Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2014, Article ID 418702, 6 pages
http://dx.doi.org/10.1155/2014/418702

Research Article

Unisoft Hypervector Spaces

Abdullah M. Al-Roqi

Department of Mathematics, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

Correspondence should be addressed to Abdullah M. Al-Roqi; amalroqi692@gmail.com

Received 26 November 2013; Accepted 6 January 2014; Published 23 February 2014

Academic Editor: M. Mursaleen

The notion of unisoft subfields, unisoft algebras over unisoft subfields, and unisoft hypervector spaces are introduced, and their properties and characterizations are considered. In connection with linear transformations, unisoft hypervector spaces are discussed.

1. Introduction

The hyperstructure theory was introduced by Marty [1] at the 8th Congress of Scandinavian Mathematicians in 1934. As a generalization of fuzzy vector spaces, the fuzzy hypervector spaces are studied by Ameri and Dehghan (see [2, 3]). Molodtsov [4] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out to several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [5] described the application of soft set theory to a decision-making problem. Maji et al. [6] also studied several operations on the theory of soft sets. Chen et al. [7] presented a new definition of soft set parametrization reduction and compared this definition to the related concept of attributes reduction in rough set theory. The algebraic structure of set theories dealing with uncertainties has been studied by some authors. Çağman et al. [8] introduced fuzzy parameterized (FP) soft sets and their related properties. They proposed a decision-making method based on FP soft set theory and provided an example which shows that the method can be successfully applied to the problems that contain uncertainties. Feng [9] considered the application of soft rough approximations in multicriteria group decision-making problems. Aktaş and Çağman [10] studied the basic concepts of soft set theory and compared soft sets to fuzzy and rough sets, providing examples to clarify their differences. They also discussed the notion of soft groups. After that, many algebraic properties of soft sets are studied (see [11–21]).

In this paper, we introduce the notion of unisoft subfields, unisoft algebras over unisoft subfields, and unisoft hypervector spaces. We study their properties and characterizations. In connection with linear transformations, we discuss unisoft hypervector spaces.

2. Preliminaries

A soft set theory introduced by Molodtsov [4] and Çağman and Enginoğlu [22] provided new definitions and various results on soft set theory.

In what follows, let U be an initial universe set and let E be a set of parameters. Let $P(U)$ denote the power set of U and $A, B, C, \ldots \subseteq E$.

Definition 1 (*see [4, 22]*). A soft set (α, A) over U is defined to be the set of ordered pairs

$$(\alpha, A) := \{(x, \alpha(x)) : x \in E, \alpha(x) \in P(U)\}, \quad (1)$$

where $\alpha : E \to P(U)$ such that $\alpha(x) = \emptyset$ if $x \notin A$.

A map $\bigcirc : H \times H \to P_*(H)$ is called a hyperoperation or join operation, where $P_*(H)$ is the set of all nonempty subsets of H. The join operation is extended to subsets of H in natural way, so that $A \bigcirc B$ is given by

$$A \bigcirc B = \bigcup \{a \bigcirc b \mid a \in A, b \in B\}. \quad (2)$$

The notations $a \bigcirc A$ and $A \bigcirc a$ are used for $\{a\} \bigcirc A$ and $A \bigcirc \{a\}$, respectively. Generally, the singleton $\{a\}$ is identified by its element a.

Definition 2 (*see [23]*). Let F be a field and $(V, +)$ be an abelian group. A *hypervector space* over F is defined to be the quadruplet $(V, +, \bigcirc, F)$, where "$\bigcirc$" is a mapping

$$\bigcirc : F \times V \longrightarrow P_*(V) \tag{3}$$

such that for all $a, b \in F$ and $x, y \in V$ the following conditions hold:

(H1) $a \bigcirc (x + y) \subseteq a \bigcirc x + a \bigcirc y$,

(H2) $(a + b) \bigcirc x \subseteq a \bigcirc x + b \bigcirc x$,

(H3) $a \bigcirc (b \bigcirc x) = (ab) \bigcirc x$,

(H4) $a \bigcirc (-x) = (-a) \bigcirc x = -(a \bigcirc x)$,

(H5) $x \in 1 \bigcirc x$.

A hypervector space $(V, +, \bigcirc, F)$ over a field F is said to be *strongly left distributive* (see [2]) if it satisfies the following condition:

$$(\forall a \in F)(\forall x, y \in V)(a \bigcirc (x + y) = a \bigcirc x + a \bigcirc y). \tag{4}$$

3. Unisoft Algebras over a Unisoft Field

In what follows let F be a field unless otherwise specified.

Definition 3. A soft set (α, F) over F is called a *unisoft subfield* of F if the following conditions are satisfied:

(1) $(\forall a, b \in F)(\alpha(a + b) \subseteq \alpha(a) \cup \alpha(b))$,

(2) $(\forall a \in F)(\alpha(-a) \subseteq \alpha(a))$,

(3) $(\forall a, b \in F)(\alpha(a) \cup \alpha(b) \supseteq \alpha(ab))$,

(4) $(\forall a \in F)(a \neq 0 \Rightarrow \alpha(a) \supseteq \alpha(a^{-1}))$.

Proposition 4. *If (α, F) is a unisoft subfield of F, then*

(1) $(\forall a \in F)(\alpha(a) \supseteq \alpha(0))$,

(2) $(\forall a \in F)(a \neq 0 \Rightarrow \alpha(a) \supseteq \alpha(1))$,

(3) $\alpha(1) \supseteq \alpha(0)$.

Proof. (1) For all $a \in F$, we have

$$\alpha(0) = \alpha(a + (-a)) \subseteq \alpha(a) \cup \alpha(-a) = \alpha(a). \tag{5}$$

(2) Let $a \in F$ be such that $a \neq 0$. Then

$$\alpha(1) = \alpha(aa^{-1}) \subseteq \alpha(a) \cup \alpha(a^{-1}) = \alpha(a). \tag{6}$$

(3) It follows from (1). □

It is easy to show that the following theorem holds.

Theorem 5. *A soft set (α, F) over F is a unisoft subfield of F if and only if the nonempty γ-exclusive set*

$$e_F(\alpha; \gamma) := \{a \in F \mid \gamma \supseteq \alpha(a)\} \tag{7}$$

of (α, F) is a subfield of F for all $\gamma \in P(U)$.

Definition 6. Let V be an algebra over F and let (α, F) be a unisoft subfield of F. A soft set (β, V) is called an *unisoft algebra* over (α, F) if it satisfies the following conditions:

(1) $(\forall x, y \in V)(\beta(x + y) \subseteq \beta(x) \cup \beta(y))$,

(2) $(\forall a \in F)(\forall x \in V)(\beta(ax) \subseteq \alpha(a) \cup \beta(x))$,

(3) $(\forall x, y \in V)(\beta(xy) \subseteq \beta(x) \cup \beta(y))$,

(4) $(\forall x \in V)(\alpha(1) \subseteq \beta(x))$.

Proposition 7. *Let V be an algebra over F and let (α, F) be a unisoft subfield of F. If (β, V) is a unisoft algebra over (α, F), then $\alpha(0) \subseteq \beta(x)$ for all $x \in V$.*

Proof. For any $x \in V$, we have $\alpha(0) \subseteq \alpha(1) \subseteq \beta(x)$. □

We provide a characterization of a unisoft algebra over (α, F).

Theorem 8. *For any algebra V over F, let (α, F) be a unisoft subfield of F. Then a soft set (β, V) is a unisoft algebra over (α, F) if and only if it satisfies (3) and (4) of Definition 6 and*

$$(\forall a, b \in F)(\forall x, y \in V) \\ \times (\beta(ax + by) \subseteq (\alpha(a) \cup \beta(x)) \cup (\alpha(b) \cup \beta(y))). \tag{8}$$

Proof. Assume that (β, V) is a unisoft algebra over (α, F). Using (1) and (2) of Definition 6, we have

$$\beta(ax + by) \subseteq \beta(ax) \cup \beta(by) \\ \subseteq (\alpha(a) \cup \beta(x)) \cup (\alpha(b) \cup \beta(y)) \tag{9}$$

for all $a, b \in F$ and $x, y \in V$.

Conversely, suppose that (β, V) satisfies (3) and (4) of Definition 6 and (8). Then

$$\beta(x + y) = \beta(1x + 1y) \\ \subseteq (\alpha(1) \cup \beta(x)) \cup (\alpha(1) \cup \beta(y)) \\ = \beta(x) \cup \beta(y). \tag{10}$$

By using Definition 6(3) and Proposition 4(3), we obtain $\alpha(0) \subseteq \alpha(1) \subseteq \beta(x)$ for all $x \in V$. Thus

$$\beta(ax) = \beta(ax + 0x) \subseteq (\alpha(a) \cup \beta(x)) \cup (\alpha(0) \cup \beta(x)) \\ = (\alpha(a) \cup \beta(x)) \cup \beta(x) = \alpha(a) \cup \beta(x) \tag{11}$$

for all $a \in F$ and $x \in V$. Therefore (β, V) is a unisoft algebra over (α, F). □

For any sets X and Y, let $\mu : X \rightarrow Y$ be a function and (α, X) and (β, Y) be soft sets over U.

(1) The soft set

$$\mu^{-1}(\beta, Y) = \{(x, \mu^{-1}(\beta)(x)) : x \in X, \mu^{-1}(\beta)(x) \in P(U)\}, \tag{12}$$

where $\mu^{-1}(\beta)(x) = \beta(\mu(x))$, is called the *unisoft preimage* of (β, Y) under μ.

(2) The soft set

$$\mu(\alpha, X) = \{(y, \mu(\alpha)(y)) : y \in Y, \mu(\alpha)(y) \in P(U)\}, \quad (13)$$

where

$$\mu(\alpha)(y) = \begin{cases} \bigcap_{x \in \mu^{-1}(y)} \alpha(x) & \text{if } \mu^{-1}(y) \neq \emptyset, \\ U & \text{otherwise,} \end{cases} \quad (14)$$

is called the *unisoft image* of (α, X) under μ.

Theorem 9. *Let V and W be algebras over F. For any algebraic homomorphism $\mu : V \to W$,*

(1) *if (β, W) is a unisoft algebra over (α, F), then the unisoft preimage $\mu^{-1}(\beta, W)$ of (β, W) under μ is also a unisoft algebra over (α, F).*

(2) *If (β, V) is a unisoft algebra over (α, F), then the unisoft image $\mu(\beta, V)$ of (β, V) under μ is also a unisoft algebra over (α, F).*

Proof. (1) For any $a, b \in F$ and $x, y \in V$, we have

$$\begin{aligned} \mu^{-1}(\beta)(ax + by) &= \beta(\mu(ax + by)) \\ &= \beta(a\mu(x) + b\mu(y)) \\ &\subseteq (\alpha(a) \cup \beta(\mu(x))) \\ &\quad \cup (\alpha(b) \cup \beta(\mu(y))) \\ &= (\alpha(a) \cup \mu^{-1}(\beta)(x)) \\ &\quad \cup (\alpha(b) \cup \mu^{-1}(\beta)(y)), \\ \mu^{-1}(\beta)(xy) &= \beta(\mu(xy)) = \beta(\mu(x)\mu(y)) \\ &\subseteq \beta(\mu(x)) \cup \beta(\mu(y)) \\ &= \mu^{-1}(\beta)(x) \cup \mu^{-1}(\beta)(y), \end{aligned} \quad (15)$$

and $\alpha(1) \subseteq \beta(\mu(x)) = \mu^{-1}(\beta)(x)$. Therefore, by Theorem 8, $\mu^{-1}(\beta, W)$ is a unisoft algebra over (α, F).

(2) Let $y_1, y_2 \in W$. If $\mu^{-1}(y_1) = \emptyset$ or $\mu^{-1}(y_2) = \emptyset$, then

$$\mu(\beta)(y_1) \cup \mu(\beta)(y_2) = U \supseteq \mu(\beta)(y_1 + y_2). \quad (16)$$

Assume that $\mu^{-1}(y_1) \neq \emptyset$ and $\mu^{-1}(y_2) \neq \emptyset$. Then $\mu^{-1}(y_1 + y_2) \neq \emptyset$, and so

$$\begin{aligned} \mu(\beta)(y_1 + y_2) &= \bigcap_{x \in \mu^{-1}(y_1 + y_2)} \beta(x) \\ &\subseteq \bigcap_{\substack{x_1 \in \mu^{-1}(y_1) \\ x_2 \in \mu^{-1}(y_2)}} \beta(x_1 + x_2) \\ &\subseteq \bigcap_{\substack{x_1 \in \mu^{-1}(y_1) \\ x_2 \in \mu^{-1}(y_2)}} (\beta(x_1) \cup \beta(x_2)) \\ &= \left(\bigcap_{x_1 \in \mu^{-1}(y_1)} \beta(x_1)\right) \cup \left(\bigcap_{x_2 \in \mu^{-1}(y_2)} \beta(x_2)\right) \\ &= \mu(\beta)(y_1) \cup \mu(\beta)(y_2). \end{aligned} \quad (17)$$

For any $y \in W$ and $a \in F$, we have

$$\begin{aligned} \mu(\beta)(ay) &= \bigcap_{\mu(x) = ay} \beta(x) = \bigcap_{\mu(x) = y} \beta(ay) \\ &\subseteq \bigcap_{\mu(x) = y} (\alpha(a) \cup \beta(y)) \\ &= \alpha(a) \cup \bigcap_{\mu(x) = y} \beta(y) \\ &= \alpha(a) \cup \mu(\beta)(y). \end{aligned} \quad (18)$$

For all $y_1, y_2 \in W$, if at least one of $\mu^{-1}(y_1)$ and $\mu^{-1}(y_2)$ is empty, then the inclusion

$$\mu(\beta)(y_1) \cup \mu(\beta)(y_2) \supseteq \mu(\beta)(y_1 y_2) \quad (19)$$

is clear. Assume that $\mu^{-1}(y_1) \neq \emptyset$ and $\mu^{-1}(y_2) \neq \emptyset$. Then

$$\begin{aligned} &\mu(\beta)(y_1) \cup \mu(\beta)(y_2) \\ &= \left(\bigcap_{x_1 \in \mu^{-1}(y_1)} \beta(x_1)\right) \cup \left(\bigcap_{x_2 \in \mu^{-1}(y_2)} \beta(x_2)\right) \\ &= \bigcap_{\substack{x_1 \in \mu^{-1}(y_1) \\ x_2 \in \mu^{-1}(y_2)}} (\beta(x_1) \cup \beta(x_2)) \\ &\supseteq \bigcap_{\substack{x_1 \in \mu^{-1}(y_1) \\ x_2 \in \mu^{-1}(y_2)}} \beta(x_1 x_2) \\ &= \bigcap_{x \in \mu^{-1}(y_1 y_2)} \beta(x) \\ &= \mu(\beta)(y_1 y_2). \end{aligned} \quad (20)$$

Since $\alpha(1) \subseteq \beta(x)$ for all $x \in V$, it follows that

$$\alpha(1) \subseteq \bigcap_{x \in \mu^{-1}(y)} \beta(x) = \mu(\beta)(y) \quad (21)$$

for all $y \in W$. Therefore $\mu(\beta, V)$ is a unisoft algebra over (α, F). □

4. Unisoft Hypervector Spaces

Definition 10. Let V be a hypervector space over F and (α, F) a unisoft subfield of F. A soft set (β, V) over V is called a *unisoft hypervector space* of V related to (α, F) if the following assertions are valid:

(1) $(\forall x, y \in V)(\beta(x+y) \subseteq \beta(x) \cup \beta(y))$,

(2) $(\forall x \in V)(\beta(-x) \subseteq \beta(x))$,

(3) $(\forall a \in F)(\forall x \in V)(\bigcup_{y \in a \bigcirc x} \beta(y) \subseteq \alpha(a) \cup \beta(x))$,

(4) $\alpha(1) \subseteq \beta(\theta)$ where θ is the zero of $(V, +)$.

Proposition 11. *Let V be a hypervector space over F and (α, F) a unisoft subfield of F. If (β, V) is a unisoft hypervector space of V related to (α, F), then*

(1) $\beta(\theta) \supseteq \alpha(0)$,

(2) $(\forall x \in V)(\beta(x) \supseteq \beta(\theta))$,

(3) $(\forall x \in V)(\beta(x) \supseteq \alpha(0))$.

Proof. It is an immediate consequence of Definition 10 and Proposition 4. □

Proposition 12. *Let V be a hypervector space over F. If (β, V) is a unisoft hypervector space of V related to a unisoft subfield (α, F) of F, then*

$$(\forall x \in V)\left(\beta(x) = \bigcup_{y \in 1 \bigcirc x} \beta(y)\right). \tag{22}$$

Proof. Let $x \in V$. Since $x \in 1 \bigcirc x$ by (H5), we have $\beta(x) \subseteq \bigcup_{y \in 1 \bigcirc x} \beta(y)$. Using Definition 10(3) we have

$$\bigcup_{y \in 1 \bigcirc x} \beta(y) \subseteq \alpha(1) \cup \beta(x) \subseteq \beta(x). \tag{23}$$

Hence $\beta(x) = \bigcup_{y \in 1 \bigcirc x} \beta(y)$ for all $x \in V$. □

Theorem 13. *Assume that a hypervector space V over F is strongly left distributive. Let (α, F) be a unisoft subfield of F. Then a soft set (β, V) over V is a unisoft hypervector space of V related to (α, F) if and only if the following conditions are true:*

(1) $\bigcup_{z \in a \bigcirc x + b \bigcirc y} \beta(z) \subseteq (\alpha(a) \cup \beta(x)) \cup (\alpha(b) \cup \beta(y))$,

(2) $\beta(x) \supseteq \alpha(1)$

for all $a, b \in F$ and all $x, y \in V$.

Proof. Assume that (β, V) is a unisoft hypervector space of V related to (α, F). The second condition follows from Proposition 11(2) and Definition 10(4). Let $a, b \in F$ and $x, y \in V$. Then

$$\begin{aligned}\bigcup_{z \in a \bigcirc x + b \bigcirc y} \beta(z) &= \bigcup_{\substack{z \in u+v \\ u \in a \bigcirc x, v \in b \bigcirc y}} \beta(z) \\ &\subseteq (\alpha(a) \cup \beta(x)) \cup (\alpha(b) \cup \beta(y)).\end{aligned} \tag{24}$$

Conversely suppose the conditions (1) and (2) are true. For all $x, y \in V$, we have

$$\begin{aligned}\beta(x+y) &\subseteq \bigcup_{z \in 1 \bigcirc x + 1 \bigcirc y} \beta(z) \\ &\subseteq (\alpha(1) \cup \beta(x)) \cup (\alpha(1) \cup \beta(y)) \\ &\subseteq \beta(x) \cup \beta(y).\end{aligned} \tag{25}$$

Since (α, F) is a unisoft subfield of F, we have $\beta(a) \supseteq \alpha(1) \supseteq \alpha(0)$ and $\beta(a) \supseteq \alpha(1) \supseteq \alpha(-1)$. Note that $0 \in 0 \bigcirc x$ for all $x \in V$. It follows that

$$\begin{aligned}\beta(-x) &\subseteq \bigcup_{y \in 0 \bigcirc x + (-1) \bigcirc x} \beta(y) \\ &\subseteq (\alpha(0) \cup \beta(x)) \cup (\alpha(-1) \cup \beta(x)) \\ &= \beta(x) \cup \beta(x) = \beta(x)\end{aligned} \tag{26}$$

for all $x \in V$. Let $a \in F$ and $x \in V$. Then

$$\begin{aligned}\bigcup_{y \in a \bigcirc x} \beta(y) &\subseteq \bigcup_{\substack{y \in u+v \\ u \in 0 \bigcirc x, v \in a \bigcirc x}} \beta(y) \\ &\subseteq (\alpha(0) \cup \beta(x)) \cup (\alpha(a) \cup \beta(x)) \\ &= \beta(x) \cup (\alpha(a) \cup \beta(x)) \\ &= \alpha(a) \cup \beta(x).\end{aligned} \tag{27}$$

Clearly, $\alpha(1) \subseteq \beta(\theta)$. Therefore (β, V) is a unisoft hypervector space of V related to (α, F). □

Theorem 14. *Let V be a hypervector space over F and (α, F) a unisoft subfield of F. If a soft set (β, V) over V is a unisoft hypervector space of V related to (α, F), then the nonempty γ-exclusive set*

$$e_V(\beta; \gamma) := \{x \in V \mid \gamma \supseteq \beta(x)\} \tag{28}$$

of (β, V) is a subhypervector space of V over the field $e_F(\alpha; \gamma)$ for all $\gamma \in P(U)$.

Proof. Let $x, y \in e_V(\beta; \gamma)$. Then $\beta(x) \subseteq \gamma$ and $\beta(y) \subseteq \gamma$. It follows that

$$\begin{aligned}\beta(x-y) &= \beta(x+(-y)) \\ &\subseteq \beta(x) \cup \beta(-y) \subseteq \beta(x) \cup \beta(y) \subseteq \gamma.\end{aligned} \tag{29}$$

Hence $x - y \in e_V(\beta; \gamma)$. Note that $e_F(\alpha; \gamma)$ is a subfield of F (see Theorem 5). Let $a \in e_F(\alpha; \gamma)$, $x \in e_V(\beta; \gamma)$ and $y \in a \bigcirc x$. Then

$$\beta(y) \subseteq \bigcup_{z \in a \bigcirc x} \beta(z) \subseteq \alpha(a) \cup \beta(x) \subseteq \gamma, \tag{30}$$

and so $y \in e_V(\beta; \gamma)$ which shows that $a \bigcirc x \subseteq e_V(\beta; \gamma)$. Therefore $e_V(\beta; \gamma)$ is a hypervector space over the field $e_F(\alpha; \gamma)$ for all $\gamma \in P(U)$. □

Let V and W be hypervector spaces over F. A mapping $T : V \to W$ is called *linear transformation* (see [3]) if it satisfies

(i) $(\forall x, y \in V)(T(x+y) = T(x) + T(y))$,

(ii) $(\forall a \in F)$ $(\forall x \in V)(T(a \bigcirc x) \subseteq a \bigcirc T(x))$.

Theorem 15. *Let V and W be hypervector spaces over F and let (α, F) be a unisoft subfield of F. For any linear transformation $T : V \to W$, if (β, W) is a unisoft hypervector space of W related to (α, F), then $T^{-1}(\beta, W)$ is a unisoft hypervector space of V related to (α, F).*

Proof. Let $a, b \in F$ and $x, y \in V$. Since T is a linear transformation, we have

$$\begin{aligned}
&\bigcup_{z\in a\bigcirc x+b\bigcirc y} T^{-1}(\beta)(z) \\
&= \bigcup_{z\in a\bigcirc x+b\bigcirc y} \beta(T(z)) \\
&= \bigcup_{u\in a\bigcirc x, v\in b\bigcirc y} \beta(T(u+v)) \\
&\subseteq \bigcup_{\substack{T(u)\in a\bigcirc T(x)\\ T(v)\in b\bigcirc T(y)}} \beta(T(u+v)) \\
&\subseteq (\beta(T(x)) \cup \alpha(a)) \cup (\beta(T(y)) \cup \alpha(b)) \\
&= (T^{-1}(\beta)(x) \cup \alpha(a)) \cup (T^{-1}(\beta)(y) \cup \alpha(b)).
\end{aligned} \quad (31)$$

Obviously, $\alpha(1) \subseteq T^{-1}(\beta)(x)$ for all $x \in V$. It follows from Theorem 13 that $T^{-1}(\beta, W)$ is a unisoft hypervector space of V related to (α, F). □

Theorem 16. *Let V and W be hypervector spaces over F and let (α, F) be a unisoft subfield of F. For any linear transformation $T : V \to W$, if (β, V) is a unisoft hypervector space of V related to (α, F), then $T(\beta, V)$ is a unisoft hypervector space of W related to (α, F).*

Proof. Let $a, b \in F$ and $x, y \in W$. If at least one of $T^{-1}(x)$ and $T^{-1}(y)$ is empty, then the inclusion

$$\begin{aligned}
&\bigcup_{z\in a\bigcirc x+b\bigcirc y} T(\beta)(z) \\
&\subseteq (\alpha(a) \cup T(\beta)(x)) \cup (\alpha(b) \cup T(\beta)(y))
\end{aligned} \quad (32)$$

is clear. Assume that $T^{-1}(x)$ and $T^{-1}(y)$ are nonempty. Then there exist $u, v \in V$ such that $T(u) = x$ and $T(v) = y$. Thus,

$$\begin{aligned}
a\bigcirc x + b\bigcirc y &= a\bigcirc T(u) + b\bigcirc T(v) \\
&\supseteq T(a\bigcirc u) + T(b\bigcirc v) \\
&= T(a\bigcirc u + b\bigcirc v)
\end{aligned} \quad (33)$$

since T is linear. Hence, $a\bigcirc u + b\bigcirc v \subseteq T^{-1}(a\bigcirc x + b\bigcirc y)$. Then

$$\begin{aligned}
&\bigcup_{w\in a\bigcirc x+b\bigcirc y} T(\beta)(w) \\
&= \bigcup_{\substack{w\in a\bigcirc x+b\bigcirc y\\ T(z)=w}} \beta(z) \\
&\subseteq \bigcup_{\substack{u\in T^{-1}(x),\, v\in T^{-1}(y)\\ z=z_1+z_2,\, z_1\in a\bigcirc u,\, z_2\in b\bigcirc v}} (\beta(z_1) \cup \beta(z_2)) \\
&\subseteq (\alpha(a) \cup \beta(u)) \cup (\alpha(b) \cup \beta(v)) \\
&= (\alpha(a) \cup T(\beta)(x)) \cup (\alpha(b) \cup T(\beta)(y)).
\end{aligned} \quad (34)$$

Obviously, $\alpha(1) \subseteq T(\beta)(x)$ for all $x \in W$. Therefore $T(\beta, V)$ is a unisoft hypervector space of W related to (α, F) by Theorem 13. □

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The author would like to express his sincere thanks to the anonymous referees for their valuable suggestions and comments that improved this paper.

References

[1] F. Marty, “Sur une generalization de la notion de groupe,” in *Proceedings of the 8th Congress des Mathematiciens Scandinaves*, pp. 45–49, Stockholm, Sweden, 1934.
[2] R. Ameri and O. Dehghan, “Fuzzy hypervector spaces based on fuzzy singletons,” *Computers & Mathematics with Applications*, vol. 61, no. 10, pp. 2933–2943, 2011.
[3] R. Ameri and O. R. Dehghan, “Fuzzy hypervector spaces,” *Advances in Fuzzy Systems*, vol. 2008, Article ID 295649, 9 pages, 2008.
[4] D. Molodtsov, “Soft set theory—first results,” *Computers & Mathematics with Applications*, vol. 37, no. 4-5, pp. 19–31, 1999.
[5] P. K. Maji, A. R. Roy, and R. Biswas, “An application of soft sets in a decision making problem,” *Computers & Mathematics with Applications*, vol. 44, no. 8-9, pp. 1077–1083, 2002.
[6] P. K. Maji, R. Biswas, and A. R. Roy, “Soft set theory,” *Computers & Mathematics with Applications*, vol. 45, no. 4-5, pp. 555–562, 2003.
[7] D. Chen, E. C. C. Tsang, D. S. Yeung, and X. Wang, “The parameterization reduction of soft sets and its applications,” *Computers & Mathematics with Applications*, vol. 49, no. 5-6, pp. 757–763, 2005.
[8] N. Çağman, F. Çitak, and S. Enginoglu, “FP-soft set theory and its applications,” *Annals of Fuzzy Mathematics and Informatics*, vol. 2, no. 2, pp. 219–226, 2011.
[9] F. Feng, “Soft rough sets applied to multicriteria group decision making,” *Annals of Fuzzy Mathematics and Informatics*, vol. 2, no. 1, pp. 69–80, 2011.
[10] H. Aktaş and N. Çağman, “Soft sets and soft groups,” *Information Sciences*, vol. 177, no. 13, pp. 2726–2735, 2007.
[11] U. Acar, F. Koyuncu, and B. Tanay, “Soft sets and soft rings,” *Computers & Mathematics with Applications*, vol. 59, no. 11, pp. 3458–3463, 2010.
[12] A. O. Atagün and A. Sezgin, “Soft substructures of rings, fields and modules,” *Computers & Mathematics with Applications*, vol. 61, no. 3, pp. 592–601, 2011.
[13] F. Feng, Y. B. Jun, and X. Zhao, “Soft semirings,” *Computers & Mathematics with Applications*, vol. 56, no. 10, pp. 2621–2628, 2008.
[14] Y. B. Jun, “Soft BCK/BCI-algebras,” *Computers & Mathematics with Applications*, vol. 56, no. 5, pp. 1408–1413, 2008.
[15] Y. B. Jun, H. S. Kim, and J. Neggers, “Pseudo d-algebras,” *Information Sciences*, vol. 179, no. 11, pp. 1751–1759, 2009.

[16] Y. B. Jun, K. J. Lee, and A. Khan, "Soft ordered semigroups," *Mathematical Logic Quarterly*, vol. 56, no. 1, pp. 42–50, 2010.

[17] Y. B. Jun, K. J. Lee, and C. H. Park, "Soft set theory applied to ideals in d-algebras," *Computers & Mathematics with Applications*, vol. 57, no. 3, pp. 367–378, 2009.

[18] Y. B. Jun, K. J. Lee, and J. Zhan, "Soft p-ideals of soft BCI-algebras," *Computers & Mathematics with Applications*, vol. 58, no. 10, pp. 2060–2068, 2009.

[19] Y. B. Jun and C. H. Park, "Applications of soft sets in ideal theory of BCK/BCI-algebras," *Information Sciences*, vol. 178, no. 11, pp. 2466–2475, 2008.

[20] C. H. Park, Y. B. Jun, and M. A. Öztürk, "Soft WS-algebras," *Korean Mathematical Society*, vol. 23, no. 3, pp. 313–324, 2008.

[21] J. Zhan and Y. B. Jun, "Soft *BL*-algebras based on fuzzy sets," *Computers & Mathematics with Applications*, vol. 59, no. 6, pp. 2037–2046, 2010.

[22] N. Çağman and S. Enginoğlu, "Soft set theory and uni–int decision making," *European Journal of Operational Research*, vol. 207, no. 2, pp. 848–855, 2010.

[23] M. S. Tallini, "Hypervector spaces," in *Proceedings of the 4th International Congress on AHA*, pp. 167–174, 1990.