

Research Article

A New Similarity Measure between Intuitionistic Fuzzy Sets and Its Application to Pattern Recognition

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As a generation of ordinary fuzzy set, the concept of intuitionistic fuzzy set (IFS), characterized both by a membership degree and by a nonmembership degree, is a more flexible way to cope with the uncertainty. Similarity measures of intuitionistic fuzzy sets are used to indicate the similarity degree between intuitionistic fuzzy sets. Although many similarity measures for intuitionistic fuzzy sets have been proposed in previous studies, some of those cannot satisfy the axioms of similarity or provide counterintuitive cases. In this paper, a new similarity measure and weighted similarity measure between IFSs are proposed. It proves that the proposed similarity measures satisfy the properties of the axiomatic definition for similarity measures. Comparison between the previous similarity measures and the proposed similarity measure indicates that the proposed similarity measure does not provide any counterintuitive cases. Moreover, it is demonstrated that the proposed similarity measure is capable of discriminating difference between patterns.

1. Introduction

Since it was proposed by Zadeh [1], the theory of fuzzy set (FS) has achieved great success due to its capability of handling uncertainty. Therefore, over the last decades, several higher order fuzzy sets have been introduced in the literature. Intuitionistic fuzzy set (IFS), as one of the higher order fuzzy sets, was proposed by Atanassov [2] to deal with vagueness. The main advantage of the IFS is its property to cope with the uncertainty that may exist due to information impression. Because it assigns to each element a membership degree, a nonmembership degree, and a hesitation degree, IFS constitutes an extension of Zadeh's fuzzy set which only assigns to each element a membership degree [3]. So IFS is regarded as a more effective way to deal with vagueness than fuzzy set. Although Gau and Buehrer later presented vague set [4], it was pointed out by Bustince and Burillo that the notion of vague sets was the same as that of IFS [5].

The definition of similarity measure between two IFSs is one of the most interesting topics in IFSs theory. A similarity measure is defined to compare the information carried by IFSs. Measures of similarity between IFSs, as

an important tool for decision making, pattern recognition, machine learning, and image processing, have received much attention in recent years [6, 7]. Many similarity measures have been proposed. A few of them come from the well-known distance measures. The first study was carried out by Szmied and Kacprzyk [8] extending the well-known distances measures, such as the Hamming distance and the Euclidean distance, to IFS environment and comparing them with the approaches used for ordinary fuzzy sets. However, Wang and Xin [9] implied that the distance measures of Szmied and Kacprzyk [8] were not effective in some cases. Therefore, several new distance measures were proposed and applied to pattern recognition. Grzegorzewski [10] also extended the Hamming distance, the Euclidean distance, and their normalized counterparts to IFS environment. Later, Chen [11] pointed out that some errors existed in Grzegorzewski [10] by showing some counterexamples. Hung and Yang [12] extended the Hausdorff distance to IFSs and proposed three similarity measures.

On the other hand, instead of extending the well-known measures, some studies defined new similarity measures for IFSs. Dengfeng and Chuntian [13] suggested a new similarity

measure for IFSs based on the membership degree and the nonmembership degree. Mitchell [14] showed that the similarity measure of Dengfeng and Chuntian [13] had some counterintuitive cases and modified the similarity measure based on statistical point of view. Moreover, Liang and Shi [15] presented some examples to show that the similarity measure of Dengfeng and Chuntian [13] was not reasonable for some conditions and therefore proposed several new similarity measures for IFSs. Li et al. [16] analyzed, compared, and summarized the existing similarity measures between IFSs/vague sets by their counterintuitive examples in pattern recognition. Ye [7] conducted a similar comparative study of the existing similarity measures between IFSs and proposed a cosine similarity measure and a weighted cosine similarity measure. Hwang et al. [17] proposed a similarity measure for IFSs in which Sugeno integral was used for aggregation. The proposed similarity measure was applied to clustering problem. Xu [18] introduced a series of similarity measures for IFSs and applied them to multiple attribute decision making problem based on intuitionistic fuzzy information. Xu and Chen [19] introduced a series of distance and similarity measures, which are various combinations and generalizations of the weighted Hamming distance, the weighted Euclidean distance, and the weighted Hausdorff distance. Xu and Yager [20] developed a similarity measure between IFSs and applied the developed similarity measure for consensus analysis in group decision making based on intuitionistic fuzzy preference relations. Xia and Xu [6] proposed a series of distance measures based on the intuitionistic fuzzy point operators. In addition to these studies, some works have been interested in relationships between distance measure, similarity measure, and entropy of IFSs. Zeng and Guo [21] investigated the relationship among the normalized distance, the similarity measure, the inclusion measure, and the entropy of interval-valued fuzzy sets. It was also showed that the similarity measure, the inclusion measure, and the entropy of interval-valued fuzzy sets could be induced by the normalized distance of interval-valued fuzzy sets based on their axiomatic definitions. Wei et al. [22] introduced an entropy measure generalizing the existing entropy measures for IFS and IFSs. Also an approach was introduced to construct similarity measures using entropy measures for IFS and IFSs. Boran and Akay [23] proposed a new general type of similarity measure for IFS with two parameters, expressing L_p norm and the level of uncertainty, respectively. This similarity measure can also make sense in terms of counterintuitive cases.

Moreover, Zhang and Yu [24] presented a new distance (or similarity) measure based on interval comparison, where the IFSs were, respectively, transformed into the symmetric triangular fuzzy numbers. Comparison with the widely used methods indicated that the proposed method contained more information, with much less loss of information. Li et al. [25] introduced an axiomatic definition of the similarity measure of IFSs. The relationship between the entropy and the similarity measure of IFS was investigated in detail. It was proved that the similarity measure and the entropy of IFS can be transformed into each other based on their axiomatic definitions. Farhadinia [26] generalized results on the entropy of interval-valued fuzzy sets (IVFSs) based on

the intuitionistic distance and its relationship with similarity measure. Based on the set of new axioms, he also proved some theorems that entropy and similarity measure for IVFSs can be transformed by each other in a general way. Li [27] defined two dissimilarity measures between intuitionistic fuzzy sets of a finite set, and it was proved that both of the measures are metrical. Papakostas et al. [28] investigated the main theoretical and computational properties of the measures as well as the relationships between them. A comparison of the distance and similarity measures was carried out by them, from a pattern recognition point of view.

Among similarity measures proposed by other authors, some of those, however, cannot satisfy the axioms of similarity or provide counterintuitive cases or are produced by complex formats. Therefore, we propose new similarity measures with relative simple expression. The proposed similarity measure depends on the triplet, membership degree, nonmembership degree, and hesitation margin. This paper proves that the proposed measures satisfy the properties of the axiomatic definition for similarity measures. In addition, several numerical examples are provided to compare the proposed measure with a number of existing measures.

The remainder of this paper is organized as follows. Section 2 recalls the definitions related to the IFSs and lists the properties that a distance measure for IFSs and a similarity measure for IFSs should possess. In Section 3, the existing similarity measures for IFSs will be recalled. The new similarity measure, together with its proofs, is presented in Section 4. Comparison between the proposed similarity measure and the existing similarity measures is carried out in Section 5. The application of the proposed similarity measure to pattern recognition is presented in Section 6, followed by the conclusion of this paper in Section 7.

2. Preliminaries

In this section, we briefly recall the basic concepts related to IFS and then list the properties of the axiomatic definition for similarity measures.

Definition 1. Let $X = \{x_1, x_2, \dots, x_n\}$ be a universe of discourse; then, a fuzzy set A in X is defined as follows [1]:

$$A = \{\langle x, \mu_A(x) \rangle \mid x \in X\}, \quad (1)$$

where $\mu_A(x) : X \rightarrow [0, 1]$ is the membership degree.

Definition 2. An IFS A in X defined by Atanassov can be written as [2]

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}, \quad (2)$$

where $\mu_A(x) : X \rightarrow [0, 1]$ and $\nu_A(x) : X \rightarrow [0, 1]$ are membership degree and nonmembership degree, respectively, with the condition

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1. \quad (3)$$

$\pi_A(x)$ determined by the following expression:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), \quad (4)$$

is called the hesitancy degree of the element $x \in X$ to the set A , and $\pi_A(x) \in [0, 1]$, for all $x \in X$.

$\pi_A(x)$ is also called the intuitionistic index of x to A . Greater $\pi_A(x)$ indicates more vagueness on x . Obviously, when $\pi_A(x) = 0$, for all $x \in X$, the IFS degenerates into an ordinary fuzzy set.

In the sequel, the couple $\langle \mu_A(x), \nu_A(x) \rangle$ is called an IFS or intuitionistic fuzzy value (IFV) for clarity. Let $\text{IFSs}(X)$ denote the set of all IFSs in X .

Definition 3. For $A \in \text{IFSs}(X)$ and $B \in \text{IFSs}(X)$, some relations between them are defined as

- (R1) $A \subset B$ if and only if for all $x \in X$ $\mu_A(x) \leq \mu_B(x), \nu_A(x) \geq \nu_B(x)$;
- (R2) $A = B$ if and only if for all $x \in X$ $\mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x)$;
- (R3) $A^C = \{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X \}$, where A^C is the complement of A .

Definition 4. Let D denote the mapping $D : \text{IFS} \times \text{IFS} \rightarrow [0, 1]$; if $D(A, B)$ satisfies the following properties, $D(A, B)$ is called a distance between $A \in \text{IFSs}(X)$ and $B \in \text{IFSs}(X)$. Consider the following:

- (DP1) $0 \leq D(A, B) \leq 1$;
- (DP2) $D(A, B) = 0$, if and only if $A = B$;
- (DP3) $D(A, B) = D(B, A)$;
- (DP4) if $A \subseteq B \subseteq C$, then $D(A, B) \leq D(A, C)$, and $D(B, C) \leq D(A, C)$.

Definition 5. The mapping $S : \text{IFS} \times \text{IFS} \rightarrow [0, 1]$ is called a degree of similarity between $A \in \text{IFSs}(X)$ and $B \in \text{IFSs}(X)$, if $S(A, B)$ satisfies the following properties:

- (SP1) $0 \leq S(A, B) \leq 1$;
- (SP2) $S(A, B) = 1$, if and only if $A = B$;
- (SP3) $S(A, B) = S(B, A)$;
- (SP4) if $A \subseteq B \subseteq C$, then $S(A, B) \geq S(A, C)$, and $S(B, C) \geq S(A, C)$.

Because distance and similarity measures are complementary concepts, similarity measures can be used to define distance measures and vice versa.

3. Existing Similarity Measures

Let $X = \{x_1, x_2, \dots, x_n\}$ be a universe of discourse. $A \in \text{IFSs}(X)$ and $B \in \text{IFSs}(X)$ are two IFSs in X , denoted by $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in X \}$, respectively.

Considering the outcome of the analysis presented by Bustince and Burillo [5], which concluded that the intuitionistic fuzzy sets and the vague sets are similar, Chen [29] proposed the first similarity measure for IFSs defined as

$$S_C(A, B) = 1 - \frac{\sum_{i=1}^n |(\mu_A(x_i) - \nu_A(x_i)) - (\mu_B(x_i) - \nu_B(x_i))|}{2n} \tag{5}$$

Hong and Kim [30] proposed the following measure to overcome the deficiency of Chen's similarity measure:

$$S_H(A, B) = 1 - \frac{\sum_{i=1}^n |(\mu_A(x_i) - \mu_B(x_i)) - (\nu_A(x_i) - \nu_B(x_i))|}{2n} \tag{6}$$

Following the work of Hong and Kim, Li and Xu [31] proposed new similarity measures:

$$S_L(A, B) = 1 - \frac{\sum_{i=1}^n |(\mu_A(x_i) - \nu_A(x_i)) - (\mu_B(x_i) - \nu_B(x_i))|}{4n} - \frac{\sum_{i=1}^n (|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|)}{4n} \tag{7}$$

Taking the membership distance and the nonmember ship distance between the two IFSs into consideration, Li et al. [32] proposed a similarity measure as follows:

$$S_O(A, B) = 1 - \sqrt{\frac{\sum_{i=1}^n ((\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2)}{2n}} \tag{8}$$

Dengfeng and Chuntian [13] pointed out that some similarity could not deal with the similarity between IFSs well. So they defined a new similarity measure for pattern recognition:

$$S_{DC}(A, B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^n |\varphi_A(x_i) - \varphi_B(x_i)|^p}{n}}, \tag{9}$$

where $\varphi_A(x_i) = (\mu_A(x_i) + 1 - \nu_A(x_i))/2$ and $\varphi_B(x_i) = (\mu_B(x_i) + 1 - \nu_B(x_i))/2$.

Mitchell [14] found that the similarity measure $S_{DC}(A, B)$ would characterize two different IFSs as identical. To overcome this drawback, he provided a more realistic strong similarity measure of the following form:

$$S_{HB}(A, B) = \frac{1}{2} (\rho_\mu(A, B) + \rho_\nu(A, B)), \tag{10}$$

where $\rho_\mu(A, B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^n |\mu_A(x_i) - \mu_B(x_i)|^p}{n}}$, $\rho_\nu(A, B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^n |\nu_A(x_i) - \nu_B(x_i)|^p}{n}}$.

Liang and Shi [15] proposed the following three similarity measures:

$$S_e^p(A, B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^n (\phi_\mu(x_i) + \phi_\nu(x_i))^p}{n}}, \quad (11)$$

where $\phi_\mu(x_i) = |\mu_A(x_i) - \mu_B(x_i)|/2$, $\phi_\nu(x_i) = |(1 - \nu_A(x_i)) - (1 - \nu_B(x_i))|/2$. Consider

$$S_s^p(A, B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^n |\psi_{s1}(x_i) + \psi_{s2}(x_i)|^p}{n}}, \quad (12)$$

where $\psi_{s1}(x_i) = |m_{A1}(x_i) - m_{B1}(x_i)|/2$, $\psi_{s2}(x_i) = |m_{A2}(x_i) - m_{B2}(x_i)|/2$, $m_{A1}(x_i) = |\mu_A(x_i) + m_A(x_i)|/2$, $m_{B1}(x_i) = |\mu_B(x_i) + m_B(x_i)|/2$, $m_{A2}(x_i) = |1 - \nu_A(x_i) + m_A(x_i)|/2$, $m_{B2}(x_i) = |1 - \nu_B(x_i) + m_B(x_i)|/2$, $m_A(x_i) = |1 - \nu_A(x_i) + \mu_A(x_i)|/2$, and $m_B(x_i) = |1 - \nu_B(x_i) + \mu_B(x_i)|/2$. Consider

$$S_h^p(A, B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^n (\eta_1(x_i) + \eta_2(x_i) + \eta_3(x_i))^p}{3n}}, \quad (13)$$

where $\eta_1(x_i) = \phi_\mu(x_i) + \phi_\nu(x_i)$ (defined in S_e^p), $\eta_2(x_i) = |\varphi_\mu(x_i) - \varphi_\nu(x_i)|$ (defined in S_{DC}), $\eta_3(x_i) = \max(l_A(x_i), l_B(x_i)) - \min(l_A(x_i), l_B(x_i))$, $l_A(x_i) = (1 - \mu_A(x_i) - \nu_A(x_i))/2$, and $l_B(x_i) = (1 - \mu_B(x_i) - \nu_B(x_i))/2$.

Hung and Yang [12] introduced three similarity measures based on the Hausdorff distance $d_H(A, B)$:

$$\begin{aligned} S_{HY}^1(A, B) &= 1 - d_H(A, B), \\ S_{HY}^2(A, B) &= \frac{e^{-d_H(A, B)} - e^{-1}}{1 - e^{-1}}, \\ S_{HY}^3(A, B) &= \frac{1 - d_H(A, B)}{1 + d_H(A, B)}, \end{aligned} \quad (14)$$

where $d_H(A, B) = (1/n) \sum_{i=1}^n \max(|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|)$.

Based on the concept of the cosine similarity measure for fuzzy sets, a cosine similarity measure was proposed by Ye [7] as follows:

$$C_{IFS}(A, B) = \frac{1}{n} \sum_{i=1}^n \frac{\mu_A(x_i) \mu_B(x_i) + \nu_A(x_i) \nu_B(x_i)}{\sqrt{(\mu_A(x_i))^2 + (\nu_A(x_i))^2} \sqrt{(\mu_B(x_i))^2 + (\nu_B(x_i))^2}}. \quad (15)$$

One of the latest results on similarity measure is the biparametric similarity measure for IFS, proposed by Boran and Akay [23]. It has the following form with two parameters:

$$\begin{aligned} S_t^p(A, B) &= 1 - \left(\sum_{i=1}^n \frac{1}{2n(1+p)} \left\{ |t(\mu_A(x_i) - \mu_B(x_i)) - (\nu_A(x_i) - \nu_B(x_i))|^p + |t(\nu_A(x_i) - \nu_B(x_i)) - (\mu_A(x_i) - \mu_B(x_i))|^p \right\} \right)^{1/p}, \end{aligned} \quad (16)$$

where $p = 1, 2, 3, \dots$ is the L_p norm and $t = 2, 3, 4, \dots$ identifies the level of uncertainty.

4. A New Similarity Measure

Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ and $B = \{(x, \mu_B(x), \nu_B(x)) \mid x \in X\}$ be two IFSs in X . We propose a new similarity measure:

$$\begin{aligned} S_Y(A, B) &= \frac{1}{2n} \sum_{i=1}^n \left(\sqrt{\mu_A(x_i) \mu_B(x_i)} + 2\sqrt{\nu_A(x_i) \nu_B(x_i)} + \sqrt{\pi_A(x_i) \pi_B(x_i)} + \sqrt{(1 - \nu_A(x_i))(1 - \nu_B(x_i))} \right), \end{aligned} \quad (17)$$

where $\pi_A(x_i)$ and $\pi_B(x_i)$ are, respectively, the hesitancy degree of the element $x_i \in X$ to the sets A and B .

Theorem 6. $S_Y(A, B)$ is the similarity measure between two IFSs A and B in X .

Proof. For the sake of simplicity, IFSs A and B are denoted by $A = \{(\mu_A(x_i), \nu_A(x_i))\}$ and $B = \{(\mu_B(x_i), \nu_B(x_i))\}$, respectively.

(SP1) For each $x, y \in [0, +\infty]$, we have $0 \leq \sqrt{xy} \leq (x+y)/2$.

Given $0 \leq \mu_A(x_i) \leq 1, 0 \leq \nu_A(x_i) \leq 1, 0 \leq \pi_A(x_i) \leq 1, 0 \leq \mu_B(x_i) \leq 1, 0 \leq \nu_B(x_i) \leq 1, 0 \leq \pi_B(x_i) \leq 1, 0 \leq 1 - \nu_A(x_i) \leq 1$, and $0 \leq 1 - \nu_B(x_i) \leq 1$, we get

$$\begin{aligned} 0 &\leq \sqrt{\mu_A(x_i) \mu_B(x_i)} \leq \frac{\mu_A(x_i) + \mu_B(x_i)}{2}, \\ 0 &\leq \sqrt{\nu_A(x_i) \nu_B(x_i)} \leq \frac{\nu_A(x_i) + \nu_B(x_i)}{2}, \end{aligned}$$

$$0 \leq \sqrt{\pi_A(x_i)\pi_B(x_i)} \leq \frac{\pi_A(x_i) + \pi_B(x_i)}{2},$$

$$0 \leq \sqrt{(1 - v_A(x_i))(1 - v_B(x_i))} \leq \frac{1 - v_A(x_i) + 1 - v_B(x_i)}{2}. \tag{18}$$

Then we have

$$0 \leq \sqrt{\mu_A(x_i)\mu_B(x_i)} + 2\sqrt{v_A(x_i)v_B(x_i)}$$

$$+ \sqrt{\pi_A(x_i)\pi_B(x_i)} + \sqrt{(1 - v_A(x_i))(1 - v_B(x_i))}$$

$$\leq \frac{\mu_A(x_i) + \mu_B(x_i)}{2} + 2 \cdot \frac{v_A(x_i) + v_B(x_i)}{2}$$

$$+ \frac{\pi_A(x_i) + \pi_B(x_i)}{2} + \frac{1 - v_A(x_i) + 1 - v_B(x_i)}{2} \tag{19}$$

$$= 1 + \frac{\mu_A(x_i) + v_A(x_i) + \pi_A(x_i)}{2}$$

$$+ \frac{\mu_B(x_i) + v_B(x_i) + \pi_B(x_i)}{2}$$

$$= 2.$$

Hence,

$$0 \leq \sum_{i=1}^n \left(\sqrt{\mu_A(x_i)\mu_B(x_i)} + 2\sqrt{v_A(x_i)v_B(x_i)} \right)$$

$$+ \sqrt{\pi_A(x_i)\pi_B(x_i)} \tag{20}$$

$$+ \sqrt{(1 - v_A(x_i))(1 - v_B(x_i))} \leq 2n.$$

Finally, we get the following inequality: $0 \leq S_Y(A, B) \leq (1/2n) \cdot 2n = 1$.

Thus, $0 \leq S_Y(A, B) \leq 1$.

(SP2) We know that \sqrt{xy} assumes its maximum value $(x + y)/2$ when $x = y$. Therefore, we have

$$S_Y(A, B) = 1 \iff \sqrt{\mu_A(x_i)\mu_B(x_i)} + 2\sqrt{v_A(x_i)v_B(x_i)}$$

$$+ \sqrt{\pi_A(x_i)\pi_B(x_i)}$$

$$+ \sqrt{(1 - v_A(x_i))(1 - v_B(x_i))} = 2$$

$$\iff \mu_A(x_i)\mu_B(x_i), v_A(x_i)v_B(x_i),$$

$$\pi_A(x_i)\pi_B(x_i), 1 - v_A(x_i) = 1 - v_B(x_i)$$

$$\iff A = B. \tag{21}$$

Thus, $S_Y(A, B) = 1$, if and only if $A = B$.

(SP3) It is easy to note that the expression of $S_Y(A, B)$ is commutative. So we have

$$S_Y(A, B) = S_Y(B, A). \tag{22}$$

(SP4) Let C be another IFS in X , denoted by $C = \{\mu_C(x_i), v_C(x_i)\}$. A, B , and C satisfy the relation $A \subseteq B \subseteq C$. Then, we have $0 \leq \mu_A(x_i) \leq \mu_B(x_i) \leq \mu_C(x_i) \leq 1$ and $0 \leq v_C(x_i) \leq v_B(x_i) \leq v_A(x_i) \leq 1$, for all $x \in X$. Based on (17), the similarity measures between (B, C) and (A, C) can be written as

$$S_Y(B, C) = \frac{1}{2n} \sum_{i=1}^n \left(\sqrt{\mu_B(x_i)\mu_C(x_i)} + \sqrt{v_B(x_i)v_C(x_i)} \right)$$

$$+ \sqrt{\pi_B(x_i)\pi_C(x_i)}$$

$$+ \sqrt{(1 - v_B(x_i))(1 - v_C(x_i))},$$

$$S_Y(A, C) = \frac{1}{2n} \sum_{i=1}^n \left(\sqrt{\mu_A(x_i)\mu_C(x_i)} + \sqrt{v_A(x_i)v_C(x_i)} \right)$$

$$+ \sqrt{\pi_A(x_i)\pi_C(x_i)}$$

$$+ \sqrt{(1 - v_A(x_i))(1 - v_C(x_i))}. \tag{23}$$

For $a, b \in [0, 1]$, $a + b \leq 1$ we can define a function f as

$$f(x, y) = \sqrt{ax} + 2\sqrt{by} + \sqrt{(1 - a - b)(1 - x - y)}$$

$$+ \sqrt{(1 - b)(1 - y)}, \tag{24}$$

where $x, y \in [0, 1]$, $x + y \in [0, 1]$.

Then we have

$$\frac{\partial f}{\partial x} = \frac{\sqrt{a}}{2\sqrt{x}} - \frac{\sqrt{1 - a - b}}{2\sqrt{1 - x - y}}, \tag{25}$$

$$\frac{\partial f}{\partial y} = \frac{\sqrt{b}}{\sqrt{y}} - \frac{\sqrt{1 - a - b}}{2\sqrt{1 - x - y}} - \frac{\sqrt{1 - b}}{2\sqrt{1 - y}}.$$

Given $a \leq x \leq 1, b \leq 1$, we have

$$\left. \frac{\partial f}{\partial x} \right|_{y=b}$$

$$= \frac{\sqrt{a}}{2\sqrt{x}} - \frac{\sqrt{1 - a - b}}{2\sqrt{1 - x - y}}$$

$$= \frac{(a - x)(1 - b)}{2\sqrt{x(1 - x - y)} \left(\sqrt{a(1 - x - y)} + \sqrt{(1 - a - b)x} \right)}$$

$$\leq 0, \tag{26}$$

which means that f is a decreasing function of x , when $y = b, x \geq a$.

For $0 \leq x \leq a, b \leq 1$, we can get $(\partial f / \partial x)|_{y=b} \geq 0$, which means that f is an increasing function of x , when $y = b, x \leq a$.

Similarly, as for $\partial f/\partial y$, we can get

$$\begin{aligned} & \left. \frac{\partial f}{\partial y} \right|_{x=a} \\ &= \frac{(b-y)(1-a)}{2\sqrt{y(1-x-y)}\left(\sqrt{b(1-x-y)} + \sqrt{(1-a-b)y}\right)} \\ &+ \frac{b-y}{2\sqrt{y(1-y)}\left(\sqrt{b(1-y)} + \sqrt{(1-b)y}\right)}, \end{aligned} \tag{27}$$

which reveals that f is an increasing function of y for $x = a$, $y \leq b$, but a decreasing function when $x = a$, $y \geq b$.

Given $a = \mu_A(x_i)$, $b = \nu_A(x_i)$, and two couples $(\mu_B(x_i), \nu_B(x_i))$, $(\mu_C(x_i), \nu_C(x_i))$, satisfying $a = \mu_A(x_i) \leq \mu_B(x_i) \leq \mu_C(x_i)$, $\nu_C(x_i) \leq \nu_B(x_i) \leq \nu_A(x_i) = b$, we can get

$$\begin{aligned} f(\mu_C(x_i), b) &\leq f(\mu_B(x_i), b) \leq f(a, b), \\ f(a, \nu_C(x_i)) &\leq f(a, \nu_B(x_i)) \leq f(a, b). \end{aligned} \tag{28}$$

Since $f(x, y)$ is a continuous concave function, it is true that $f(\mu_C(x_i), \nu_C(x_i)) \leq f(\mu_B(x_i), \nu_B(x_i))$; that is,

$$\begin{aligned} & \sqrt{\mu_A(x_i)\mu_C(x_i)} + \sqrt{\nu_A(x_i)\nu_C(x_i)} + \sqrt{\pi_A(x_i)\pi_C(x_i)} \\ &+ \sqrt{(1-\nu_A(x_i))(1-\nu_C(x_i))} \\ &\leq \sqrt{\mu_A(x_i)\mu_B(x_i)} + 2\sqrt{\nu_A(x_i)\nu_B(x_i)} \\ &+ \sqrt{\pi_A(x_i)\pi_B(x_i)} + \sqrt{(1-\nu_A(x_i))(1-\nu_B(x_i))}, \end{aligned} \tag{29}$$

which indicates that $S_Y(A, B) \geq S_Y(A, C)$.

Similarly, if we suppose $a = \mu_C(x_i)$, $b = \nu_C(x_i)$, considering another two couples $(\mu_A(x_i), \nu_A(x_i))$, $(\mu_B(x_i), \nu_B(x_i))$, where $\mu_A(x_i) \leq \mu_B(x_i) \leq \mu_C(x_i) = a$, $b = \nu_C(x_i) \leq \nu_B(x_i) \leq \nu_A(x_i)$, we have

$$\begin{aligned} f(\mu_A(x_i), b) &\leq f(\mu_B(x_i), b) \leq f(a, b), \\ f(a, \nu_A(x_i)) &\leq f(a, \nu_B(x_i)) \leq f(a, b). \end{aligned} \tag{30}$$

Considering the continuity and concavity of $f(x, y)$, we can finally get $f(\mu_A(x_i), \nu_A(x_i)) \leq f(\mu_B(x_i), \nu_B(x_i))$, also written as

$$\begin{aligned} & \sqrt{\mu_A(x_i)\mu_C(x_i)} + \sqrt{\nu_A(x_i)\nu_C(x_i)} + \sqrt{\pi_A(x_i)\pi_C(x_i)} \\ &+ \sqrt{(1-\nu_A(x_i))(1-\nu_C(x_i))} \\ &\leq \sqrt{\mu_B(x_i)\mu_C(x_i)} + 2\sqrt{\nu_B(x_i)\nu_C(x_i)} \\ &+ \sqrt{\pi_B(x_i)\pi_C(x_i)} + \sqrt{(1-\nu_B(x_i))(1-\nu_C(x_i))}. \end{aligned} \tag{31}$$

So we have $S_Y(B, C) \geq S_Y(A, C)$.

Thus, $S_Y(A, B)$ satisfies (SP4).

So the similarity measure $S_Y(A, B)$ satisfies all properties in Definition 5. It is a similarity measure between IFSSs. \square

Considering the weights of x_i , we can define the weighted similarity between two IFSSs as follows:

$$\begin{aligned} S_{WY}(A, B) &= \frac{1}{2} \sum_{i=1}^n w_i \left(\sqrt{\mu_A(x_i)\mu_B(x_i)} \right. \\ &+ 2\sqrt{\nu_A(x_i)\nu_B(x_i)} + \sqrt{\pi_A(x_i)\pi_B(x_i)} \\ &+ \left. \sqrt{(1-\nu_A(x_i))(1-\nu_B(x_i))} \right), \end{aligned} \tag{32}$$

where w_i is the weights factor of the features x_i , $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$.

Theorem 7. $S_{WY}(A, B)$ is the similarity measure between two IFSSs A and B in X .

Proof. (SP1) From the proof of (SP1) in the last theorem, we can get

$$\begin{aligned} 0 &\leq \sum_{i=1}^n w_i \left(\sqrt{\mu_A(x_i)\mu_B(x_i)} + 2\sqrt{\nu_A(x_i)\nu_B(x_i)} \right. \\ &+ \sqrt{\pi_A(x_i)\pi_B(x_i)} \\ &+ \left. \sqrt{(1-\nu_A(x_i))(1-\nu_B(x_i))} \right) \end{aligned} \tag{33}$$

$$\leq \sum_{i=1}^n 2w_i = 2 \cdot \sum_{i=1}^n w_i = 2.$$

Therefore, $0 \leq S_{WY}(A, B) \leq 1$.

(SP2) Considering the implication rule in the proof of (SP2) for Theorem 6, we have

$$\begin{aligned} S_{WY}(A, B) = 1 &\iff w_i \left(\sqrt{\mu_A(x_i)\mu_B(x_i)} \right. \\ &+ 2\sqrt{\nu_A(x_i)\nu_B(x_i)} \\ &+ \sqrt{\pi_A(x_i)\pi_B(x_i)} \\ &+ \left. \sqrt{(1-\nu_A(x_i))(1-\nu_B(x_i))} \right) \\ &= 2w_i \end{aligned}$$

$$\begin{aligned}
 &\iff \sqrt{\mu_A(x_i)\mu_B(x_i)} + 2\sqrt{v_A(x_i)v_B(x_i)} \\
 &\quad + \sqrt{\pi_A(x_i)\pi_B(x_i)} \\
 &\quad + \sqrt{(1-v_A(x_i))(1-v_B(x_i))} = 2 \\
 &\iff \mu_A(x_i)\mu_B(x_i), v_A(x_i)v_B(x_i), \\
 &\quad \pi_A(x_i)\pi_B(x_i), 1-v_A(x_i) = 1-v_B(x_i) \\
 &\iff A = B.
 \end{aligned} \tag{34}$$

So we get $S_{WY}(A, B) = 1 \iff A = B$.
 (SP3) It is straightforward that $S_{WY}(A, B)$ satisfies (SP3).
 (SP4) Since all $w_i \geq 0$, we can get

$$\begin{aligned}
 &w_i \left(\sqrt{\mu_A(x_i)\mu_C(x_i)} + \sqrt{v_A(x_i)v_C(x_i)} \right. \\
 &\quad \left. + \sqrt{\pi_A(x_i)\pi_C(x_i)} + \sqrt{(1-v_A(x_i))(1-v_C(x_i))} \right) \\
 &\leq w_i \left(\sqrt{\mu_A(x_i)\mu_B(x_i)} + 2\sqrt{v_A(x_i)v_B(x_i)} \right. \\
 &\quad \left. + \sqrt{\pi_A(x_i)\pi_B(x_i)} + \sqrt{(1-v_A(x_i))(1-v_B(x_i))} \right), \\
 &w_i \left(\sqrt{\mu_A(x_i)\mu_C(x_i)} + \sqrt{v_A(x_i)v_C(x_i)} \right. \\
 &\quad \left. + \sqrt{\pi_A(x_i)\pi_C(x_i)} + \sqrt{(1-v_A(x_i))(1-v_C(x_i))} \right) \\
 &\leq w_i \left(\sqrt{\mu_B(x_i)\mu_C(x_i)} + 2\sqrt{v_B(x_i)v_C(x_i)} \right. \\
 &\quad \left. + \sqrt{\pi_B(x_i)\pi_C(x_i)} + \sqrt{(1-v_B(x_i))(1-v_C(x_i))} \right).
 \end{aligned} \tag{35}$$

Finally we have $S_{WY}(A, B) \geq S_{WY}(A, C), S_{WY}(B, C) \geq S_{WY}(A, C)$.

So far, we have finished the proof that $S_{WY}(A, B)$ is a similarity measure between IFSs A and B . \square

5. Numerical Comparisons

In order to illustrate the superiority of the proposed similarity measure, a comparison between the proposed similarity measure and all the existing similarity measures is conducted based on the numerical cases in [23]. Table 1 presents a comprehensive comparison of the similarity measures for IFS with counterintuitive examples ($p = 1$ for $S_{HB}, S_e^p, S_s^p, S_h^p$ and $p = 1, t = 2$ for S_t^p).

We can see that $S_C(A, B) = S_{DC}(A, B) = C_{IFS}(A, B) = 1$ for two different IFSs $A = \langle 0.3, 0.3 \rangle$ and $B = \langle 0.4, 0.4 \rangle$. This indicates that the second axiom of similarity measure (SP2) is not satisfied by $S_C(A, B), S_{DC}(A, B)$, and $C_{IFS}(A, B)$. This also can be illustrated by $S_C(A, B) = S_{DC}(A, B) = 1$ when $A = \langle 0.5, 0.5 \rangle, B = \langle 0, 0 \rangle$ and $A = \langle 0.4, 0.2 \rangle, B = \langle 0.5, 0.3 \rangle$.

As for $S_H, S_O, S_{HB}, S_e^p, S_s^p$, and S_h^p , different pairs of A, B may provide the identical results, which cannot satisfy the application of pattern recognition. It can be read from Table 1 that $S_{HB} = 0.9$ for both $A = \langle 0.3, 0.3 \rangle, B = \langle 0.4, 0.4 \rangle$ and $A = \langle 0.3, 0.4 \rangle, B = \langle 0.4, 0.3 \rangle$. Such situation seems to be going from bad to worse for S_{HY}^1 , where all the cases take the same similarity degree except Case 3 and Case 4. S_{HY}^2 and S_{HY}^3 are also stuck with this problem. S_t^p seems to be reasonable without any counterintuitive results. However, we can notice an interesting situation when comparing Case 3 and Case 4. Suppose two vote results, “all in favor” and “all abstentions,” which, respectively, represent $A = \langle 1, 0 \rangle$ and $B = \langle 0, 0 \rangle$. It is reasonable to take the similarity degree between them as 0.5. $A' = \langle 0.5, 0.5 \rangle$ and $B' = \langle 0, 0 \rangle$ can be interpreted as “the vote for resolution is half in favor, half against” and “the vote for resolution is all abstentions,” respectively. The similarity degree between them is 0.833, which is greater than 0.5. From the view of vote practice, the similarity degree between A and B should be equal to or not less than the similarity between A' and B' . Furthermore, it is worth considering the determination of parameters p and t in S_t^p . In such a sense, the proposed similarity measure is the most reasonable one with a relative simple expression and has none of the counterintuitive cases.

In order to study the effectiveness of the proposed similarity measure for IFS in the application of pattern recognition, we consider the pattern recognition problem discussed in [7, 13].

Suppose there are m patterns, which can be represented by IFSs $A_j = \{ \langle x_i, \mu_{A_j}(x_i), v_{A_j}(x_i) \rangle \mid x_i \in X \}, A_j \in IFSs(X), j = 1, 2, \dots, m$. Let the sample to be recognized be denoted by $B = \{ \langle x_i, \mu_B(x_i), v_B(x_i) \rangle \mid x_i \in X \}$. According to the recognition principle of maximum degree of similarity between IFSs, the process of assigning B to A_k is described by [7]:

$$k = \arg \max_{j=1,2,\dots,m} \{ S(A_j, B) \}. \tag{36}$$

Example 8. Assume that there exist three known patterns A_1, A_2 , and A_3 , with class labels C_1, C_2 , and C_3 , respectively. Each pattern can be expressed by IFS in $X = \{x_1, x_2, x_3\}$ as follows:

$$\begin{aligned}
 A_1 &= \{ \langle x_1, 1, 0 \rangle, \langle x_2, 0.8, 0 \rangle, \langle x_3, 0.7, 0.1 \rangle \}, \\
 A_2 &= \{ \langle x_1, 0.8, 0.1 \rangle, \langle x_2, 1, 0 \rangle, \langle x_3, 0.9, 0 \rangle \}, \\
 A_3 &= \{ \langle x_1, 0.6, 0.2 \rangle, \langle x_2, 0.8, 0 \rangle, \langle x_3, 1, 0 \rangle \}.
 \end{aligned} \tag{37}$$

The sample B which needs to be recognized is as follows:

$$B = \{ \langle x_1, 0.5, 0.3 \rangle, \langle x_2, 0.6, 0.2 \rangle, \langle x_3, 0.8, 0.1 \rangle \}. \tag{38}$$

The similarity degree between $A_i (i = 1, 2, 3)$ and B calculated by (17) is

$$\begin{aligned}
 S_Y(A_1, B) &= 0.887, \\
 S_Y(A_2, B) &= 0.913, \\
 S_Y(A_3, B) &= 0.936.
 \end{aligned} \tag{39}$$

TABLE 1: The comparison of similarity measures (counterintuitive cases are in bold type).

	1	2	3	4	5	6
A	$\langle 0.3, 0.3 \rangle$	$\langle 0.3, 0.4 \rangle$	$\langle 1, 0 \rangle$	$\langle 0.5, 0.5 \rangle$	$\langle 0.4, 0.2 \rangle$	$\langle 0.4, 0.2 \rangle$
B	$\langle 0.4, 0.4 \rangle$	$\langle 0.4, 0.3 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	$\langle 0.5, 0.3 \rangle$	$\langle 0.5, 0.2 \rangle$
S_C	1	0.9	0.5	1	1	0.95
S_H	0.9	0.9	0.5	0.5	0.9	0.95
S_L	0.95	0.9	0.5	0.75	0.95	0.95
S_O	0.9	0.9	0.3	0.5	0.9	0.93
S_{DC}	1	0.9	0.5	1	1	0.95
S_{HB}	0.9	0.9	0.5	0.5	0.9	0.95
S_e^p	0.9	0.9	0.5	0.5	0.9	0.95
S_s^p	0.95	0.9	0.5	0.75	0.95	0.95
S_h^p	0.933	0.933	0.5	0.67	0.933	0.95
S_{HY}^1	0.9	0.9	0	0.5	0.9	0.9
S_{HY}^2	0.85	0.85	0	0.38	0.85	0.85
S_{HY}^3	0.82	0.82	0	0.33	0.82	0.82
C_{IFS}	1	0.96	0	0	0.9971	0.9965
S_t^p	0.967	0.9	0.5	0.833	0.937	0.95
S_Y	0.985	0.994	0.5	0.354	0.936	0.896

It can be observed that pattern B should be classified to A_3 with a class label C_3 . According to the recognition principle of maximum degree of similarity between IFSs, this result is in agreement with the one obtained in [7, 13].

Let us assume that the weights of x_1 , x_2 , and x_3 are 0.5, 0.3, and 0.2, respectively, as they were assumed in [7]. Considering (32), we can get

$$\begin{aligned} S_{WY}(A_1, B) &= 0.853, \\ S_{WY}(A_2, B) &= 0.919, \\ S_{WY}(A_3, B) &= 0.949. \end{aligned} \quad (40)$$

According to (36), B can be recognized as A_3 , which is identical to the result obtained in [7, 13].

To make our similarity measure more transparent and comparable with the measures proposed earlier by other authors, the example analyzed in [17] will be discussed next.

Example 9. Assume that there are three IFS patterns in $X = \{x_1, x_2, x_3\}$. The three patterns are denoted as follows:

$$\begin{aligned} A_1 &= \{\langle x_1, 0.3, 0.3 \rangle, \langle x_2, 0.2, 0.2 \rangle, \langle x_3, 0.1, 0.1 \rangle\}, \\ A_2 &= \{\langle x_1, 0.2, 0.2 \rangle, \langle x_2, 0.2, 0.2 \rangle, \langle x_3, 0.2, 0.2 \rangle\}, \\ A_3 &= \{\langle x_1, 0.4, 0.4 \rangle, \langle x_2, 0.4, 0.4 \rangle, \langle x_3, 0.4, 0.4 \rangle\}. \end{aligned} \quad (41)$$

Assume that a sample $B = \{\langle x_1, 0.3, 0.3 \rangle, \langle x_2, 0.2, 0.2 \rangle, \langle x_3, 0.1, 0.1 \rangle\}$ is to be classified.

The similarity degrees of $S(A_1, B)$, $S(A_2, B)$, and $S(A_3, B)$ are calculated by (5)–(16) for all existing similarity measures and shown in Table 2, where $p = 1$ for S_{HB} , S_e^p , S_s^p , S_h^p and $p = 1, t = 2$ for S_t^p .

The proposed similarity measure S_Y can be calculated by (17) as follows:

$$\begin{aligned} S_Y(A_1, B) &= 1, \\ S_Y(A_2, B) &= 0.990, \\ S_Y(A_3, B) &= 0.932. \end{aligned} \quad (42)$$

It is obvious that B is equal to A_1 . This indicates that sample B should be classified to A_1 . However, the similarity degrees of $S(A_1, B)$, $S(A_2, B)$, and $S(A_3, B)$ are equal to each other when S_C , S_H , S_{DC} , and C_{IFS} are employed. These four similarity measures are not capable of discriminating difference between the three patterns. Fortunately, the results of $S_Y(A_i, B)$ ($i = 1, 2, 3$) can be used to make correct classification conclusion. This means that the proposed similarity measure shows an identical performance with majority of the existing measures.

6. Applications in Pattern Recognition

Along with the previous investigation of classification capabilities of the proposed measure, an additional experiment discussed in [7, 22, 23, 33–36], will be presented as an application in pattern recognition. In this paper, we propose an alternative approach to medical diagnosis using the newly defined similarity measure.

Let us consider the same data as in [7, 22, 23, 33–36]. Suppose that there are four patients Al, Bob, Joe, Ted, represented as $P = \{\text{Al, Bob, Joe, Ted}\}$. Their symptoms are $S = \{\text{Temperature, Headache, Stomach pain, Cough, Chest pain}\}$. The set of diagnoses is defined as $D = \{\text{Viral fever, Malaria, Typhoid, Stomach problem, Chest problem}\}$. The intuitionistic fuzzy relation $P \rightarrow S$ is presented in Table 3. Table 4 gives the intuitionistic fuzzy relation $S \rightarrow D$. Each element of the tables is given in the form of IFV, which is

TABLE 2: The similarity measures between the known patterns and the unknown patterns in Example 9 (patterns not discriminated are in bold type).

	$S(A_1, B)$	$S(A_2, B)$	$S(A_3, B)$		$S(A_1, B)$	$S(A_2, B)$	$S(A_3, B)$
S_C	1	1	1	S_s^p	1	0.967	0.900
S_H	1	1	1	S_h^p	1	0.956	0.867
S_L	1	0.967	0.9	S_{HY}^1	1	0.967	0.8
S_O	1	0.918	0.784	S_{HY}^2	1	0.898	0.713
S_{DC}	1	1	1	S_{HY}^3	1	0.875	0.667
S_{HB}	1	0.933	0.8	C_{IFS}	1	1	1
S_e^p	1	0.933	0.8	S_t^p	1	0.978	0.933

TABLE 3: Symptoms characteristic for the patients.

	Temperature	Headache	Stomach pain	Cough	Chest pain
Al	(0.8, 0.1)	(0.6, 0.1)	(0.2, 0.8)	(0.6, 0.1)	(0.1, 0.6)
Bob	(0, 0.8)	(0.4, 0.4)	(0.6, 0.1)	(0.1, 0.7)	(0.1, 0.8)
Joe	(0.8, 0.1)	(0.8, 0.1)	(0.0, 0.6)	(0.2, 0.7)	(0.0, 0.5)
Ted	(0.6, 0.1)	(0.5, 0.4)	(0.3, 0.4)	(0.7, 0.2)	(0.3, 0.4)

TABLE 4: Symptoms characteristic for the diagnoses.

	Viral fever	Malaria	Typhoid	Stomach problem	Chest pain problem
Temperature	(0.4, 0.0)	(0.7, 0.0)	(0.3, 0.3)	(0.1, 0.7)	(0.1, 0.8)
Headache	(0.3, 0.5)	(0.2, 0.6)	(0.6, 0.1)	(0.2, 0.4)	(0, 0.8)
Stomach pain	(0.1, 0.7)	(0.0, 0.9)	(0.2, 0.7)	(0.8, 0.0)	(0.2, 0.8)
Cough	(0.4, 0.3)	(0.7, 0.0)	(0.2, 0.6)	(0.2, 0.7)	(0.2, 0.8)
Chest pain	(0.1, 0.7)	(0.1, 0.8)	(0.1, 0.9)	(0.2, 0.7)	(0.8, 0.1)

TABLE 5: The proposed similarity measure S_Y between each patient's symptoms and the considered set of possible diagnoses.

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Al	0.9347	0.9228	0.9223	0.7673	0.7490
Bob	0.8124	0.6775	0.8997	0.9760	0.8211
Joe	0.9152	0.8271	0.9188	0.7917	0.7456
Ted	0.9576	0.9034	0.9060	0.8577	0.8122

a pair of numbers corresponding to the membership and nonmembership values, respectively.

In order to make a proper diagnosis for each patient, we calculate the similarity degree between each patient and each diagnose. According to the principle of maximum similarity degree, the higher similarity degree indicates a proper diagnosis. In Table 5, the similarity degree S_Y between patients and diagnoses is presented. According to the similarity degrees in Table 5, conclusion can be made that Al suffers from Viral fever, Bob suffers from Stomach problem, Joe suffers from Typhoid, and Ted suffers from Viral fever. The diagnosis results for this case obtained in previous study have been presented in [23]. It is clear that our proposed method provides the same results obtained by Vlachos and Sergiadis in [33], Own in [34], and Boran and Akay in [23]. Moreover, our proposed similarity measure is calculated based on the IFNs, without any other parameters such as p, t in [23]. So it can reduce the computation complexity.

7. Conclusion

Since most of the existing similarity measures for IFs have provided counterintuitive results, a new similarity measure and weighted similarity measure between IFs were proposed in this paper. The new similarity measure is calculated based on the membership degree $\mu_A(x)$, nonmembership degree $\nu_A(x)$, hesitancy degree $\pi_A(x)$, and the upper bound of membership $1 - \nu_A(x)$. In some special cases where some of the existing similarity measures cannot provide reasonable results, the proposed similarity measure shows great capacity for discriminating IFs. Moreover, investigation of the new measure's classification capability was carried out based on two numerical examples and medical diagnosis. It has been illustrated that the proposed similarity measure performs as well as or better than previous measures. Further research will be focused on its applications in other practical fields.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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