that any shock-wave arising from disturbance propagation should dampen as it travels away from the source. In other words the closed loop interconnected system for the formation needs to be stable with constraint condition.

Automatic vehicle longitudinal following control is an important issue for coordinated control for a group of unmanned autonomous vehicles in automated highway system (for short, AHS). In AHS, vehicles are dynamically coupled by feedback control laws. The control objective is to dramatically improve the traffic flow capacity on a highway by enabling vehicles to travel together in tightly spaced platoons [15]. Therefore, the controller design for vehicle longitudinal following systems (for short, VLFS) is an interesting and challenging problem. Some significant research on the string stability analysis for VLFS has been done; see [17, 18]. Nevertheless, uncertain disturbance factors were not considered in [17, 18]. Uncertainties inevitably exist in the vehicle operating environment and vehicle systematic itself. In [19], some sufficient conditions, which assure the string stability of a class of stochastic VLFS with infinite dimensions, were obtained by using the vector Lyapunov function method. Since the Cauchy inequality technique was applied to deduce the stability conditions for the systems, the obtained criteria were relatively conservative. Besides, the controller design for VLFS was not studied in there. In [20], the problem of stabilization control system for a single vehicle in response to the exogenous impulsive disturbances was studied. The obtained results cannot be used to analyze the stability and controller design for the VFLS directly.

To design the controller for the VFLS, there are various approaches, such as fuzzy control [21], sliding mode control [18], adaptive control [15], adaptive-sliding mode control [22], and fuzzy-sliding mode control [23]. However, the factor of uncertainties to the systems was not considered in the above references. On the other hand, the number of vehicles in VFLS is indeterminate as vehicles enter into or leave the platoon randomly. Therefore, the VLFS can only be described as infinite dimensional interconnected system.

To sum up, this paper will present some sufficient conditions for assuring the stability with mode constraint for a class of infinite dimensional look-ahead interconnected systems with impulsive and stochastic disturbances by using the vector Lyapunov function approach. Furthermore, the controller for a class of look-ahead VLFS with the above uncertainties is constructed by the sliding mode control method. Based on the obtained new stability conditions, the domain of the control parameters of the systems is proposed.

2. Mathematical Preliminaries

For convenience, some notations are introduced as follows:

$$\begin{gathered}
\|y_i\|_\infty^p = \|y_i(\cdot)\|_\infty^p = \sup_{t\ge 0} E\left[|y_i(t)|^p\right], \\
\|y(0)\|_\infty^p = \sup_i E\left[|y_i(0)|^p\right], \\
\|y_i\|_p = \|y_i(\cdot)\|_p = \left(\int_0^\infty E\left[|y_i(t)|^p\right]dt\right)^{1/p}, \\
\|y(0)\|_p = \left(\sum_{i=1}^{\infty} E\left[|y_i(0)|^p\right]\right)^{1/p}, \\
\|y(0)\|_\infty^{i-1} = \max_{1\le j\le i-1} \|y_j(0)\|, \\
\|y(t)\|_\infty^{i-1} = \max_{1\le j\le i-1} \sup_{t\ge 0} \|y_j(t)\|,
\end{gathered} \tag{1}$$

where $|\cdot|$ is the Euclidean norm and $0 < p < \infty$, $i \in \mathbf{N}$, E denotes the expectation of stochastic process, and $\mathbf{N}$ denotes the set of natural numbers.

2.1. Model Description. Consider a class of general nonlinear infinite dimensional look-ahead interconnected systems with stochastic and impulsive disturbances described by

$$\begin{gathered}
dy_i = f_i(y_i, y_{i-1}, \dot{y}_{i-1})\,dt + q_i(y_i, y_{i-1}, \dot{y}_{i-1})\,d\xi_i, \quad t \ne t_k \\
\Delta y_i(t_k) = y_i(t_k^+) - y_i(t_k^-), \quad t = t_k;
\end{gathered} \tag{2}$$

here, $t \in [t_0, +\infty)$ and $y_i \in R^n$ denotes the state of the ith subsystem, $i \in \mathbf{N}$. $\Delta y_i(t_k)$ is the impulsive strength at discrete moment t_k, and the discrete set $\{t_k\}$ is assumed to be satisfied that $0 \le t_0 < t_1 < \cdots < t_k < \cdots$, and $k \to \infty$ as $t_k \to \infty$, $k \in \mathbf{N}$. It is assumed that $y_i(t)$ is right, continuous with $y_i(t_k) = y_i(t_k^+)$, and $y_i(t_k^-) = \lim_{t\to t_k^-} y_i(t)$. Consider f_i, q_i: $R^n \times R^n \times R^n \to R^n$; let $f_i(0,0,0) = q_i(0,0,0) = 0$. Assume that ξ_i is the one-dimensional independent standard Wiener processes defined on space (Ω, F, P); here Ω denotes sample space, F denotes σ algebra of subset of the sample space, and P denotes probability measures.

The system (2) can be treated as an interconnection of isolated subsystems x_i given by

$$\begin{gathered}
dy_i = f_i(y_i, 0, 0)\,dt + q_i(y_i, 0, 0)\,d\xi_i, \quad t \ne t_k, \\
\Delta y_i(t_k) = y_i(t_k^+) - y_i(t_k^-), \quad t = t_k, \\
i, k \in \mathbf{N}.
\end{gathered} \tag{3}$$

2.2. Definitions and Assumptions. Let $y_i = 0$ ($i \in \mathbf{N}$) be the unique zero solution of system (2).

Definition 1 (see [19]). If, for $\forall\varepsilon > 0$, $\exists\delta > 0$ such that $\|y(0)\|_\infty^2 < \delta \Rightarrow \sup_i \|y_i(t)\|_\infty^2 < \varepsilon$, then $y_i = 0$ ($i \in \mathbf{N}$) is string stable in the mean square sense.

It should be noted that the string stability in this paper is defined for look-ahead interconnected system, which is a special class of interconnected systems. The string stability could guarantee that the state of every subsystem is uniformly bounded if the initial states of the subsystems are bounded.

Definition 2 (see [19]). The zero solution of system (2) $y_i = 0$ $(i \in \mathbf{N})$ is string exponentially stable in the mean square sense if $y_i = 0$ is string stable, and there exist constants $M > 0$ and $\lambda > 0$ such that $\|y_i(t)\|_\infty^2 < M\|y_i(0)\|^2 e^{-\lambda t}$ holds.

Definition 3 (see [13]). The zero solution of system (2) $y_i = 0$ $(i \in \mathbf{N})$ is string exponentially stable with mode constraint in the mean square sense if $y_i = 0$ is string exponentially stable in the mean square sense, and the inequality $\|y_i(t)\|_\infty^2 \le (\|y(t)\|_\infty^{i-1})^2$ holds, $t \ge 0$, $i \ge 2$.

Next, some assumptions will be given for the system (2) and the system (3).

Assumption A1. There exist positive constants k_l^f, k_l^q $(l = 1, 2)$ and d_1^f, d_1^q such that f and q are globally Lipschitz in their arguments; that is,

$$\begin{aligned} &|f(y_1, y_2, y_3) - f(z_1, z_2, z_3)| \\ &\quad \le k_1^f |y_1 - z_1| + k_2^f |y_2 - z_2| + d_1^f |y_3 - z_3|, \\ &|q(y_1, y_2, y_3) - q(z_1, z_2, z_3)| \\ &\quad \le k_1^q |y_1 - z_1| + k_2^q |y_2 - z_2| + d_1^q |y_3 - z_3|. \end{aligned} \tag{4}$$

Assumption A2. For every isolated subsystem (3), there exists positive definite function $V_i(t, y_i(t))$, $y_i \in R^n$, which is continuously twice differentiable with respect to y_i, and there exist positive constants α_m $(m = 1, 2, 3, 4, 5)$ such that

(i) $\alpha_1 |y_i|^2 \le V_i(t, y_i(t)) \le \alpha_2 |y_i|^2$;

(ii) $\chi_{(2)} V_i(t, y_i(t)) \le -\alpha_3 |y_i|^2$;

(iii) $|\partial V_i(t, y_i(t))/\partial y_{i,j}| \le \alpha_4 |x_i|$, $|\partial^2 V_i(t, y_i(t))/\partial y_{i,l} y_{i,j}| \le \alpha_5$,

where $\chi_{(2)}(\cdot)$ is an operator associated with (3) defined by

$$\begin{aligned} \chi_{(2)}(\cdot) = {} & \frac{\partial(\cdot)}{\partial t} + \sum_{j=1}^{n} \frac{\partial(\cdot)}{\partial x_{i,j}} f_j(y_i, 0, 0) \\ & + 0.5 \sum_{l=1}^{n} \sum_{j=1}^{n} \frac{\partial^2(\cdot)}{\partial y_{i,j} \partial y_{i,l}} \sigma_{lj}(y_i, 0, 0), \end{aligned} \tag{5}$$

where $\sigma_{lj}(y_i, 0, 0) = q_l(y_i, 0, 0) q_j(y_i, 0, 0)$, $l, j = 1, 2, \dots, n$, $i \in \mathbf{N}$.

Let V_i denote $V_i(y_i(t))$ if there is no confusion, $i \in \mathbf{N}$.

Assumption A3. Let $\Delta y_i(t_k) = I_{ik}(y_i(t_k^-))$. The function $I_{ik}(\cdot)$ with $I_{ik}(0) = 0$ denotes the strength of the intensity of impulsive disturbances, which is assumed to be continuous with t. Suppose that there exist positive constants $\underline{\gamma}_{ik}$ and $\overline{\gamma}_{ik}$ such that $\underline{\gamma}_{ik} |y_i(t_k^-)| \le |I_{ik}(y_i(t_k^-)) + y_i(t_k^-)| \le \overline{\gamma}_{ik} |y_i(t_k^-)|$ holds; here, $0 < \underline{\gamma}_{ik} < 1 < \overline{\gamma}_{ik}$, $i, k \in \mathbf{N}$.

Next a lemma established by us in [24] is given which will be used in the proof of the following theorem.

Lemma 4. *Suppose that $W_i(t) \ge 0$, and $E[W_i(t)] < \infty$, $t \ge 0$, $i \in \mathbf{N}$. Consider the following inequalities:*

$$\begin{aligned} & E\chi_{(1)}(W_i) \\ & \quad \le g_i(E(W_i), E(W_{i-1}), \dots, E(W_1), t) \\ & \qquad \times \left\{ -\beta_{i0} E(W_i^{m_{ii}}) + \sum_{j=1}^{\infty} \beta_{ij} E(W_{i-j}^{m_{ij}}) \right. \\ & \qquad \left. + [E(W_i^{m_{ii}})]^{-1} \left[\sum_{j=1}^{\infty} \beta'_{ij} E(W_{i-j}^{m_{ij}}) \right]^2 \right\} \end{aligned} \tag{6}$$

for $W_i > 0$; it is assumed that $g_i(\cdot) > 0$. Let $\beta_{i0} > 0$, $\beta_{ij} \ge 0$, $\beta'_{ij} \ge 0$, and $\beta_{ij} = \beta'_{ij} = 0$ $(j \ge i)$, $m_{ii} \le m_{ij}$, $i, j \in \mathbf{N}$. $\chi_{(1)}(\cdot)$ is an operator associated with (2) given by

$$\begin{aligned} \chi_{(1)}(\cdot) = {} & \frac{\partial(\cdot)}{\partial t} + \sum_{j=1}^{n} \frac{\partial(\cdot)}{\partial y_{i,j}} f_j(y_i, y_{i-1}, \dot{y}_{i-1}) \\ & + 0.5 \sum_{l=1}^{n} \sum_{j=1}^{n} \frac{\partial^2(\cdot)}{\partial y_{i,j} \partial y_{i,l}} \sigma_{lj}(y_i, y_{i-1}, \dot{y}_{i-1}), \\ \sigma_{lj}(y_i, y_{i-1}, \dot{y}_{i-1}) = {} & q_l(y_i, y_{i-1}, \dot{y}_{i-1}) q_j(y_i, y_{i-1}, \dot{y}_{i-1}), \\ & l, j = 1, 2, \dots, n, \end{aligned} \tag{7}$$

where $y_i = [y_{i1}, y_{i2}, \dots, y_{in}] \in R^n$.

If there exists $W = (E(W_{10}), E(W_{20}), \dots)$ such that

$$\begin{aligned} & -\beta_{i0} E(W_{i,0}^{m_{ii}}) + \sum_{j=1}^{\infty} \beta_{ij} E(W_{i-j,0}^{m_{ij}}) \\ & + \left[E(W_{i,0}^{m_{ii}}) \right]^{-1} \left[\sum_{j=1}^{\infty} \beta'_{ij} E(W_{i-j,0}^{m_{ij}}) \right]^2 < 0, \end{aligned} \tag{8}$$

then, for $\forall \varepsilon > 0$, $\exists \delta > 0$ such that $\|W(0)\|_\infty < \delta \Rightarrow \sup_i \|W_i(t)\|_\infty < \varepsilon$.

3. Stability Results

In this section, some sufficient conditions for judging the string exponential stability with mode constraint in the mean square for system (2) will be established.

Theorem 5. *Suppose that Assumptions A1–A3 are satisfied. If there exist constants $\eta > 0$ and $\xi > 0$ such that $2\ln\overline{\eta}_k/(t_k - t_{k-1}) \le \eta < \xi$ and if the following inequality holds,*

$$\begin{aligned}&(-\alpha_3+\xi\alpha_2)\left(\underline{\eta}_k\right)^2\alpha_2^{-1/2}+(\overline{\eta}_k)^2 n\alpha_4\alpha_1^{-1/2}\left(k_2^f+d_1^f k_1^f\right)\\&\quad\times\sum_{j=1}^{i-1}\left(d_1^f\right)^{j-1}+\frac{1}{2}(\overline{\eta}_k)^2 n^2\alpha_5\\&\quad\times\left[\alpha_1^{-1/2}\left(k_2^q+d_1^q k_1^q\right)\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\right]\\&\quad\times\left[2k_1^q+\left(\frac{\alpha_2}{\alpha_1}\right)^{1/2}\left(k_2^q+d_1^q k_1^q\right)\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\right]<0,\end{aligned}\tag{9}$$

where $\overline{\eta}_k = \sup_i\{\overline{\gamma}_{ik}\}$, $\underline{\eta}_k = \inf_i\{\underline{\gamma}_{ik}\}$, $k \in \mathbf{N}$, then the zero solution of system (2) is string exponentially stable with mode constraint in the mean square sense.

Proof. As mentioned in Assumption A2, the function V_i is the vector Lyapunov function of the ith isolated subsystem of the interconnected system (2). According to the vector Lyapunov function theory, in order to obtain the exponential string stability conditions with mode constraint for system (2), we choose the following vector Lyapunov function:

$$W_i = e^{\xi t}V_i, \quad i \in \mathbf{N}. \tag{10}$$

When $t \ne t_k$, $k \in \mathbf{N}$, calculating the operator $\chi_{(1)}W_i$ along the zero solution of system (2) and applying Assumptions A1-A2, we get

$$\begin{aligned}\chi_{(1)}W_i &= \xi e^{\xi t}V_i + e^{\xi t}\\&\quad\times\left[\sum_{j=1}^{n}\frac{\partial V_i}{\partial y_{i,j}}f_j\left(y_i,y_{i-1},\dot{y}_{i-1}\right)\right.\\&\qquad\left.+\frac{1}{2}\sum_{l=1}^{n}\sum_{j=1}^{n}\frac{\partial^2 V_i}{\partial y_{i,l}\partial y_{i,j}}\sigma_{lj}\left(y_i,y_{i-1},\dot{y}_{i-1}\right)\right]\\&\le \xi e^{\xi t}V_i+e^{\xi t}\\&\quad\times\left\{-\alpha_3\left|y_i\right|^2+n\alpha_4\left|y_i\right|\left(k_2^f+d_1^f k_1^f\right)\right.\\&\qquad\times\sum_{j=1}^{i-1}\left(d_1^f\right)^{j-1}\left|y_{i-j}\right|+0.5n^2\alpha_5\\&\qquad\times\left[2k_1^q\left|y_i\right|+\left(k_2^q+d_1^q k_1^q\right)\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\left|y_{i-j}\right|\right]\\&\qquad\left.\times\left[\left(k_2^q+d_1^q k_1^q\right)\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\left|y_{i-j}\right|\right]\right\}\\&\le\left(\left|y_i\right|e^{\xi t/2}\right)\\&\quad\times\left\{\xi\alpha_2 e^{\xi t/2}\left|y_i\right|-\alpha_3 e^{\xi t/2}\left|y_i\right|\right.\\&\qquad+n\alpha_4\left(k_2^f+d_1^f k_1^f\right)\\&\qquad\times\sum_{j=1}^{i-1}\left(d_1^f\right)^{j-1}e^{\xi t/2}\left|y_{i-j}\right|\\&\qquad+0.5n^2\alpha_5\left[2k_1^q+\left(k_2^q+d_1^q k_1^q\right)\right.\\&\qquad\qquad\left.\times\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\frac{\left|y_{i-j}\right|}{\left|y_i\right|}\right]\\&\qquad\left.\times\left[\left(k_2^q+d_1^q k_1^q\right)\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}e^{\xi t/2}\left|y_{i-j}\right|\right]\right\}.\end{aligned}\tag{11}$$

It follows from Assumption A2 and $W_i = e^{\xi t}V_i$ that

$$\begin{aligned}\chi_{(1)}W_i &\le \left(\left|y_i\right|e^{\xi t/2}\right)\\&\quad\times\left\{\left(-\alpha_3+\xi\alpha_2\right)\left(\frac{W_i}{\alpha_2}\right)^{1/2}+n\alpha_4\left(k_2^f+d_1^f k_1^f\right)\right.\\&\qquad\times\sum_{j=1}^{i-1}\left(d_1^f\right)^{j-1}\left(\frac{W_{i-j}}{\alpha_1}\right)^{1/2}+\frac{1}{2}n^2\alpha_5\\&\qquad\times\left[2k_1^q+\left(k_2^q+d_1^q k_1^q\right)\right.\\&\qquad\qquad\left.\times\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\left(\frac{W_{i-j}}{W_i}\right)^{1/2}\left(\frac{\alpha_2}{\alpha_1}\right)^{1/2}\right]\\&\qquad\left.\times\left[\left(k_2^q+d_1^q k_1^q\right)\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\left(\frac{W_{i-j}}{\alpha_1}\right)^{1/2}\right]\right\}.\end{aligned}\tag{12}$$

From the properties of the operator $\chi_{(1)}$ [25], we can take the expectation of inequality (12) and rewrite it as

$$\begin{aligned}&E\chi_{(1)}W_i\\&\quad\le E\left[\left(\left|y_i\right|\right)\right]e^{\xi t/2}\end{aligned}$$

$$\times\left\{(-\alpha_3+\xi\alpha_2)E\left[\left(\frac{W_i}{\alpha_2}\right)^{1/2}\right]\right.$$
$$+n\alpha_4\left(k_2^f+d_1^f k_1^f\right)$$
$$\times\sum_{j=1}^{i-1}\left(d_1^f\right)^{j-1}E\left[\left(\frac{W_{i-j}}{\alpha_1}\right)^{1/2}\right]+\frac{1}{2}n^2\alpha_5$$
$$\times\left[2k_1^q+(k_2^q+d_1^q k_1^q)\sum_{j=1}^{i-1}(d_1^q)^{j-1}\right.$$
$$\left.\times E\left[\left(\frac{W_{i-j}}{W_i}\right)^{1/2}\right]\left(\frac{\alpha_2}{\alpha_1}\right)^{1/2}\right]$$
$$\left.\times\left[(k_2^q+d_1^q k_1^q)\sum_{j=1}^{i-1}(d_1^q)^{j-1}E\left[\left(\frac{W_{i-j}}{\alpha_1}\right)^{1/2}\right]\right]\right\}. \tag{13}$$

Taking $E(W_{i-j,0})=1$, $j=0,1,\ldots,i-1$, $i\in\mathbf{N}$. Substituting them into (13), we get

$$E\chi_{(1)}W_i$$
$$\leq E\left[\left(|y_i|\right)\right]e^{\xi t/2}$$
$$\times\left\{(-\alpha_3+\xi\alpha_2)(\alpha_2)^{-1/2}+n\alpha_4\left(k_2^f+d_1^f k_1^f\right)\right.$$
$$\times\sum_{j=1}^{i-1}\left(d_1^f\right)^{j-1}(\alpha_1)^{-1/2}+\frac{1}{2}n^2\alpha_5$$
$$\times\left[2k_1^q+(k_2^q+d_1^q k_1^q)\right. \tag{14}$$
$$\left.\times\sum_{j=1}^{i-1}(d_1^q)^{j-1}\left(\frac{\alpha_2}{\alpha_1}\right)^{1/2}\right]$$
$$\left.\times\left[(k_2^q+d_1^q k_1^q)\sum_{j=1}^{i-1}(d_1^q)^{j-1}(\alpha_1)^{-1/2}\right]\right\}.$$

Since inequality (12) implies that the following inequality holds,

$$(-\alpha_3+\xi\alpha_2)\alpha_2^{-1/2}+n\alpha_4\alpha_1^{-1/2}\left(k_2^f+d_1^f k_1^f\right)$$
$$\times\sum_{j=1}^{i-1}\left(d_1^f\right)^{j-1}+\frac{1}{2}n^2\alpha_5\left[\alpha_1^{-1/2}(k_2^q+d_1^q k_1^q)\sum_{j=1}^{i-1}(d_1^q)^{j-1}\right]$$
$$\times\left[2k_1^q+\left(\frac{\alpha_2}{\alpha_1}\right)^{1/2}(k_2^q+d_1^q k_1^q)\sum_{j=1}^{i-1}(d_1^q)^{j-1}\right]<0, \tag{15}$$

it can be concluded that $E[\chi_{(1)}W_i]<0$, $i\in\mathbf{N}$. Therefore, it follows from Lemma 4 that, for $\forall\varepsilon_0>0$, $\exists\delta_0>0$ such that $\|W_i(0)\|_\infty<\delta_0\Rightarrow\sup_i\|W_i(t)\|_\infty<\varepsilon_0$. Let $\varepsilon>0$, and satisfy $\varepsilon_0=\alpha_1\varepsilon^2$. Take $\delta_0=\alpha_2\delta^2$. When $\sup_i\|x_i(0)\|<\delta$, we get $W_i(0)=V_i(0)\leq\alpha_2\|x_i(0)\|^2\leq\alpha_2\delta^2=\delta_0$. Furthermore, we have $\sup_i\|W_i(t)\|_\infty=\sup_i\|V_i(t)\|_\infty e^{\xi t}<\varepsilon_0$; namely,

$$\sup_i\|y_i(t)\|_\infty\leq\sqrt{\frac{\sup_i\|V_i(t)\|_\infty e^{\xi t}}{\alpha_1}}e^{-\xi t/2}$$
$$<\sqrt{\frac{\varepsilon_0}{\alpha_1}}e^{-\xi t/2}=\varepsilon e^{-\xi t/2}, \tag{16}$$
$$0\leq t<t_1.$$

Next, we will use the mathematical induction method to prove that

$$\sup_i\|y_i(t)\|_\infty<\overline{\eta}_0\overline{\eta}_1\overline{\eta}_2\cdots\overline{\eta}_{k-1}\varepsilon e^{-\xi t/2},$$
$$t_{k-1}\leq t<t_k,\quad i,k\in\mathbf{N}, \tag{17}$$

hold, where $\overline{\eta}_0=1$; namely,

$$\sup_i\|y_i(t)\|_\infty^2<\left(\overline{\eta}_0\overline{\eta}_1\overline{\eta}_2\cdots\overline{\eta}_{k-1}\right)^2\varepsilon^2e^{-\xi t},$$
$$t_{k-1}\leq t<t_k,\quad i,k\in\mathbf{N}. \tag{18}$$

When $k=1$, it can be seen from (16) that (18) holds. Suppose that the following inequalities hold:

$$\sup_i\|y_i(t)\|_\infty^2<\left(\overline{\eta}_0\overline{\eta}_1\overline{\eta}_2\cdots\overline{\eta}_{k-1}\right)^2\varepsilon^2e^{-\xi t},$$
$$t_{p-1}\leq t<t_p,\quad p=1,2,\ldots,k. \tag{19}$$

From Assumption A3 and (19), we have

$$\|y_i(t_k^+)\|_\infty^2=\|I_{ik}(y_i(t_k^-))+y_i(t_k^-)\|_\infty^2\leq\overline{\gamma}_{ik}^2\|y_i(t_k^-)\|_\infty^2$$
$$\leq\overline{\eta}_k^2\|y_i(t_k^-)\|_\infty^2, \tag{20}$$
$$i,k\in\mathbf{N}.$$

Due to $\overline{\eta}_k>1$, we have

$$\|y_i(t)\|_\infty^2<\left(\overline{\eta}_0\overline{\eta}_1\overline{\eta}_2\cdots\overline{\eta}_{k-1}\overline{\eta}_k\right)^2\varepsilon^2e^{-\xi t},$$
$$t_{k-1}\leq t\leq t_k,\quad i,k\in\mathbf{N}. \tag{21}$$

We claim that (21) implies that

$$\|y_i(t)\|_\infty^2<\left(\overline{\eta}_0\overline{\eta}_1\overline{\eta}_2\cdots\overline{\eta}_{k-1}\overline{\eta}_k\right)^2\varepsilon^2e^{-\xi t},$$
$$t_k\leq t<t_{k+1},\quad i,k\in\mathbf{N}. \tag{22}$$

If inequality (22) does not hold, there exist some i and $t^*\in[t_k,t_{k+1})$ such that

$$\|y_i(t^*)\|_\infty^2=\left(\overline{\eta}_0\overline{\eta}_1\overline{\eta}_2\cdots\overline{\eta}_{k-1}\overline{\eta}_k\right)^2\varepsilon^2e^{-\xi t}, \tag{23}$$

$\chi_{(1)}W_i(t^*) = D^+W_i(t^*) \geq 0$, and

$$\left\|y_j(t)\right\|_\infty^2 \leq \left(\overline{\eta}_0\overline{\eta}_1\overline{\eta}_2\cdots\overline{\eta}_{k-1}\overline{\eta}_k\right)^2\varepsilon^2 e^{-\xi t}, \quad t \in \left[t_k, t^*\right],\ j = 1,2,\ldots,i-1. \tag{24}$$

Substituting (23) and inequality (24) into inequality (11), together with the properties of the operator χ, we get

$$\begin{aligned} E\left[\chi_{(1)}W_i\left(t^*\right)\right] &\leq \left\|y_i\right\|\Bigg\{\left(-\alpha_3+\xi\alpha_2\right)+n\alpha_4\left(k_2^f+d_1^f k_1^f\right) \\ &\quad\times\sum_{j=1}^{i-1}\left(d_1^f\right)^{j-1}+n^2\alpha_5 k_1^q\left(k_2^q+d_1^q k_1^q\right) \\ &\quad\times\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}+\frac{1}{2}n^2\alpha_5 \\ &\quad\times\left[\left(k_2^q+d_1^q k_1^q\right)\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\right]^2\Bigg\} \\ &\quad\times\varepsilon\overline{\eta}_0\overline{\eta}_1\overline{\eta}_2\cdots\overline{\eta}_{k-1}\overline{\eta}_k. \end{aligned} \tag{25}$$

Since condition (12) implies that

$$\begin{aligned} &\left(-\alpha_3+\xi\alpha_2\right)+n\alpha_4\left(k_2^f+d_1^f k_1^f\right)\sum_{j=1}^{i-1}\left(d_1^f\right)^{j-1} \\ &\quad+n^2\alpha_5 k_1^q\left(k_2^q+d_1^q k_1^q\right)\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1} \\ &\quad+\frac{1}{2}n^2\alpha_5\left[\left(k_2^q+d_1^q k_1^q\right)\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\right]^2<0, \end{aligned} \tag{26}$$

$E[\chi_{(1)}W_i(t^*)] < 0$. This is a contradiction with $\chi_{(1)}W_i(t^*) \geq 0$. By the mathematical induction method, it can be concluded that

$$\sup_i\left\|y_i(t)\right\|_\infty < \overline{\eta}_0\overline{\eta}_1\overline{\eta}_2\cdots\overline{\eta}_{k-1}\overline{\eta}_k\varepsilon e^{-\xi t/2}, \quad t_{k-1}\leq t<t_k, \quad k\in\mathbf{N}. \tag{27}$$

From condition $2\ln\overline{\eta}_k/(t_k-t_{k-1}) \leq \eta < \xi$, we get that $\overline{\eta}_k \leq e^{(1/2)\eta(t_k-t_{k-1})}$, $k \in \mathbf{N}$. Furthermore,

$$\begin{aligned} \sup_i\left\|y_i(t)\right\|_\infty &< e^{(1/2)\eta(t_1-t_0)}e^{(1/2)\eta(t_2-t_1)}\cdots e^{(1/2)\eta(t_{k-1}-t_{k-2})} \\ &\quad\times\varepsilon e^{-(1/2)\xi t}\leq\varepsilon e^{-(1/2)(\xi-\eta)t}. \end{aligned} \tag{28}$$

That is, $\sup_i\|y_i(t)\|_\infty^2 < \varepsilon^2 e^{-(\xi-\eta)t}$, $t \geq t_0$.

According to **Definition 2**, the zero solution of system (2) is the string exponentially stable in the mean square sense with convergence rate $\xi - \eta$.

Next, we proceed to prove that the zero solution of system (2) is stable with the mode constraint. From the previous analysis, when $t \neq t_k$, $k \in \mathbf{N}$, it is easy to obtain

$$\begin{aligned} \chi_{(1)}V_i &\leq -\alpha_3\left|y_i\right|^2+n\alpha_4\left|y_i\right|\left(k_2^f+d_1^f k_1^f\right)\left|y\right|_\infty^{i-1}\sum_{j=1}^{i-1}\left(d_1^f\right)^{j-1} \\ &\quad+0.5n^2\alpha_5\left[2k_1^q\left|y_i\right|+\left(k_2^q+d_1^q k_1^q\right)\left|y\right|_\infty^{i-1}\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\right] \\ &\quad\times\left[\left(k_2^q+d_1^q k_1^q\right)\left|y\right|_\infty^{i-1}\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\right] \\ &\leq -\alpha_3\left|y_i\right|^2+n\alpha_4\left(k_2^f+d_1^f k_1^f\right) \\ &\quad\times\sum_{j=1}^{i-1}\left(d_1^f\right)^{j-1}\frac{\left[\left|y_i\right|_\infty^2+\left(\left|y\right|_\infty^{i-1}\right)^2\right]}{2} \\ &\quad+n^2\alpha_5 k_1^q\left(k_2^q+d_1^q k_1^q\right) \\ &\quad\times\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\frac{\left[\left|y_i\right|_\infty^2+\left(\left|y\right|_\infty^{i-1}\right)^2\right]}{2} \\ &\quad+\frac{1}{2}n^2\alpha_5\left(k_2^q+d_1^q k_1^q\right)^2\left(\left|y\right|_\infty^{i-1}\right)^2\left[\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\right]^2 \\ &=\left[-\alpha_3+\frac{1}{2}n\alpha_4\left(k_2^f+d_1^f k_1^f\right)\sum_{j=1}^{i-1}\left(d_1^f\right)^{j-1}\right. \\ &\quad\left.+\frac{1}{2}n^2\alpha_5 k_1^q\left(k_2^q+d_1^q k_1^q\right)\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\right]\left(\left|y_i\right|_\infty^2\right) \\ &\quad+\left\{\frac{1}{2}n\alpha_4\left(k_2^f+d_1^f k_1^f\right)\sum_{j=1}^{i-1}\left(d_1^f\right)^{j-1}\right. \\ &\quad+\frac{1}{2}n^2\alpha_5 k_1^q\left(k_2^q+d_1^q k_1^q\right)\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1} \\ &\quad\left.\times\frac{1}{2}n^2\alpha_5\left(k_2^q+d_1^q k_1^q\right)^2\left[\sum_{j=1}^{i-1}\left(d_1^q\right)^{j-1}\right]^2\right\}\left(\left|y\right|_\infty^{i-1}\right)^2. \end{aligned} \tag{29}$$

Let $c = (\alpha_2/\alpha_1)^{1/2} \geq 1$, and

$$g_m = \theta_k \left(|y(t)|_\infty^{k-1} \right)^2 \times \left(\left\{ \alpha_3 - 0.5 n \alpha_4 \left(k_2^f + d_1^f k_1^f \right) \sum_{j=1}^{k-1} \left(d_1^f \right)^{j-1} - \frac{1}{2} n^2 \alpha_5 k_1^q \left(k_2^q + d_1^q k_1^q \right) \sum_{j=1}^{m-1} \left(d_1^q \right)^{j-1} \right\} \right)^{-1}, \quad t \neq t_k, \quad k \in \mathbf{N}, \tag{30}$$

where

$$\theta_m = \frac{1}{2} n \alpha_4 \left(k_2^f + d_1^f k_1^f \right) \sum_{j=1}^{m-1} \left(d_1^f \right)^{j-1} + \frac{1}{2} n^2 \alpha_5 k_1^q \left(k_2^q + d_1^q k_1^q \right) \sum_{j=1}^{m-1} \left(d_1^q \right)^{j-1} + \frac{1}{2} n^2 \alpha_5 \left(k_2^q + d_1^q k_1^q \right)^2 \left[\sum_{j=1}^{m-1} \left(d_1^q \right)^{j-1} \right]^2. \tag{31}$$

Define sets in state space: $\underline{\Pi}_i = \{x_i : |y_i(t)|^2 \leq g_i\}$ and $\overline{\Pi}_i = \{y_i : |y_i(t)|^2 \leq c g_i\}$, $i \in \mathbf{N}$. It is obvious that if $y_i \in R^n \setminus \underline{\Pi}_i$, then $\chi_{(1)} V_i < 0$. Therefore, for all $y_i \in R^n \setminus \overline{\Pi}_i$ and $z_i \in R^n \setminus \overline{\Pi}_i$, we have

$$\begin{aligned} V_i(z_i(t)) &\leq \alpha_2 |z_i(t)|^2 < \left(c^2 \right)^{-1} \alpha_2 |z_i(t)|^2 \\ &= \alpha_1 |y_i(t)|^2 \leq V_i(y_i(t)), \\ &\qquad t \neq t_k, \quad k \in \mathbf{N}. \end{aligned} \tag{32}$$

This implies that, for $y_i(0) \in \underline{\Pi}_i$, we have $y_i(t) \in \overline{\Pi}_i$, $t > 0$; that is,

$$|y_i(t)|^2 \leq \left(\frac{\alpha_2}{\alpha_1} \right)^{1/2} \theta_i \left(|y(t)|_\infty^{i-1} \right)^2 \times \left(\left\{ \alpha_3 - 0.5 n \alpha_4 \left(k_2^f + d_1^f k_1^f \right) \sum_{j=1}^{i-1} \left(d_1^f \right)^{j-1} - 0.5 n^2 \alpha_5 k_1^q \left(k_2^q + d_1^q k_1^q \right) \sum_{j=1}^{i-1} \left(d_1^q \right)^{j-1} \right\} \right)^{-1}, \quad t \neq t_k, \quad i, k \in \mathbf{N}. \tag{33}$$

Note that condition (9) implies that the following inequality holds:

$$\begin{aligned} &\left[-\alpha_3 + \frac{1}{2} n \alpha_4 \left(k_2^f + d_1^f k_1^f \right) \sum_{j=1}^{i-1} \left(d_1^f \right)^{j-1} + \frac{1}{2} n^2 \alpha_5 k_1^q \left(k_2^q + d_1^q k_1^q \right) \sum_{j=1}^{i-1} \left(d_1^q \right)^{j-1} \right] \alpha_2^{-1/2} \\ &+ \left[\frac{1}{2} n \alpha_4 \left(k_2^f + d_1^f k_1^f \right) \sum_{j=1}^{k-1} \left(d_1^f \right)^{j-1} + \frac{1}{2} n^2 \alpha_5 k_1^q \left(k_2^q + d_1^q k_1^q \right) \sum_{j=1}^{k-1} \left(d_1^q \right)^{j-1} + \frac{1}{2} n^2 \alpha_5 \left(k_2^q + d_1^q k_1^q \right)^2 \left[\sum_{j=1}^{i-1} \left(d_1^q \right)^{j-1} \right]^2 \right] \alpha_1^{-1/2} \\ &< 0; \end{aligned} \tag{34}$$

namely,

$$\left(\frac{\alpha_2}{\alpha_1} \right)^{1/2} \theta_i \times \left(\left\{ \alpha_3 - \frac{1}{2} n \alpha_4 \left(k_2^f + d_1^f k_1^f \right) \sum_{j=1}^{i-1} \left(d_1^f \right)^{j-1} - \frac{1}{2} n^2 \alpha_5 k_1^q \left(k_2^q + d_1^q k_1^q \right) \sum_{j=1}^{i-1} \left(d_1^q \right)^{j-1} \right\} \right)^{-1} < 1. \tag{35}$$

Let

$$r = \left(\frac{\alpha_2}{\alpha_1} \right)^{1/2} \theta_i \times \left(\left\{ \alpha_3 - \frac{1}{2} n \alpha_4 \left(k_2^f + d_1^f k_1^f \right) \sum_{j=1}^{i-1} \left(d_1^f \right)^{j-1} - \frac{1}{2} n^2 \alpha_5 k_1^q \left(k_2^q + d_1^q k_1^q \right) \sum_{j=1}^{i-1} \left(d_1^q \right)^{j-1} \right\} \right)^{-1}. \tag{36}$$

Furthermore, inequality (33) can be rewritten as $|y_i(t)|^2 < r(|y(t)|_\infty^{i-1})^2$; that is,

$$\|y_i(t)\|_\infty^2 \leq r \left(\|y(t)\|_\infty^{i-1} \right)^2 < \left(\|y(t)\|_\infty^{i-1} \right)^2, \quad t_{k-1} \leq t < t_k, \quad i, k \in \mathbf{N}. \tag{37}$$

When $t = t_k$, $k \in \mathbf{N}$, it follows from Assumption A3 that

$$\begin{aligned} |y_i(t_k^+)|_\infty &= |I_{ik}(y_i(t_k^-)) + y_i(t_k^-)|_\infty \\ &\leq \overline{\eta}_k |y_i(t_k^-)|_\infty < \overline{\eta}_k \sqrt{r} \left(|y(t_k^-)|_\infty^{i-1} \right), \\ &\qquad i, k \in \mathbf{N}. \end{aligned} \tag{38}$$

Note that

$$\begin{aligned}\left|y\left(t_k^-\right)\right|_\infty^{i-1} &= \max_{1\le j\le i-1}\left\{y_j\left(t_k^-\right)\right\}\\ &\le \max_{1\le j\le i-1}\left\{\left(\underline{r}_{jk}\right)^{-1}\left|y_j\left(t_k^+\right)\right|\right\}\\ &\le \left(\underline{\eta}_k\right)^{-1}\max_{1\le j\le i-1}\left\{\left|y_j\left(t_k^+\right)\right|\right\}.\end{aligned} \tag{39}$$

Substituting (39) into (38), we have $|y_i(t_k^+)|_\infty < \overline{\eta}_k(\underline{\eta}_k)^{-1}\sqrt{r}\max_{1\le j\le i-1}\{|y_j(t_k^+)|\}$, $i,k \in \mathbf{N}$. According to condition (9), we have $\overline{\eta}_k(\underline{\eta}_k)^{-1}\sqrt{r} < 1$, so

$$\left|y_i\left(t_k^+\right)\right|_\infty < \max_{1\le j\le i-1}\left\{\left|y_j\left(t_k^+\right)\right|\right\}; \tag{40}$$

namely, $\|y_i(t_k^+)\|_\infty^2 < (\|y(t_k^+)\|_\infty^{i-1})^2$, $i,k \in \mathbf{N}$. This along with (39) means that the zero solution of system (2) is stable with mode constraint.

Combining (28), (37), and (40), it follows from Definition 3 that the zero solution of system (2) is string exponentially stable with mode constraint in the mean square. □

When d_1^f and d_1^q in Assumption A3 satisfy $0 < d_1^f < 1$, $0 < d_1^q < 1$, the stability of system (2) can be judged by the following corollary.

Corollary 6. *Consider the system* (2). *Suppose that Assumptions A1–A3 are satisfied. If there exist constants* $\eta > 0$ *and* $\xi > 0$ *such that* $2\ln\overline{\eta}_k/(t_k - t_{k-1}) \le \eta < \xi$ *and if the following inequality holds,*

$$\begin{aligned}&\left(-\alpha_3 + \xi\alpha_2\right)\underline{\eta}_k\alpha_2^{-1/2} + n\overline{\eta}_k\alpha_4\alpha_1^{-1/2}\left(k_2^f + d_1^f k_1^f\right)\\ &\quad\times\left(1-d_1^f\right)^{-1} + \frac{1}{2}\overline{\eta}_k n^2\alpha_5\\ &\quad\times\left[\alpha_1^{-1/2}\left(k_2^q + d_1^q k_1^q\right)\left(1-d_1^q\right)^{-1}\right]\\ &\quad\times\left[2k_1^q + \left(\frac{\alpha_2}{\alpha_1}\right)^{1/2}\left(k_2^q + d_1^q k_1^q\right)\left(1-d_1^q\right)^{-1}\right] < 0,\end{aligned} \tag{41}$$

where $\overline{\eta}_k = \sup_i\{\overline{\gamma}_{ik}\}$, $\underline{\eta}_k = \inf_i\{\underline{\gamma}_{ik}\}$, $k \in \mathbf{N}$, $0 < d_1^f < 1$, *and* $0 < d_1^q < 1$, *then the zero solution of system* (2) *is string exponentially stable with mode constraint in the mean square sense.*

The proof of Corollary 6 can be done by induction as the proof of Theorem 5, and so we omit it here.

Remark 7. By using the vector Lyapunov function method, the stability of a class of infinite dimensional look-ahead interconnected systems is studied in this paper. It should be noted that a comparison system aggregated by Lyapunov functions is usually a linear system. So when applying the stability condition of such a linear system to the original nonlinear system, "super-sufficient" stability conditions are obtained in general, as analyzed in [17, 19]. That is to say, the obtained stability conditions are relatively conservative. It can be seen from inequality (14) that the comparison systems aggregated by the Lyapunov functions in this paper are still nonlinear systems, which means that our obtained results are less conservative than the existing ones.

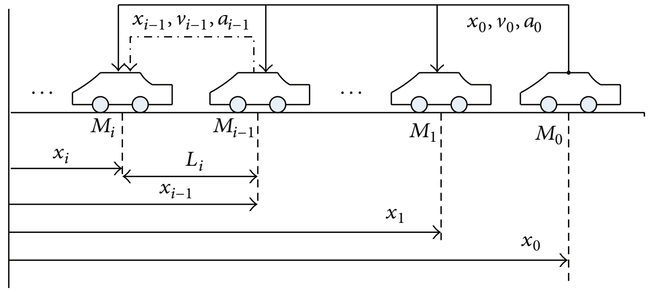

Figure 1: Spacing errors in a platoon.

Remark 8. The dynamic behavior of some stochastic look-ahead interconnected systems without considering impulsive disturbance has been analyzed in [19, 24], and some sufficient conditions ensuring the string stability of the system have been obtained by the vector Lyapunov function methods. The obtained conditions in [19, 24] cannot be used to judge the stability with mode constraint for the systems. On the other hand, the models studied in [19, 24] are derived from the context of the controller design problem of look-ahead vehicle longitudinal following systems. However, the authors did not further study how to find the suitable parameters domain of the controller for the systems based on their established stability conditions.

Remark 9. Some research has been studied by us in [13] on the string stability with mode constraint for the system (2) without impulsive disturbance. It is easy to see that the results in [13] are contained in the obtained results in Theorem 5 in this paper.

The research studied in [13, 19, 24] did not pay attention to how to choose parameters domains of the controller for the look-ahead VLFS by using their obtained stability conditions. In the next section, we will design the controller for a class of look-ahead VLFS with stochastic and impulsive disturbances. Based on the obtained stability results in this paper, we will give the domains of the control parameters chosen for the system.

4. Controller Design for Vehicle Following System

Consider a platoon of vehicles using a longitudinal control system for vehicle following, as shown in Figure 1. The position, velocity, and acceleration of the ith following vehicle are x_i, v_i, and a_i, respectively. The constant spacing policy in [18] is employed by all automated vehicles in the platoon. Let L_i be the desired intervehicular distance of the ith following vehicle.

4.1. Dynamic Model. The rolling resistance friction is considered as the stochastic factor of the system. Let $F_i = \overline{F}_i + \breve{F}_i$, where $\overline{F}_i$ is the certain part and $\breve{F}_i$ is the stochastic part. It is assumed that $\breve{F}_i/M_i$ is white noise process with mean value 0 and mean square error σ^2. Let $d(\breve{F}_i/M_i) = g(\dot{x}_i)d\xi$. Therefore, the model of the longitudinal dynamics of a member vehicle with impulsive and stochastic disturbances is given by

$$\begin{aligned} d\dot{x}_i(t) &= \frac{-c_i\dot{x}_i^2(t) + u_i(t) - \overline{F}_i}{M_i} + g(\dot{x}_i)\,d\xi, \quad t \neq t_k \\ \Delta v_i(t_k) &= v_i(t_k^+) - v_i(t_k^-) = I_{ik}(v_i(t_k^-)), \quad t = t_k, \end{aligned} \tag{42}$$

where, $i, k \in \mathbf{N}$, x_i, $\dot{x}_i$, $d\dot{x}_i$, $c_i\dot{x}_i^2$, u_i, $\overline{F}_i$, and M_i denote the position, velocity, acceleration, effective aerodynamic drag, control effort, certain rolling resistance friction, and mass of the ith following vehicle, respectively. Let $v_i(t_k)$ be the velocity of the impulsive moment of the ith following vehicle. Let the initial position of the ith following vehicle be $x_i(0)$.

In order to avoid the collision among vehicles in the platoon in the presence of the impulsive disturbances, it is assumed that $\underline{\gamma}_{ik}v_i(t_k^-) \leq v_i(t_k^+) \leq \overline{\gamma}_{ik}v_i(t_k^-)$ and $0 < \underline{\gamma}_{ik} < 1 < \overline{\gamma}_{ik}$; here, $\underline{\gamma}_{ik}$ and $\overline{\gamma}_{ik}$ are positive constants, $i, k \in \mathbf{N}$.

Let $\varepsilon_i(t)$ be the spacing error of the ith vehicle, which is given by

$$\varepsilon_i(t) = x_i(t) - x_{i-1}(t) + L_i, \quad i \in \mathbf{N}. \tag{43}$$

Obviously, we have $\dot{\varepsilon}_i(t) = \dot{x}_i(t) - \dot{x}_{i-1}(t)$ and $\ddot{\varepsilon}_i(t) = \ddot{x}_i(t) - \ddot{x}_{i-1}(t)$. Furthermore, (42) can be rewritten as

$$\begin{aligned} d\dot{\varepsilon}_i(t) &= \frac{u_i(t) - c_i[\dot{\varepsilon}_i(t) + \dot{x}_{i-1}(t)]^2 - \overline{F}_i}{M_i}dt \\ &\quad + g(\dot{x}_i)\,d\xi - d\dot{x}_{i-1}(t), \\ \Delta\dot{\varepsilon}_i(t_k) &= \dot{\varepsilon}_i(t_k^+) - \dot{\varepsilon}_i(t_k^-), \quad k \in \mathbf{N}, \\ &\qquad i \in \mathbf{N}. \end{aligned} \tag{44}$$

When $t = t_k$, $k \in \mathbf{N}$, due to $\underline{\gamma}_{ik}v_i(t_k^-) \leq v_i(t_k^+) \leq \overline{\gamma}_{ik}v_i(t_k^-)$, there exists $\underline{\eta}_k \geq \sup_i\{\underline{\gamma}_{ik}\}$ and $\overline{\eta}_k \geq \sup_i\{\overline{\gamma}_{ik}\}$ such that

$$\begin{aligned} \dot{\varepsilon}_i(t_k^+) &= v_i(t_k^+) - v_{i-1}(t_k^+) \\ &= I_{ik}(v_i(t_k^-)) + v_i(t_k^-) - I_{(i-1)k} \\ &\quad \times (v_{i-1}(t_k^-)) - v_{i-1}(t_k^-) \\ &\leq \overline{\gamma}_{ik}\left[v_i(t_k^-) - \left(\frac{\underline{\gamma}_{(i-1)k}}{\overline{\gamma}_{ik}}\right)v_{i-1}(t_k^-)\right] \\ &\leq \overline{\eta}_k[v_i(t_k^-) - v_{i-1}(t_k^-)], \\ \dot{\varepsilon}_i(t_k^+) &= v_i(t_k^+) - v_{i-1}(t_k^+) = I_{ik}(v_i(t_k^-)) + v_i(t_k^-) \\ &\quad - I_{(i-1)k}(v_{i-1}(t_k^-)) - v_{i-1}(t_k^-) \\ &\geq \underline{\gamma}_{(i-1)k}\left[\left(\frac{\overline{\gamma}_{ik}}{\underline{\gamma}_{(i-1)k}}\right)v_i(t_k^-) - v_{i-1}(t_k^-)\right] \\ &\geq \underline{\eta}_k[v_i(t_k^-) - v_{i-1}(t_k^-)]; \end{aligned} \tag{45}$$

namely,

$$\underline{\eta}_k\dot{\varepsilon}_i(t_k^-) \leq \dot{\varepsilon}_i(t_k^+) \leq \overline{\eta}_k\dot{\varepsilon}_i(t_k^-), \quad i, k \in \mathbf{N}. \tag{46}$$

Assume that there exists a constant $\eta > 0$ such that $2\ln\overline{\eta}_k/(t_k - t_{k-1}) \leq \eta$.

It is well known that real number set R is a measurable set and $t \in [0, \infty) \subset R^+$. Consider that $v_i(t)$ is continuous on interval (t_{k-1}^+, t_k^-) and is a simple function in the set $\{t_k^-, t_k^+\}$, and so $v_i(t)$ is measurable on interval $(t_{k-1}^+, t_k^+]$; here, $k \in \mathbf{N}$. Furthermore, $v_i(t)$ is Lebesgue integral on $(t_{k-1}^+, t_k^+]$, $k \in \mathbf{N}$. Let $E_k = (t_{k-1}^+, t_k^+] = E_k^1 \bigcup E_k^2 \bigcup E_k^3$; here, $E_k^1 = (t_{k-1}^+, t_k^-)$, $E_k^2 = \{t_k^-\}$, and $E_k^3 = \{t_k^+\}$. Obviously, $E_k^1 \bigcap E_k^2 \bigcap E_k^3 = \Phi$. Let $(L)\int_{E_k} v_i(t)dt$ denote the integral of $v_i(t)$ in the set E_k, and let $\mathrm{Mes}(\cdot)$ denote the measure; we have

$$\begin{aligned} x_i(t_k^+) &= x_i(t_{k-1}^-) + (L)\int_{E_k} v_i(t)\,dt \\ &= x_i(t_{k-1}^-) + (L)\int_{E_k^1} v_i(t)\,dt \\ &\quad + (L)\int_{E_k^2} v_i(t)\,dt + (L)\int_{E_k^3} v_i(t)\,dt, \\ x_i(t_k^-) &= x_i(t_{k-1}^-) + (L)\int_{E_k^1} v_i(t)\,dt + (L)\int_{E_k^2} v_i(t)\,dt. \end{aligned} \tag{47}$$

Due to the fact that $\mathrm{Mes}(E_k^2) = \mathrm{Mes}(E_k^3) = 0$, $x_i(t_k^+) = x_i(t_k^-)$. Furthermore,

$$\begin{aligned} \varepsilon_i(t_k^+) &= x_i(t_k^+) - x_{i-1}(t_k^+) + L_i \\ &= x_i(t_k^-) - x_{i-1}(t_k^-) + L_i < \overline{\eta}_k\varepsilon_i(t_k^-) \\ \varepsilon_i(t_k^+) &= x_i(t_k^+) - x_{i-1}(t_k^+) + L_i \\ &= x_i(t_k^-) - x_{i-1}(t_k^-) + L_i > \underline{\eta}_k\varepsilon_i(t_k^-); \end{aligned} \tag{48}$$

that is,

$$\underline{\eta}_k\varepsilon_i(t_k^-) < \varepsilon_i(t_k^+) < \overline{\eta}_k\varepsilon_i(t_k^-), \quad i, k \in \mathbf{N}. \tag{49}$$

4.2. Controller Design. Define an auxiliary error given by the following equation:

$$S_i(t) = q_1[\dot{x}_i(t) - \dot{x}_{i-1}(t)] + q_2[x_i(t) - x_{i-1}(t) + L_i] + q_3[\dot{x}_i(t) - v_0(t)] + q_4\left[x_i(t) - x_0(t) + \sum_{j=1}^{i} L_j\right] + \sigma_i(t), \quad (50)$$

where $\sigma_i(t)$ satisfies

$$d\sigma_i(t) = q_1 d\dot{x}_{i-1} + q_3 dv_0 - (q_1 + q_3) g(\dot{x}_i) d\xi \quad (51)$$

and $\dot{S}_i$ is independent on ξ; that is to say, S_i is a finite variable [26]. It is assumed that $\sigma_i(t_k^+) = \sigma_i(t_k^-)$, $i, k \in \mathbf{N}$.

The expression of control law $u_i(t)$ is chosen as follows:

$$u_i(t) = u_{iequ}(t) + u_{iN}(t), \quad i \in \mathbf{N}, \quad (52)$$

where

$$u_{iequ}(t) = c_i\dot{x}_i^2(t) + \overline{F}_i - \frac{M_i}{(q_1+q_3)} \times [q_2(\dot{x}_i(t) - \dot{x}_{i-1}(t)) + q_4(\dot{x}_i(t) - v_0(t))],$$
$$u_{iN}(t) = -\frac{M_i}{q_1+q_3}\lambda S_i(t). \quad (53)$$

Here, $\lambda > 0$ with $\lambda \geq \eta > 0$ is the control parameter and will be chosen later.

4.3. Stability Analysis

4.3.1. Reachability of Slide Mode. In this section, we will analyze the fact that the slide mode is asymptotically reachable.

Choose the control vector Lyapunov function $V_i(t) = 0.5S_i^2(t)$, $t > 0$, $i \in \mathbf{N}$.

(i) When $t \neq t_k$, $k \in \mathbf{N}$, calculating the right upper derivative of $V_i(t)$ along (44), we get

$$D^+V_i(t) = S_i\dot{S}_i = S_i\left[(q_1+q_3)\frac{-c_i\dot{x}_i^2 + u_i - \overline{F}_i}{M_i} + (q_2+q_4)\dot{x}_i - q_2\dot{x}_{i-1} - q_4 v_0\right], \quad i \in \mathbf{N}. \quad (54)$$

Substituting (52) into the above equation, we obtain $D^+V_i(t) = -\lambda S_i^2(t)$, $i \in \mathbf{N}$. It is obvious that when $S_i(t) \neq 0$, $D^+V_i(t) < 0$. Therefore, $V_i(t)$ is strictly monotone decreasing, $t > 0$, $t \neq t_k$, $i, k \in \mathbf{N}$.

(ii) When $t = t_k$, $k \in \mathbf{N}$, according to (50) and $\sigma_i(t_k^+) = \sigma_i(t_k^-)$, we obtain

$$\begin{aligned} S_i(t_k^+) &< q_1\eta_k[v_i(t_k^-) - v_{i-1}(t_k^-)] + q_2[x_i(t_k^-) - x_{i-1}(t_k^-) + L_i] + q_3\eta_k[v_i(t_k^-) - v_0(t_k^-)] + q_4 \\ &\quad \times \left[x_i(t_k^-) - x_0(t_k^-) + \sum_{j=1}^{i} L_j\right] + \eta_k\sigma_i(t_k^+) \\ &< \eta_k S_i(t_k^-), \quad i, k \in \mathbf{N}. \end{aligned} \quad (55)$$

Due to the fact that $\dot{S}_i(t) + \lambda S_i(t) = 0$, $S_i(t_{k+1}^+) \leq \overline{\eta}_{k+1} S_i(t_k^+) \exp[-\lambda(t_{k+1}^+ - t_k^+)]$. From the condition $2\ln\overline{\eta}_k/(t_k - t_{k-1}) \leq \eta$, $\lambda \geq \eta > 0$, we get $\eta_{k+1}\exp[-\lambda(t_{k+1}^+ - t_k^+)] \leq \exp[-(\lambda - 0.5\eta)(t_{k+1}^+ - t_k^+)] < 1$, and so

$$S_i(t_{k+1}^+) < S_i(t_k^+), \quad i, k \in \mathbf{N}. \quad (56)$$

From (56), the impulsive sequence $\{S_i(t_k^+)\}$ is strictly monotone decreasing, $i, k \in \mathbf{N}$. On the other hand, we know from the preceding analysis that when $t \neq t_k$, $S_i(t)$ is strictly monotone decreasing. To sum up, it can be concluded that $S_i(t) \to 0$ as $t \to \infty$. Namely, the slide mode $S_i(t)$ is asymptotically reachable, $i \in \mathbf{N}$.

4.3.2. Stability of Slide Mode Motion. In this section, the stability domain of the control parameters will be proposed based on the new stability conditions established in Section 3.

Substituting the expression of control law (52) into the system (44), we can obtain the following slide mode motion equation given by

$$\begin{aligned} d\dot{x}_i(t) &= (q_1+q_3)^{-1} \times [q_1 d\dot{x}_{i-1}(t) + q_3 dv_0(t) - q_2(\dot{x}_i(t) - \dot{x}_{i-1}(t)) \\ &\quad - q_4(\dot{x}_i(t) - v_0(t)) - \lambda S_i(t)]\,dt + g(\dot{x}_i(t))\,d\xi, \quad t \neq t_k, \\ \Delta v_i(t_k) &= v_i(t_k^+) - v_i(t_k^-), \quad t = t_k, \\ & i, k \in \mathbf{N}. \end{aligned} \quad (57)$$

By the same way, we get

$$\begin{aligned} d\dot{x}_{i-1}(t) &= (q_1+q_3)^{-1} \\ &\quad \times [q_1 d\dot{x}_{i-2}(t) + q_3 dv_0(t) \\ &\qquad - q_2(\dot{x}_{i-1}(t) - \dot{x}_{i-2}(t)) \\ &\qquad - q_4(\dot{x}_{i-1}(t) - v_0(t)) - \lambda S_{i-1}(t)]\, dt \\ &\quad + g(\dot{x}_{i-1}(t))\, d\xi, \quad t \neq t_k, \\ \Delta v_{i-1}(t_k) &= v_{i-1}(t_k^+) - v_{i-1}(t_k^-), \quad t = t_k, \\ &\qquad i, k \in \mathbf{N}. \end{aligned} \tag{58}$$

In the surface of the slide mode, due to the fact that $S_i(t) = S_{i-1}(t) = 0$, $d(S_i(t) - S_{i-1}(t)) = 0$, $i \in \mathbf{N}$. Furthermore, according to (44) and the assumption conditions, we have

$$\begin{aligned} d\dot{\varepsilon}_i(t) &= \left[-\frac{q_2+q_4}{q_1+q_3}\dot{\varepsilon}_i(t) + \frac{q_2}{q_1+q_3}\dot{\varepsilon}_{i-1}(t)\right] dt \\ &\quad + [g(\dot{x}_i(t)) - g(\dot{x}_{i-1}(t))]\, d\xi, \quad t \neq t_k, \\ \Delta\dot{\varepsilon}_i(t_k) &= \dot{\varepsilon}_i(t_k^+) - \dot{\varepsilon}_i(t_k^-), \quad t = t_k, \\ &\qquad i, k \in \mathbf{N}. \end{aligned} \tag{59}$$

Let $y_i(t) = \dot{\varepsilon}_i(t) = \dot{x}_i(t) - \dot{x}_{i-1}(t)$, $\varepsilon_0(t) = 0$, $\dot{\varepsilon}_0(t) = 0$, and $\ddot{\varepsilon}_0(t) = 0$. Set $l_1 = (q_2+q_4)/(q_1+q_3)$ and $l_2 = q_2/(q_1+q_3)$. From (59), we have

$$\begin{aligned} dy_1(t) &= -l_1 y_1(t)\, dt + [g(\dot{x}_1(t)) - g(\dot{x}_0(t))]\, d\xi, \\ &\qquad t \neq t_k, \\ dy_i(t) &= [-l_1 y_i(t) + l_2 y_{i-1}(t)]\, dt \\ &\quad + [g(\dot{x}_i(t)) - g(\dot{x}_{i-1}(t))]\, d\xi, \quad t \neq t_k, \\ \Delta y_i(t_k) &= y_i(t_k^+) - y_i(t_k^-), \quad t = t_k, \\ &\qquad i, k \in \mathbf{N}. \end{aligned} \tag{60}$$

Consider that $y_i(t) = \dot{\varepsilon}_i(t) = \dot{x}_i(t) - \dot{x}_{i-1}(t)$; (60) can be rewritten as

$$\begin{aligned} dy_1(t) &= -l_1 y_1(t)\, dt + g(y_1(t))\, d\xi, \quad t \neq t_k \\ dy_i(t) &= [-l_1 y_i(t) + l_2 y_{i-1}(t)]\, dt \\ &\quad + [g(y_i(t) + y_{i-1}(t) + \cdots + y_1(t)) \\ &\quad - g(y_{i-1}(t) + y_{i-2}(t) + \cdots + y_1(t))]\, d\xi, \\ &\qquad t \neq t_k, \\ \Delta y_i(t_k) &= y_i(t_k^+) - y_i(t_k^-), \quad t = t_k, \\ &\qquad i, k \in \mathbf{N}, \end{aligned} \tag{61}$$

where $\underline{\eta}_k |y_i(t_k^-)| \le |y_i(t_k^+)| \le \overline{\eta}_k |y_i(t_k^-)|$.

The isolate subsystems of (61) are of the following forms:

$$\begin{aligned} \dot{y}_i(t) &= -l_1 y_i(t) + g(y_i(t))\, d\xi, \quad t \neq t_k, \\ \Delta y_i(t_k) &= y_i(t_k^+) - y_i(t_k^-), \quad t = t_k, \\ &\qquad i, k \in \mathbf{N}. \end{aligned} \tag{62}$$

Obviously, (61) with isolate subsystems (62) is a special case of (2) with isolate subsystems (3).

Taking the vector function Lyapunov $V_i(t) = y_i^2(t)$, $i \in \mathbf{N}$. It is easy to get $\alpha_1 = 1$, $\alpha_2 = 1$, and $\alpha_4 = \alpha_5 = 2$. From Theorem 5, for judging the stability for (2), some sufficient conditions for assuring the string exponential stability with mode constraint in the mean square sense for (61) can be established as follows:

(i) it is assumed that g satisfies the global Lipschitz condition; that is, there exists constant $\theta > 0$ such that $|g(y) - g(z)| \le \theta |y - z|$;

(ii) $\alpha_3 \ge 2l_1 - \theta^2$;

(iii) there exists constant $\lambda > 0$ such that $2\ln \overline{\eta}_k/(t_k - t_{k-1}) \le \eta < \lambda$, $k \in \mathbf{N}$;

(iv) $\underline{\eta}_k(\lambda - 2l_1 + \theta^2) + 2\overline{\eta}_k l_2 < 0$, $k \in \mathbf{N}$.

By the same analysis with the proof of Theorem 5, it is easy to obtain that the zero solution $y_i(t) = 0$ of (61) is string exponentially stable with mode constraint in the mean square. From the relations among $y_i(t)$, $\varepsilon_i(t)$, and $S_i(t)$, it is easy to conclude that the vehicle following errors $\dot{\varepsilon}_i(t)$ and $\varepsilon_i(t)$ converge to zero, $i \in \mathbf{N}$.

Remark 10. The stability with mode constraint for the vehicle following error system (59) could guarantee that the error of every following vehicle is not only uniformly bounded, but also less than the error of the leading vehicle.

5. Numerical Simulations

In this section, some numerical simulations are performed for a four-vehicle platoon with one leading vehicle and three following vehicles. The reference model of VLFS is referred to in Section 4 in this paper (Figure 1).

Let $g(\dot{x}_i) = \dot{x}_i$, $i = 1, 2, 3$. The control laws for every following vehicle are of the forms in (56). According to the conditions in Section 4.3.2, the control parameters are designed as $q_1 = 1$, $q_2 = 2$, $q_3 = 3$, $q_4 = 4$, and $\lambda = 1.4$. The leading vehicle in the platoon makes the following acceleration maneuver [18]: when $0\,\text{s} \le t < 4\,\text{s}$, $a_0 = 0\,\text{m/s}^2$; when $4\,\text{s} \le t < 7\,\text{s}$, $a_0 = -0.25(t-2)\,\text{m/s}^2$; when $7\,\text{s} \le t < 10\,\text{s}$, $a_0 = -0.75\,\text{m/s}^2$; when $10\,\text{s} \le t < 16\,\text{s}$, $a_0 = 0.25(t-10) - 0.75\,\text{m/s}^2$; when $16\,\text{s} \le t < 19\,\text{s}$, $a_0 = 0.75\,\text{m/s}^2$; when $19\,\text{s} \le t < 22\,\text{s}$, $a_0 = 0.25(19-t) + 0.75\,\text{m/s}^2$; when $22\,\text{s} \le t \le 30\,\text{s}$, $a_0 = 0\,\text{m/s}^2$.

It is assumed that the desired intervehicular distance of the three following vehicles is $L_i = 8\,\text{m}$, $i = 1, 2, 3$. Table 1 shows the initial states of vehicles in the platoon. From (46), it can be obtained that $e_1(0) = 0.5\,\text{m}$, $e_2(0) = 0.4\,\text{m}$, and $e_3(0) = 0.3\,\text{m}$. Table 2 shows the parameters of the following

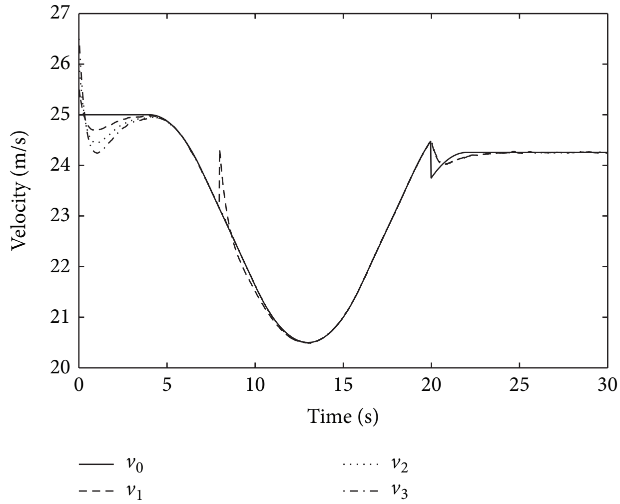

FIGURE 2: Velocity curves of vehicles.

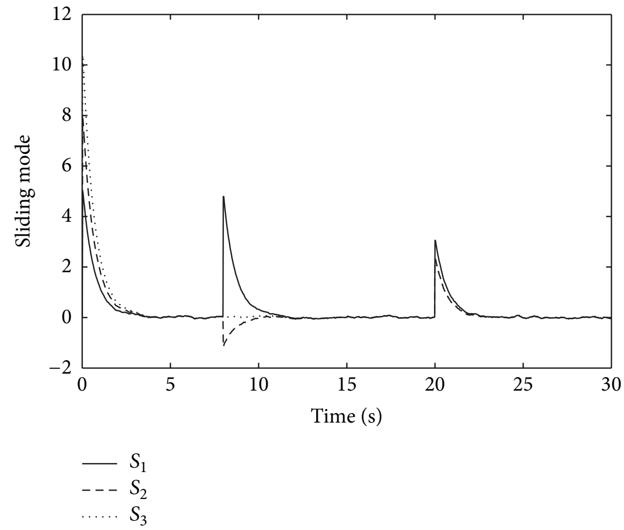

FIGURE 4: Slide mode curves of vehicles.

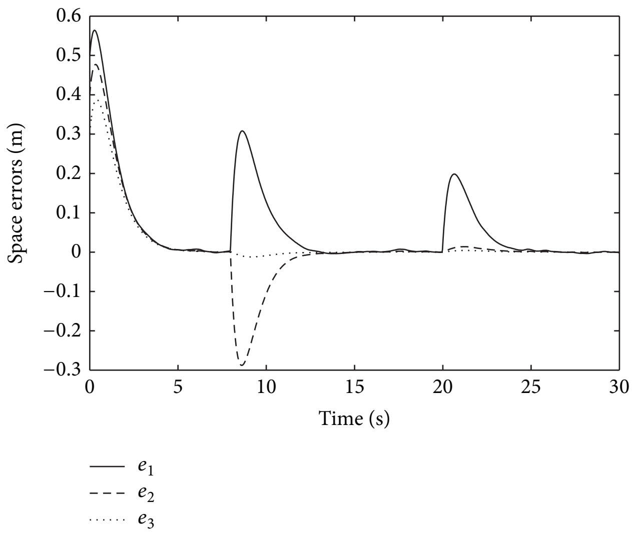

FIGURE 3: Spacing error curves of vehicles.

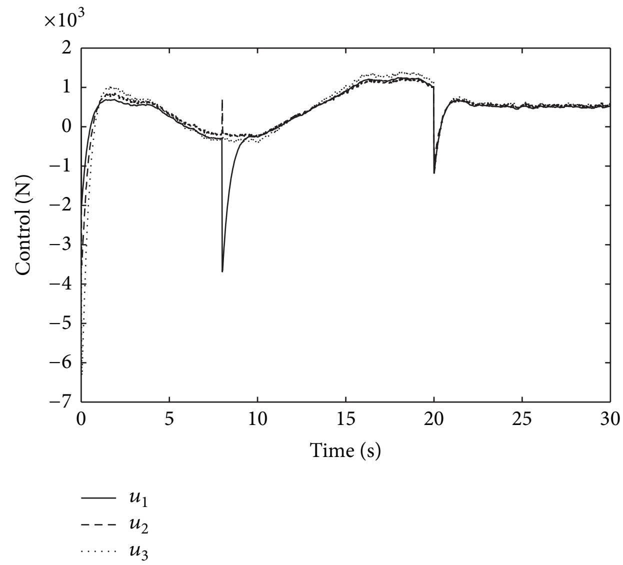

FIGURE 5: Control input curves of vehicles.

TABLE 1: Initial states of vehicles.

Vehicle number i	$a_i(0)$ (m/s^2)	$v_i(0)$ (m/s)	$x_i(0)$ (m)
0	0	25.0	50.0
1	0	25.5	42.5
2	0	26.0	34.9
3	0	26.5	27.2

TABLE 2: Parameters of vehicles.

Vehicle number i	M_i (kg)	c_i (Ns^2/m^2)	F_i (N)
1	1000	0.50	200
2	900	0.60	180
3	1100	0.55	220

vehicles in the platoon. It is assumed that when $t_k = 8$ s, $v_1(t_k^+) = 1.05v_1(t_k^-)$; when $t_k = 20$ s, $v_0(t_k^+) = 0.97v_0(t_k^-)$. The approach in [27] is used to generate Brown motion trajectory.

The simulation results are shown in Figures 2, 3, 4, and 5. From the simulation results, it can be seen that the errors of following vehicles not only converge to zero but also satisfy the mode constraint condition. That is to say, the stability results obtained in this paper are correct and practical. Since the domains of control parameters are large, the controllers are easy to be designed in practice.

6. Conclusions

In this paper, the problem of string exponential stability with mode constraint of infinite dimensional nonlinear interconnected systems with stochastic and impulsive disturbances has been studied by using the vector Lyapunov function method. Sufficient conditions of string exponential stability with mode constraint have been derived for a class of general infinite dimensional look-ahead interconnected systems with impulsive and stochastic disturbances. Moreover, the controller for a class of look-ahead vehicle longitudinal following systems with the above uncertainties has been proposed by the sliding mode control method. Based on the obtained new stability conditions, the domain of the control parameters of the systems has been obtained, and the domain of the control parameters of the systems is enlarged. A numerical example with simulations has been given to show the effectiveness and correctness of the obtained results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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