

## Research Article

# Robust Adaptive Stabilization of Nonholonomic Mobile Robots with Bounded Disturbances

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Received 6 August 2014; Accepted 24 August 2014; Published 17 November 2014

Academic Editor: Shen Yin

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The stabilization problem of nonholonomic mobile robots with unknown system parameters and environmental disturbances is investigated in this paper. Considering the dynamic model and the kinematic model of mobile robots, the transverse function approach is adopted to construct an additional control parameter, so that the closed-loop system is not underactuated. Then the adaptive backstepping method and the parameter projection technique are applied to design the controller to stabilize the system. At last, simulation results demonstrate the effectiveness of our proposed controller schemes.

## 1. Introduction

The motion control of nonholonomic mobile robots has been an active research field in the past few decades and remains a challenging control problem. For nonholonomic mobile robots, it cannot be stabilized by any static continuous state feedback [1], due to the Brockett's condition [2]. To handle the stability of nonholonomic systems, it has to resort to either a time-varying [3, 4], a discontinuous [5, 6], or a dynamic state feedback controller [7, 8].

The early work in [9] studies the tracking problem based on backstepping method for both a kinematic model and a dynamic model of mobile robot. In [10], a neural network adaptive controller based on input-output linearization design is presented to guide a mobile robot during trajectory tracking. However, these methods are only applicable when the system dynamic parameters are known. In order to handle system disturbances that are inevitable in real applications, an adaptive tracking controller has been proposed for a class of mobile robots with uncertainties [11]. Based on Lyapunov's direct method and backstepping technique [12], a time-varying global adaptive controller at the torque level that simultaneously solves both tracking and stabilization problems in the case of unknown dynamic

parameters has been developed. Similar work can be found in [13]. Moreover, a robust adaptive controller is designed for a mobile robot with bounded unknown disturbances in [14]. The works in [15, 16] extend the control law design for more general uncertain nonholonomic systems. Subsequent related works on the stabilization and tracking control of nonholonomic mobile robots include, but are not limited to, [17–21] and many references therein. However, all the stabilization schemes mentioned above need the assistance of the sinusoidal function; thus, the convergence rate of stabilization is slow.

To overcome this problem, in this paper, we consider the stabilization control of the nonholonomic mobile robot with unknown parameters and bounded external disturbances. In contrast to the aforementioned results, all the system parameters as well as the disturbance are assumed to be unknown. Based on the transverse function approach [22] and backstepping techniques, a robust adaptive controller will be developed to guarantee the asymptotical stability of the nonholonomic mobile robot. By employing the transverse function approach, an “auxiliary manipulated variable” is introduced, with which the difficulty encountered in controlling an underactuated system can be overcome. The rest of this paper is arranged as follows. In Section 2, we present

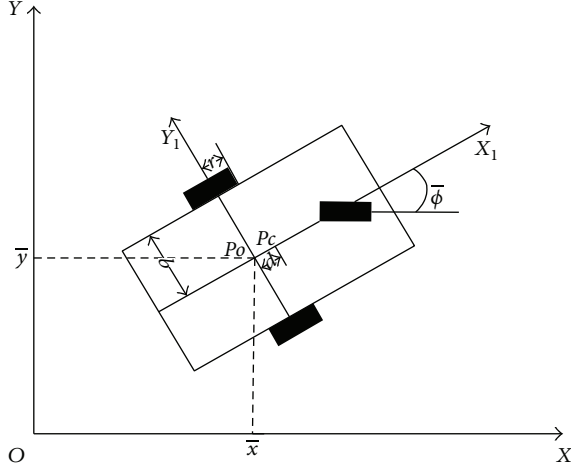


FIGURE 1: A two-wheeled nonholonomic mobile robot.

the model of nonholonomic mobile robots and problem formulation. Then we propose an adaptive control scheme to achieve stabilization in Section 3, where the stability of the overall system is discussed. Simulation results are provided in Section 4. Finally, some concluding remarks are drawn in Section 5.

## 2. Problem Formulation

A unicycle mobile robot is considered, which consists of two driving wheels located at the same axis and a passive self-adjusted supporting wheel. The actuated two wheels are driven by two DC servomotors independently. As shown in Figure 1, the geometric center and center of mass of the mobile robot do not coincide. The origin of  $Po-X_1Y_1$  frame is the geometric center  $Po$ , the center of mass  $Pc$  is on  $X_1$  axis, and the distance to the origin  $Po$  is  $d$ . The position of  $Po$  in global coordinate frame  $O-XY$  is  $(\bar{x}, \bar{y})$  and  $\bar{\phi}$  is the orientation of the local frame  $Po-X_1Y_1$ . For the sake of simplicity, it is assumed that the robot does not slip and there is no sliding between the tire and the road; that is, there is no Coulomb-like friction. Then the system can be described by the following dynamic model and kinematic model [23], where the parameters are shown in Table 1:

$$\dot{\eta} = J(\eta) \omega, \quad (1)$$

$$M\dot{\omega} + C(\dot{\eta}) \omega + D\omega = \tau + \tau_d, \quad (2)$$

where  $\eta = (\bar{x}, \bar{y}, \bar{\phi})^T$  denotes the position and orientation of the robot,  $\omega = (\omega_1, \omega_2)^T$  denotes the velocities of the left and right wheels,  $\tau = (\tau_1, \tau_2)^T$  represents the control torques applied to the wheels,  $M$  is a symmetric, positive definite inertia matrix,  $C(\dot{\eta})$  is the centripetal and Coriolis matrix,  $D$  denotes the surface friction, and  $\tau_d$  is the bounded unknown

TABLE 1: Definition of parameters of mobile robot.

| Symbols              | Description  |
|----------------------|--|
| $m_c$                | Mass of body   |
| $m_w$                | Mass of wheel with a motor   |
| $I_c$                | Moment of inertia of the body about the vertical axis through $Pc$ |
| $I_w$                | Moment of inertia of the wheel with a motor about the wheel axis   |
| $I_m$                | Moment of inertia of the wheel with a motor about the diameter     |
| $b$                  | Half width of the mobile robot                                     |
| $r$                  | Radius of the wheel  |
| $d_{11}, d_{22}$     | Damping coefficients   |
| $\omega_1, \omega_2$ | Velocities of the left and right wheels                            |
| $\tau_1, \tau_2$     | Control torques applied to the wheels                              |
| $\tau_d$             | External disturbance   |

external disturbance. Matrices  $J(\eta)$ ,  $M$ , and  $C(\dot{\eta})$  are the same as those in [12], which are given as

$$J(\eta) = \frac{r}{2} \begin{bmatrix} \cos \bar{\phi} & \cos \bar{\phi} \\ \sin \bar{\phi} & \sin \bar{\phi} \\ b^{-1} & -b^{-1} \end{bmatrix}, \quad M = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{11} \end{bmatrix},$$

$$C(\dot{\eta}) = \begin{bmatrix} 0 & \frac{r^2}{2b} m_c \dot{\bar{\phi}} \\ -\frac{r^2}{2b} m_c \dot{\bar{\phi}} & 0 \end{bmatrix}, \quad D = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}, \quad (3)$$

$$m_{11} = \frac{r^2}{4b^2} (mb^2 + I) + I_w, \quad m_{12} = \frac{r^2}{4b^2} (mb^2 - I),$$

$$m = m_c + 2m_w, \quad I = m_c d^2 + 2m_w b^2 + I_c + 2I_m.$$

The upper bound of the external disturbance is assumed to satisfy

$$\|\tau_d\| \leq \tau_{d\max}, \quad (4)$$

where  $\tau_{d\max}$  is an unknown positive constant.

Two control vectors are introduced as  $u_1 = 0.5(\omega_1 + \omega_2)$  and  $u_2 = 0.5(\omega_1 - \omega_2)$ , and then the kinematic model (1) can be written as

$$\begin{aligned} \bar{x} &= r \cos \bar{\phi} u_1, \\ \bar{y} &= r \sin \bar{\phi} u_1, \\ \bar{\phi} &= \frac{r}{b} u_2. \end{aligned} \quad (5)$$

**Assumption 1.** The parameters  $r$  and  $b$  fall in known compact sets; that is, there exist some known positive constants  $\bar{r}, \underline{r}, \bar{b}$ , and  $\underline{b}$ , such that  $\underline{r} < r < \bar{r}$  and  $\underline{b} < b < \bar{b}$ .

**Remark 2.** The degrees of freedom of the nonholonomic mobile robot are three, but there are only two independent control inputs, so the system (5) is underactuated.

### 3. Controller Design

In this section, the transverse function approach is adopted firstly to perform suitable change of coordinates. With the change of coordinates, an additional controller  $\xi$  will be created and thus the kinematic model (5) is no longer underactuated. In the second step, the adaptive controller and parameter estimator are designed such that the system is stabilized.

**3.1. Control Objective.** Design the control inputs  $\tau$  to stabilize the nonholonomic mobile robot, as modeled in (2) and (5).

**3.1.1. Coordinates Transformation.** The new coordinates  $(x, y, \phi)$  and additional controller  $\xi$  are introduced, and the kinematic model (5) is transformed as follows:

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + R(\phi) \begin{bmatrix} f_1(\xi) \\ f_2(\xi) \end{bmatrix}, \\ \phi &= \bar{\phi} - f_3(\xi), \end{aligned} \quad (6)$$

where

$$R(\phi) = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}, \quad (7)$$

and  $f_l(\xi)$ , for  $l = 1, 2, 3$ , are functions of  $\xi$  designed as

$$\begin{aligned} f_1(\xi) &= \varepsilon_1 \sin(\xi) \frac{\sin(f_3)}{f_3}, \\ f_2(\xi) &= \varepsilon_1 \sin(\xi) \frac{1 - \cos(f_3)}{f_3}, \\ f_3(\xi) &= \varepsilon_2 \cos(\xi), \end{aligned} \quad (8)$$

where  $\varepsilon_1$  and  $\varepsilon_2$  are arbitrarily small positive constants and  $\varepsilon_2$  satisfies  $0 < \varepsilon_2 < \pi/2$ . Then the following properties can be shown:

$$|f_1| < \varepsilon_1, \quad |f_2| < \varepsilon_1, \quad |f_3| < \varepsilon_2. \quad (9)$$

Taking the derivatives of  $x$ ,  $y$ , and  $\phi$  yields

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} &= Q \begin{bmatrix} ru_1 \\ \dot{\xi} \end{bmatrix} + \frac{\partial R(\phi)}{\partial \phi} \begin{bmatrix} f_1(\xi) \\ f_2(\xi) \end{bmatrix} \dot{\phi}, \\ \dot{\phi} &= rb^{-1}u_2 - \frac{\partial f_3(\xi)}{\partial \xi} \dot{\xi}, \end{aligned} \quad (10)$$

where

$$Q = \begin{bmatrix} \begin{pmatrix} \cos(\bar{\phi}) \\ \sin(\bar{\phi}) \end{pmatrix} R(\phi) \begin{pmatrix} \frac{\partial f_1(\xi)}{\partial \xi} \\ \frac{\partial f_2(\xi)}{\partial \xi} \end{pmatrix} \end{bmatrix} \quad (11)$$

is ensured to be invertible [17]. Different from  $(\bar{x}, \bar{y}, \bar{\phi})$ , the transformed coordinates  $(x, y, \phi)$  can be controlled separately by  $u_1$ ,  $u_2$ , and  $\xi$ , where  $\xi$  is an auxiliary manipulated variable to be designed; thus, (10) are not underactuated.

**3.1.2. Controller Design.** By replacing (5) with (10), the closed-loop system composed by (2) and (10) is of strict feedback form. Therefore, the backstepping technique [24, 25] method is applied to design the control inputs  $\tau$ . Obviously, the design procedure can be divided into two steps. In the first step, the virtual controls  $u_{1d}$  and  $u_{2d}$  together with the “auxiliary manipulated variable”  $\xi$  will be constructed to stabilize the system. In the second step, the actual control signals for  $\tau$  will be delivered such that  $u_1$  and  $u_2$  in (10) can approach the virtual controls  $u_{1d}$  and  $u_{2d}$ , respectively. Apart from these, the adaptive laws for the unknown system parameters will also be provided.

**Step 1.** Define the parameter estimation errors  $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ ,  $i = 1, 2$ , where  $\hat{\theta}_1$  and  $\hat{\theta}_2$  are the estimated values of  $r$  and  $rb^{-1}$ , respectively.

Let  $q = [x, y]^T$ , and choose the Lyapunov function candidate in this step as

$$V_1 = \frac{1}{2}q^T q + \frac{1}{2}\phi^2, \quad (12)$$

and then the derivative of  $V_1$  is  $\dot{V}_1 = q^T \dot{q} + \phi \dot{\phi}$ .

We then introduce two new error variables:

$$u_{1e} = u_1 - u_{1d}, \quad u_{2e} = u_2 - u_{2d}, \quad (13)$$

where  $u_{1d}$  and  $u_{2d}$  are the virtual controls for  $u_1$  and  $u_2$ , respectively.  $u_{1d}$ ,  $u_{2d}$ , and  $\dot{\phi}$  are chosen as

$$\begin{aligned} \begin{bmatrix} u_{1d} \\ \dot{\xi} \end{bmatrix} &= \begin{bmatrix} \hat{\theta}_1^{-1} & 0 \\ 0 & 1 \end{bmatrix} Q^{-1} \left( -k_1 q - R'(\phi) \begin{bmatrix} f_1(\xi) \\ f_2(\xi) \end{bmatrix} \times (-k_2 \phi) \right), \\ u_{2d} &= \hat{\theta}_2^{-1} \left( -k_2 \phi + \frac{\partial f_3(\xi)}{\partial \xi} \dot{\xi} \right), \end{aligned} \quad (14)$$

where  $k_1$  and  $k_2$  are positive constants. The above design delivers the following results:

$$\begin{aligned} \dot{q} &= -k_1 q + Q \begin{bmatrix} \bar{\theta}_1 u_1 + \hat{\theta}_1 u_{1e} \\ 0 \end{bmatrix} \\ &+ \frac{\partial R(\phi)}{\partial \phi} \begin{bmatrix} f_1(\xi) \\ f_2(\xi) \end{bmatrix} \times (\bar{\theta}_2 u_2 + \hat{\theta}_2 u_{2e}), \\ \dot{\phi} &= -k_2 \phi + \bar{\theta}_2 u_2 + \hat{\theta}_2 u_{2e}. \end{aligned} \quad (15)$$

The parameter estimators for  $r$  and  $rb^{-1}$  are designed as

$$\begin{aligned} \dot{\hat{\theta}}_1 &= \text{Proj}(\hat{\theta}_1, \gamma_{\theta_1} \pi_1 u_1), \\ \dot{\hat{\theta}}_2 &= \text{Proj}(\hat{\theta}_2, \gamma_{\theta_2} \pi_2 u_2), \end{aligned} \quad (16)$$

where  $\pi_1 = x \cos(\bar{\phi}) + y \sin(\bar{\phi})$ ,  $\pi_2 = q(\partial R(\phi)/\partial \phi) \begin{bmatrix} f_1(\xi) \\ f_2(\xi) \end{bmatrix} + \phi$ . Note that  $\text{Proj}(\cdot)$  denotes a Lipschitz continuous projection operator about which the design details and properties can be found in [26] and the following results are then obtained.

**Lemma 3.** If  $|\hat{b}(t_0)| \leq b_M$ , then the projection satisfies  $\tilde{b} \text{Proj}(a, \hat{b}) \geq \tilde{b}a$ , where  $\tilde{b} = b - \hat{b}$ .

Choose the Lyapunov function candidate in this step as

$$V_2 = V_1 + \frac{1}{2\gamma_{\theta_1}} \tilde{\theta}_1^2 + \frac{1}{2\gamma_{\theta_2}} \tilde{\theta}_2^2. \quad (17)$$

We obtain that

$$\dot{V}_2 \leq -k_1 q^T q - k_2 \phi^2 + \pi_1 \hat{\theta}_1 u_{1e} + \pi_2 \hat{\theta}_2 u_{2e}. \quad (18)$$

Step 2. We are at the position to derive the actual control torque  $\tau$ .

Define  $z_i = \omega_i - \omega_{id}$ ,  $i = 1, 2$ , where  $\omega_{1d} = u_{1d} + u_{2d}$ ,  $\omega_{2d} = u_{1d} - u_{2d}$ . Let  $z = [z_1, z_2]^T$ ; thus, we have

$$z = \omega - \begin{bmatrix} \omega_{1d} \\ \omega_{2d} \end{bmatrix}. \quad (19)$$

Multiplying the derivatives of both sides of (19) by  $M$  and combining it with (14), we obtain that

$$M\dot{z} = -Dz + \chi^T \vartheta + \tau + \tau_d, \quad (20)$$

where

$$\begin{aligned} \vartheta &= \begin{bmatrix} \frac{r^3}{2b^2} m_c d & d_{11} & d_{22} & m_{11} & m_{12} & m_{11}r & m_{12}r & m_{11}rb^{-1} & m_{12}rb^{-1} \end{bmatrix}, \\ \chi &= \begin{bmatrix} -\omega_2 u_2 & -\omega_{1d} & 0 & -\Delta_{11} & -\Delta_{12} & -\Delta_{21} & -\Delta_{22} & -\Delta_{31} & -\Delta_{32} \\ \omega_1 u_2 & 0 & -\omega_{2d} & -\Delta_{12} & -\Delta_{11} & -\Delta_{22} & -\Delta_{21} & -\Delta_{32} & -\Delta_{31} \end{bmatrix}, \\ \Delta_{1k} &= \frac{\partial \omega_{kd}}{\partial \hat{\theta}_1} \hat{\theta}_1 + \frac{\partial \omega_{kd}}{\partial \hat{\theta}_2} \hat{\theta}_2, \\ \Delta_{2k} &= u_1 \left( \frac{\partial \omega_{kd}}{\partial \bar{x}} \cos \bar{\phi} + \frac{\partial \omega_{kd}}{\partial \bar{y}} \sin(\bar{\phi}) \right), \\ \Delta_{3k} &= \frac{\partial \omega_{kd}}{\partial \bar{\phi}} u_2, \quad k = 1, 2. \end{aligned} \quad (21)$$

Introduce the estimate  $\hat{\vartheta}$  for unknown parameter vector  $\vartheta$ . Then the local control torque and adaptive law are designed as

$$\tau = -Kz - \chi^T \hat{\vartheta} - 0.5\Xi - \text{sgn}(z^T) * \hat{\tau}_{d\max}, \quad (22)$$

$$\dot{\hat{\vartheta}} = \Gamma \chi z, \quad (23)$$

where  $K$  is a given positive matrix and  $\hat{\tau}_{d\max}$  is the estimate of the upper bound of  $\tau_d$ ,  $\Xi = [\Xi_1, \Xi_2]^T$  with

$$\begin{aligned} \Xi_1 &= \pi_1 \hat{\theta}_1 + \pi_2 \hat{\theta}_2, \\ \Xi_2 &= \pi_1 \hat{\theta}_1 - \pi_2 \hat{\theta}_2. \end{aligned} \quad (24)$$

The update law for  $\hat{\tau}_{d\max}$  is chosen as

$$\dot{\hat{\tau}}_{id\max} = |z_i|, \quad i = 1, 2. \quad (25)$$

Choose the Lyapunov function for the overall system as

$$V_3 = V_2 + \frac{1}{2} (z^T Mz + \tilde{\vartheta}^T \Gamma^{-1} \tilde{\vartheta} + \|\tau_d\|_2^2), \quad (26)$$

where  $\Gamma$  is a symmetric and positive definite matrix and  $\tilde{\vartheta} = \vartheta - \hat{\vartheta}$ . We obtain that

$$\dot{V}_3(t) \leq -k_1 q_e^T q_e - k_2 \phi_e^2 - z^T (K + D) z. \quad (27)$$

The main results in this section are formally presented in the following theorem.

**Theorem 4.** Consider the nonholonomic mobile robot system (1) and (2), with the controller (22) and parameter update laws (23) and (25) under Assumption 1. Then the closed-loop system is stable and satisfies

$$\begin{aligned} \lim_{t \rightarrow \infty} \bar{x}(t) &\leq \sqrt{2}\varepsilon_1, \\ \lim_{t \rightarrow \infty} \bar{y}(t) &\leq \sqrt{2}\varepsilon_1, \\ \lim_{t \rightarrow \infty} \bar{\phi}(t) &\leq \varepsilon_2. \end{aligned} \quad (28)$$

*Proof.* Considering the projection operation,  $\tilde{\theta}_i$ ,  $i = 1, 2$  are bounded. Thus from (27), all signals in  $V_3$  are bounded. Hence,  $x$ ,  $y$ , and  $\phi$  are bounded. From (14), it is easy to check that  $u_{1d}$ ,  $u_{2d}$ , and  $\dot{\xi}$  are bounded. Thus  $u_1$  and  $u_2$  are bounded. From (26), the boundedness of  $\tau$  is concluded.

From (6) and (8), we obtain that

$$\|(x - \bar{x}, y - \bar{y})\| \leq \sqrt{2}\varepsilon_1, \quad |\phi - \bar{\phi}| \leq \varepsilon_2. \quad (29)$$

It then follows that

$$\begin{aligned} |\bar{x}| &\leq |\bar{x} - x| + |x|, \\ |\bar{y}| &\leq |\bar{y} - y| + |y|, \\ |\bar{\phi}| &\leq |\bar{\phi} - \phi| + |\phi|. \end{aligned} \quad (30)$$

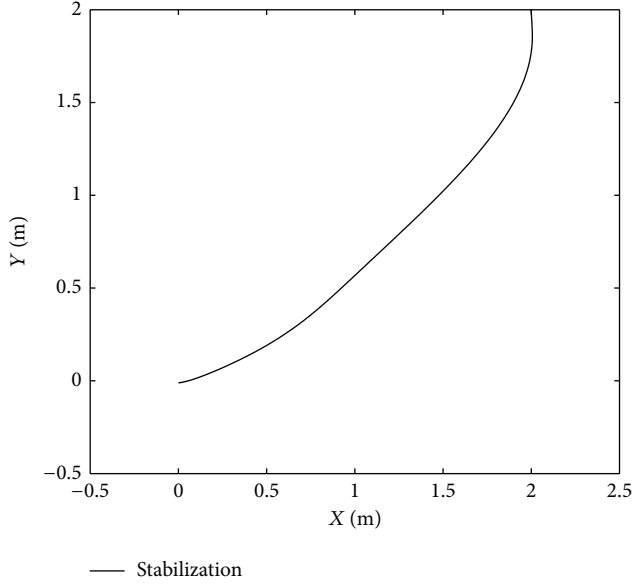


FIGURE 2: The position of the robot.

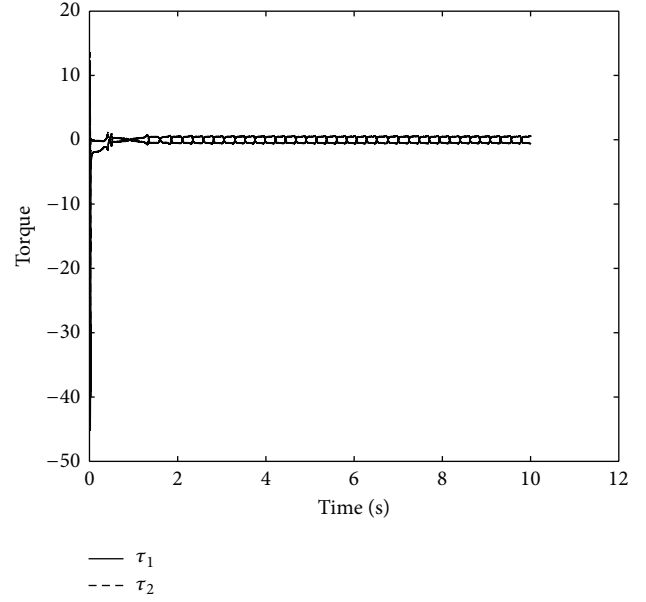


FIGURE 4: Control signals.

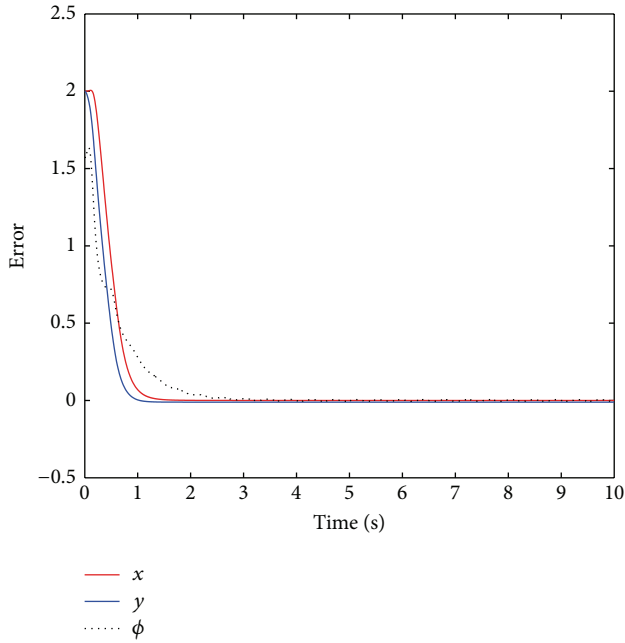
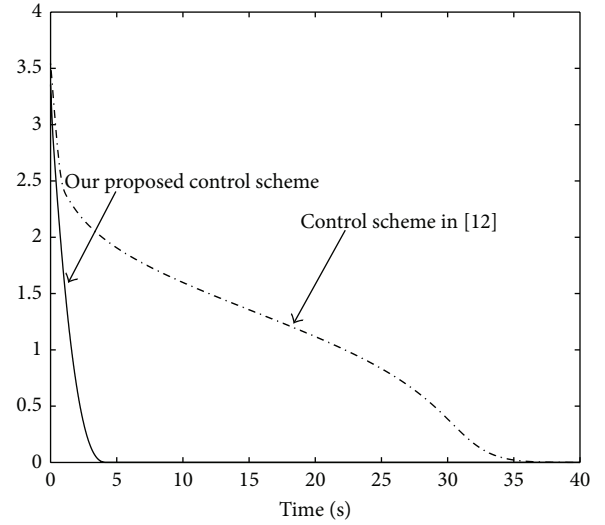
FIGURE 3: The trajectory of  $x$ ,  $y$ , and  $\phi$ .

FIGURE 5: Comparison of our proposed control scheme with the scheme in [12].

Since  $x$ ,  $y$ , and  $\phi$  will converge to zero asymptotically, (28) hold.  $\square$

**Remark 5.** Since  $\varepsilon_1$  and  $\varepsilon_2$  are arbitrarily small positive constants; thus from (28) we know the stabilization errors are also arbitrarily small. The convergence rate of the system is actually close related to the design parameters  $k_1$ ,  $k_2$ , and  $D$ ; thus, compared to the results mentioned in the Introduction, the convergence rate can be much faster.

TABLE 2: Simulation parameters of mobile robots.

| Parameters      |  |
|-----------------|--|
| Kinematic model | $b = 0.75, r = 0.15$   |
| Dynamic model   | $I_c = 15.625, I_w = 0.005, I_m = 0.00025, m_c = 30,$<br>$m_w = 1, d = 0.3, d_{11} = d_{22} = 5$ |

#### 4. Simulation Results

In this section, we illustrate the design procedure and how to compute the bound of the control torque based on design parameters in Table 2 using Matlab.

The initial estimates for the unknown parameters are set to be 75% of their true values. The bounds of the external disturbances are assumed to be 10. The design parameters are chosen as follows:  $k_1 = 2$ ,  $k_2 = 3$ ,  $K = 3I$ ,  $\varepsilon_1 = 0.05$ ,  $\varepsilon_2 = 0.02$ . The initial conditions are set as  $(2, 2, \pi/2)$ . The results on the mobile robot position and tracking errors evolving with time are shown in Figures 2 and 3, respectively. It can be seen that the system stabilized in 3 seconds with the torque in Figure 4. To show that our proposed control scheme renders a faster convergence rate, we make a comparison between our control scheme and the scheme proposed in [12] in terms of  $\|e\| = \sqrt{\bar{x}^2 + \bar{y}^2 + \bar{\phi}^2}$  with the same control design parameters, as shown in Figure 5. We can clearly see that the convergence rate of our proposed scheme is much faster than that of [12].

## 5. Conclusions

In this paper, stabilization problem of nonholonomic mobile robot with unknown system parameters and external disturbances is investigated. By considering the kinematic model and dynamic model of the system, the traverse function approach and the backstepping method are used to stabilize the mobile robot.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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