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Research Article

On the Strong Convergence of a Sufficient Descent Polak-Ribière-Polyak Conjugate Gradient Method

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Recently, Zhang et al. proposed a sufficient descent Polak-Ribière-Polyak (SDPRP) conjugate gradient method for large-scale unconstrained optimization problems and proved its global convergence in the sense that $\liminf_{k\to\infty}\|\nabla f(x_k)\| = 0$ when an Armijo-type line search is used. In this paper, motivated by the line searches proposed by Shi et al. and Zhang et al., we propose two new Armijo-type line searches and show that the SDPRP method has strong convergence in the sense that $\lim_{k\to\infty}\|\nabla f(x_k)\| = 0$ under the two new line searches. Numerical results are reported to show the efficiency of the SDPRP with the new Armijo-type line searches in practical computation.

1. Introduction

In this paper, we are concerned with the following unconstrained minimization problem:

$$\min f(x), \quad x \in R^n, \tag{1}$$

where $f : R^n \to R^1$ is a smooth function whose gradient $\nabla f(x)$ is often denoted by $g(x)$. The related problem is called large-scale minimization problem when its dimension n is very large (e.g., $n > 10^6$). For solving large-scale minimization problems, the matrices-free methods are quite efficient. Among such methods, the conjugate gradient method is very famous for its excellent numerical performance in the practical computation. Much progress has been achieved in the study of global convergence of the various conjugate gradient methods, such as the Polak-Ribière-Polyak (PRP) [1, 2], the Fletcher-Reeves (FR) [3], the Hestenes-Stiefel (HS) [4, 5], and the Dai-Yuan (DY) [6] conjugate gradient methods, et al.

Recently, Zhang et al. [7] presented a sufficient descent Polak-Ribière-Polyak (SDPRP) conjugate gradient method for solving large-scale problem (1), whose most important property is that its generated direction is always a sufficient descent direction for the objective function. Moreover, this property is independent of the line search used, and it reduces to the classical PRP method when the exact line search is used. The iterative process of the SDPRP method is given by

$$x_{k+1} = x_k + \alpha_k d_k, \quad k = 0, 1, \ldots, \tag{2}$$

where x_k is the current iterate, $\alpha_k > 0$ is called the stepsize which can be obtained by some line search techniques, such as the Armijo line search, the Goldstein line search, and the (strong) Wolfe line search, and d_k is the search direction determined by

$$d_k = \begin{cases} -g_k, & \text{if } k = 0, \\ -g_k + \beta_k^{\mathrm{PRP}} d_{k-1} - \theta_k y_{k-1}, & \text{if } k \geq 1, \end{cases} \tag{3}$$

with

$$\beta_k^{\mathrm{PRP}} = \frac{g_k^\top y_{k-1}}{\|g_{k-1}\|^2}, \qquad \theta_k = \frac{g_k^\top d_{k-1}}{\|g_{k-1}\|^2}, \tag{4}$$

where $y_{k-1} = g_k - g_{k-1}$. It is easy to deduce from (3) and (4) that

$$g_k^\top d_k = -\|g_k\|^2, \tag{5}$$

which indicates that d_k is a sufficient descent direction of $f(x)$ at the current iterate x_k if $\|g_k\| \neq 0$; that is, x_k is not a stationary point of the objective function $f(x)$. It has been proved that SDPRP method has global convergence under an Armijo-type line search [7] in the sense that

$$\liminf_{k \to \infty} \|g(x_k)\| = 0, \quad (6)$$

which means that at least one cluster point of the sequence $\{x_k\}$ is a stationary point if it is bounded.

In another recent paper, Shi and Shen [8] showed that the classical PRP method in [1] has strong convergence and linear convergence rate under a customized Armijo-type line search, which is somewhat complicated. The new Armijo-type line search ensures that the search direction generated by the classical PRP method possesses the sufficient descent property, which is helpful to prove the global convergence.

In this paper, motivated by the Armijo-type line search in [8], we first propose a similar but simple line search, which can ensure that the SDPRP method has strongly global convergence in the sense that

$$\lim_{k \to \infty} \|g(x_k)\| = 0, \quad (7)$$

that is, any cluster point of the sequence $\{x_k\}$ is a stationary point of the objective function $f(x)$. Noting that the above new line search needs to estimate the Lipschitz constant, which is not easy even for linear function, we present another Armijo-type line search, which is motivated by the line search in [7]. This new line search can also guarantee the global convergence of the SDPRP method in the above sense.

The remainder of the paper is organized as follows. In Section 2 we introduce the two new Armijo-type line searches and present the strongly convergent SDPRP method. The global convergence is established under the above two new Armijo-type line searches in Section 3. Some numerical results are presented in Section 4, and in the last section, we conclude the paper with some remarks.

2. Strongly Convergent SDPRP Method

First, we give the following basic assumptions on the objection function $f(x)$.

Assumption 1. Consider the following.

(H1) The objective function $f(x)$ has a lower bound on the level set $L_0 = \{x \in R^n \mid f(x) \leq f(x_0)\}$, where x_0 is the starting point.

(H2) In some neighborhood N of L_0, the gradient $g(x)$ is Lipschitz continuous on an open convex set B that contains L_0, that is, there exists a constant $L > 0$ such that

$$\|g(x) - g(y)\| \leq L\|x - y\|, \quad \text{for any } x, y \in B. \quad (8)$$

(H3) The level set $L_0 = \{x \in R^n \mid f(x) \leq f(x_0)\}$ is bounded.

Although $g(x)$ is Lipschitz continuous, the Lipschitz constant L is usually unknown in practice, even for the linear function $g(x)$. Therefore, we need to estimate the Lipschitz constant L. Here, we adopt one of three estimating approaches proposed in [9]. More specifically, if $k \geq 1$, then we can set

$$L \cong L_k = \max\left\{L_{k-1}, \frac{\|y_{k-1}\|}{\|s_{k-1}\|}\right\}, \quad (9)$$

with $L_0 > 0$, and $s_{k-1} = x_k - x_{k-1}$.

Armijo-Type Line Search I. Set $\mu \in (0,1)$, $\rho \in (0,1)$, $c \in (0,1)$, and the initial stepsize $\delta_k = (1-c)\|g_k\|^2/(L_k\|d_k\|^2)$, where L_k is determined by (9). Let α_k be the largest α in $\{\delta_k, \rho\delta_k, \rho^2\delta_k, \ldots\}$ such that

$$f(x_k + \alpha d_k) - f(x_k) \leq -\mu\alpha\|g_k\|^2. \quad (10)$$

Armijo-Type Line Search II. Set $\mu > 0$, $\rho \in (0,1)$. Let α_k be the largest α in $\{1, \rho, \rho^2, \ldots\}$ such that

$$f(x_k + \alpha d_k) \leq f(x_k) - \mu\alpha^2\|d_k\|^4. \quad (11)$$

Now we begin to describe the strongly convergent SDPRP method.

Algorithm 2 (strongly convergent SDPRP method).

Step 0. Given an initial point $x_0 \in R^n$, $\mu \in (0,1/2)$ and $\rho \in (0,1)$, $c \in (0,1)$ and set $d_0 = -g_0$, $k := 0$.

Step 1. If $\|g_k\| = 0$ then stop; otherwise go to Step 2.

Step 2. Compute the descent direction d_k by (3) and (4). Determine the stepsize α_k by the Armijo-type line search (10) or (11).

Step 3. Set $x_{k+1} = x_k + \alpha_k d_k$, and $k := k+1$; go to Step 1.

Lemma 3. *Assume that (H1) and (H2) hold, then there exist $m_0 > 0$ and $M_0 > 0$ such that for any $k \geq 0$, one has*

$$m_0 \leq L_k \leq M_0, \quad (12)$$

where L_k is defined by (9).

Proof. See [9, Lemma 2.1]. □

Lemma 4. *Assume that (H1) and (H2) hold. If $\|g_k\| > 0$, then the new Armijo-type line search I is well-defined for the index k.*

Proof. The proof is easy; for completeness, we give the proof here. In fact, we can prove this lemma by contradiction. Suppose that the conclusion does not hold; then for k, the inequality (10) does not hold for any nonnegative integer m; that is,

$$f(x_k) - f(x_k + \delta_k\rho^m d_k) < \mu\delta_k\rho^m\|g_k\|^2, \quad \forall m. \quad (13)$$

Thus,

$$\frac{f(x_k) - f(x_k + \delta_k \rho^m d_k)}{\delta_k \rho^m} < \mu \|g_k\|^2, \quad \forall m. \tag{14}$$

Letting $m \to +\infty$, by the continuity of $f(x)$ and $-g_k^\top d_k = \|g_k\|^2$, we can obtain

$$\|g_k\|^2 \le \mu \|g_k\|^2. \tag{15}$$

This and $\mu \in (0,1)$ yield that

$$\|g_k\| = 0, \tag{16}$$

which contradicts to $\|g_k\| > 0$. The proof is completed. □

Lemma 5. *Assume that (H2) and (H3) hold. If $\|g_k\| > 0$, then the new Armijo-type line search II is well-defined for the index k.*

Proof. The lemma is also proved by contradiction. Suppose that the conclusion does not hold; then for k, the inequality (11) does not hold for any nonnegative integer m; that is,

$$f(x_k + \rho^m d_k) > f(x_k) - \mu \rho^{2m} \|d_k\|^4, \quad \forall m. \tag{17}$$

That is,

$$\frac{f(x_k + \rho^m d_k) - f(x_k)}{\rho^m} > -\mu \rho^m \|d_k\|^4, \quad \forall m. \tag{18}$$

Letting $m \to +\infty$, by the continuity of $f(x)$ and $-g_k^\top d_k = \|g_k\|^2$, we can obtain

$$-\|g_k\|^2 \ge 0, \tag{19}$$

that is,

$$\|g_k\| = 0, \tag{20}$$

which contradicts to $\|g_k\| > 0$. The proof is completed. □

3. Strongly Global Convergence

Throughout this section, we assume that $\|g_k\| > 0$, for all $k \ge 0$; otherwise a stationary point of the objective function $f(x)$ has been found.

3.1. Global Convergence of SDPRP Method with the Line Search I. We first prove the global convergent of SDPRP method with the Armijo-type line search I.

Lemma 6. *For all $k \ge 0$, one has*

$$\|d_k\| \le \left(1 + \frac{2L(1-c)}{m_0}\right) \|g_k\|, \quad \forall k, \tag{21}$$

where m_0 is defined in Lemma 3.

Proof. If $k = 0$ then

$$\|d_k\| = \|g_k\| \le \left(1 + \frac{2L(1-c)}{m_0}\right) \|g_k\|. \tag{22}$$

If $k \ge 1$ then, from (3), (4), and (H2), we can get that

$$\begin{aligned} \|d_k + g_k\| &= \left\|\beta_k^{\mathrm{PRP}} d_{k-1} - \theta_k y_{k-1}\right\| \\ &\le \frac{2\|g_k - g_{k-1}\| \|d_{k-1}\|}{\|g_{k-1}\|^2} \|g_k\| \\ &\le \frac{2L\alpha_{k-1}\|d_{k-1}\|^2}{\|g_{k-1}\|^2} \|g_k\| \\ &\le \frac{2L\delta_{k-1}\|d_{k-1}\|^2}{\|g_{k-1}\|^2} \|g_k\| \\ &\le \frac{2L(1-c)}{m_0} \|g_k\|, \end{aligned} \tag{23}$$

which together with the triangular inequality implies that

$$\|d_k\| \le \|d_k + g_k\| + \|g_k\| \le \left(1 + \frac{2L(1-c)}{m_0}\right) \|g_k\|. \tag{24}$$

This completes the proof. □

The following lemma shows that the stepsize sequence $\{\alpha_k\}$ generated by the Armijo-type line search I is bounded from below.

Lemma 7. *For all $k \ge 0$, there exists a constant $C > 0$, such that*

$$\alpha_k \ge C, \tag{25}$$

in which α_k is generated by the Armijo-type line search I.

Proof. We divide the proof into two cases: $\alpha_k = \delta_k$ and $\alpha_k < \delta_k$. For the first case, by (12) and (21), we get

$$\alpha_k \ge \frac{(1-c)}{M_0}\left(1 + \frac{2L(1-c)}{m_0}\right)^{-2}. \tag{26}$$

For the second case, that is $\alpha_k < \delta_k$, which indicates that α_k/ρ does not satisfy (10); that is,

$$f\left(x_k + \frac{\alpha_k d_k}{\rho}\right) > f(x_k) - \frac{\mu \alpha_k \|g_k\|^2}{\rho}. \tag{27}$$

Using the mean value theorem in the above inequality, we obtain $\theta_k \in (0,1)$, such that

$$\left[g\left(x_k + \frac{\theta_k \alpha_k d_k}{\rho}\right) - g_k\right]^\top d_k > (1-\mu)\|g_k\|^2. \tag{28}$$

This inequality and (H2), (21) show that

$$\begin{aligned}\frac{L\alpha_k}{\rho} &\geq \frac{\|g(x_k+\theta_k\alpha_k d_k/\rho)-g_k\|}{\|d_k\|}\\ &= \frac{\|g(x_k+\theta_k\alpha_k d_k/\rho)-g_k\|\cdot\|d_k\|}{\|d_k\|^2}\\ &\geq \frac{[g(x_k+\theta_k\alpha_k d_k/\rho)-g_k]^\top d_k}{\|d_k\|^2}\\ &\geq (1-\mu)\frac{\|g_k\|^2}{\|d_k\|^2}\\ &\geq (1-\mu)\left(1+\frac{2L(1-c)}{m_0}\right)^{-2}.\end{aligned} \tag{29}$$

Therefore, we have that

$$\alpha_k \geq \frac{(1-\mu)\rho}{L}\left(1+\frac{2L(1-c)}{m_0}\right)^{-2}. \tag{30}$$

Obviously, (26) and (30) show that (25) holds with

$$C=\min\left\{\frac{(1-c)}{M_0},\frac{(1-\mu)\rho}{L}\right\}\left(1+\frac{2L(1-c)}{m_0}\right)^{-2}. \tag{31}$$

This completes the proof. □

We are now ready to establish the strong convergence of SDPRP method using the Armijo-type line search I.

Theorem 8. *Suppose that (H1) and (H2) hold. Then*

$$\lim_{k\to\infty}\|g_k\|=0. \tag{32}$$

Proof. Since the generated sequence $\{x_k\} \subseteq L_0$ and the objection function $f(x)$ is bounded below on the level set L_0, by (10) and (25), we have

$$\sum_{k=0}^{\infty}C\mu\|g_k\|^2 \leq \sum_{k=0}^{\infty}(f_k-f_{k+1}) < f_0. \tag{33}$$

Thus

$$\lim_{k\to\infty}\|g_k\|=0. \tag{34}$$

This completes the proof. □

3.2. Global Convergence of SDPRP Method with the Line Search II. Then, we prove the strongly global convergent of SDPRP method with the Armijo-type line search II. It is obvious that $x_k \in L_0$ for all $k \geq 0$. Therefore, from the line search II, we have

$$\lim_{k\to\infty}\alpha_k\|d_k\|^2=0. \tag{35}$$

This together with (5) implies that

$$\lim_{k\to\infty}\alpha_k\|g_k\|^2 \leq \lim_{k\to\infty}\alpha_k\|d_k\|^2=0. \tag{36}$$

In addition, (H3) implies that there is a constant $M>0$ such that

$$\|g_k\| \leq M, \quad \forall k \geq 0. \tag{37}$$

Lemma 9. *Suppose that (H2) and (H3) hold. Then for all $k \geq 0$, one has*

$$\alpha_k \geq \min\left\{1,\frac{\rho\|g_k\|^2}{(L+\mu\|d_k\|^2)\|d_k\|^2}\right\}. \tag{38}$$

Proof. If $\alpha_k \neq 1$, then $\alpha_k'=\alpha_k/\rho$ does not satisfy (11); that is

$$f(x_k+\alpha_k' d_k) > f(x_k)-\mu(\alpha_k')^2\|d_k\|^4. \tag{39}$$

From the mean value theorem and (H2), there exists a constant $\theta_k \in (0,1)$, such that

$$\begin{aligned}&f(x_k+\alpha_k' d_k)-f(x_k)\\ &=\alpha_k' g(x_k+\theta_k\alpha_k' d_k)^\top d_k\\ &=\alpha_k' g_k^\top d_k+(g(x_k+\theta_k\alpha_k' d_k)-g_k)^\top d_k\\ &\leq -\alpha_k'\|g_k\|^2+(\alpha_k')^2 L\|d_k\|^2,\end{aligned} \tag{40}$$

which together with (36) shows that (35) holds. This completes the proof. □

We are now ready to establish the strong convergence of SDPRP method using the Armijo-type line search II. The proof is motivated by the proof of Theorem 2.2 in [10].

Theorem 10. *Suppose that (H2) and (H3) hold. Then*

$$\lim_{k\to\infty}\|g_k\|=0. \tag{41}$$

Proof. For the sake of contradiction, we suppose that the conclusion is not right. Then there exist a constant $\epsilon>0$ and an infinite index set K such that

$$\|g_{k-1}\| \geq \epsilon, \quad \forall k \in K. \tag{42}$$

Moreover, the fact $\alpha_k \leq 1$, (35) and (H2) imply that

$$\|g_k-g_{k-1}\|^2 \leq L^2\alpha_{k-1}^2\|d_{k-1}\|^2 \leq L^2\alpha_{k-1}\|d_{k-1}\|^2 \longrightarrow 0. \tag{43}$$

This and (42) indicate that there exists a positive constant ϵ_1 such that for sufficiently large $k \in K$, we have

$$\|g_k\| \geq \epsilon_1. \tag{44}$$

Then by (36) and (44), we can get

$$\lim_{k\to\infty,k\in K}\alpha_k=0. \tag{45}$$

Table 1: The results for the methods on the tested problems.

P	n	SDPRPI	SDPRPII	TTPRP
FREUROTH	50	45/265/0.0313	152/1778/0.1719	62/633/0.0625
Extended trigonometric	1000	30/112/0.2188	218/2870/4.4375	26/122/0.2344
	3000	72/116/0.8906	F	82/227/1.1875
SROSENBR	500	850/2472/0.5625	1567/4536/0.9375	1151/3288/0.7344
Extended White and Holst	1000	125/485/0.3906	170/2170/1.2031	71/760/0.4531
	5000	133/561/1.9688	411/6741/18.1563	65/701/2.0625
BEALE	1000	131/453/0.2813	49/282/0.1406	64/380/0.1563
	5000	116/370/1.1875	41/233/0.6250	55/329/0.7813
Extended penalty	1000	38/352/0.1094	F	26/250/0.0781
	3000	29/328/0.2813	F	26/277/0.2500
Perturbed quadratic	1000	340/2761/0.6875	350/3850/0.8750	283/3062/0.6719
	5000	699/6693/6.2656	1161/19049/16.5625	725/9442/8.2500
Raydan 1	500	168/611/0.2188	214/1177/0.3125	186/1017/0.2813
Raydan 2	1000	5/6/0.0313	4/6/0.0625	5/6/0.0625
	5000	5/6/0.2969	5/8/0.3281	5/6/0.2969
	10000	5/6/1.1250	6/13/1.1563	5/6/1.1406
Diagonal1	100	87/353/0.0625	88/451/0.0625	92/476/0.0781
Diagonal2	1000	6755/6756/10.9531	6753/6754/10.5625	6753/6754/10.4063
Diagonal3	100	84/435/0.0938	103/678/0.1250	103/678/0.1250
Hager	500	55/221/0.1875	61/280/0.2188	48/218/0.1563
Generalized tridiagonal-1	1000	46/236/0.3125	41/231/0.3281	46/262/0.3438
	5000	43/213/1.7656	42/234/1.8438	49/277/2.1094
Extended tridiagonal-1	1000	58/104/0.1406	64/120/0.2031	58/105/0.1563
	5000	60/110/0.9219	214/366/2.1406	68/121/0.9688
Extended three exponential	1000	25/ 81/0.0938	39/160/0.1563	39/160/0.1719
	3000	29/ 95/0.3438	38/146/0.4844	38/146/0.4688
Generalized tridiagonal-2	1000	47/278/0.3438	55/441/0.6250	76/614/0.8438

By (3), (4), and (H2), for all $k \in K$, we have

$$\begin{aligned} \|d_k\| &\le \|g_k\| + \left|\beta_k^{\mathrm{PRP}}\right| \|d_{k-1}\| + |\theta_k| \|y_{k-1}\| \\ &\le \|g_k\| + \frac{2L\alpha_{k-1}\|d_{k-1}\|^2}{\|g_{k-1}\|^2}\|g_k\| \\ &\le \left(1 + \frac{2L\alpha_{k-1}\|d_{k-1}\|^2}{\epsilon^2}\right)\|g_k\|. \end{aligned} \tag{46}$$

From (35), for all sufficiently large $k \in K$, there exist a constant $r > 0$, such that

$$\alpha_{k-1}\|d_{k-1}\|^2 \le r. \tag{47}$$

Therefore $\|d_k\| \le M_2\|g_k\|$ with $M_2 = 1 + (2Lr/\epsilon^2)$. Thus, for $k \in K$, this and (37) imply that

$$\|d_k\| \le MM_2. \tag{48}$$

Thus, from (38) and (48), we get

$$\alpha_k \ge \min\left\{1, \frac{\rho}{(L + \mu M^2 M_2^2) M_2^2}\right\}, \quad \forall k \in K, \tag{49}$$

which contradicts (45). The proof is then completed. □

4. Numerical Results

In this section, we present some numerical results to compare the performance of SDPRP method with the two new Armijo-type line searches I and II and the three-term PRP method in [7].

(i) SDPRPI: the SDPRP method with the line search (10), with $\mu = 10^{-4}$, $\rho = 0.5$, $c = 0.2$;

(ii) SDPRPII: the SDPRP method with the line search (11), with $\mu = 10^{-4}$, $\rho = 0.5$.

(iii) TTPRP: the two-term PRP method with the following Armijo-type line search: let α_k be the largest α in $\{1, \rho, \rho^2, \ldots\}$ such that

$$f(x_k + \alpha d_k) \le f(x_k) - \mu\alpha^2\|d_k\|^2, \tag{50}$$

where $\mu = 10^{-4}$, $\rho = 0.5$.

All codes were written in Matlab 7.1 and run on a portable computer. We stopped the iteration if the number of iteration exceeds 10000 or $\|g_k\| < 10^{-5}$. Tables 1 and 2 list the numerical results for solving some test problems numbered from 1 to 30 in [11] with different dimension n. Our numerical results are listed in the form NI/NF/CPU, where the symbols NI, NF, and

Table 2: The results for the methods on the tested problems.

P	n	SDPRPI	SDPRPII	TTPRP
	5000	195/1606/8.7656	66/533/3.8125	188/1764/9.3438
Diagonal4 function	1000	36/156/0.0625	57/401/0.0781	60/418/0.1094
	5000	36/154/0.4219	74/596/0.7031	78/532/0.5781
Diagonal5	1000	3/4/0.0469	3/4/0.0156	3/4/0.0469
	5000	4/5/0.4688	4/5/0.4688	4/5/0.5000
HIMMELBC	1000	48/295/0.0938	47/329/0.0781	48/313/0.0938
	5000	53/326/0.7344	88/716/0.9844	52/338/0.7344
Generalized PSC1	1000	292/540/0.5313	326/726/0.7188	422/756/0.7656
	5000	373/733/3.6094	352/1355/7.5469	373/733/3.6094
Extended PSC1	1000	28/80/0.0938	28/138/0.1094	18/59/0.0625
	5000	27/78/0.6250	56/502/1.9063	18/59/0.5781
Extended Powell	1000	213/1548/1.1094	415/5098/2.7188	207/1498/1.0781
	5000	133/979/3.5781	F	145/1055/3.8281
Extended block diagonal	1000	25/128/0.0625	37/213/0.1094	37/213/0.1094
	5000	30/151/0.6406	41/216/0.7656	41/216/0.7813
Extended Maratos	500	36/257/0.0469	48/1451/0.1563	48/466/0.0625
Extended Cliff	1000	41/235/0.1406	633/3098/1.7031	71/363/0.2188
	5000	47/255/1.0625	963/3585/10.2188	68/357/1.2344
Quadratic diagonal perturbed	1000	842/4828/1.2969	496/5415/1.1406	709/7359/1.5313
	5000	1049/7363/5.3906	1031/15570/9.2500	767/9203/5.5156
Extended Wood	1000	237/1519/0.3594	F	299/2940/0.5313
	5000	222/1483/1.7188	F	174/1738/1.8750
Extended Hiebert	1000	582/2850/0.7188	F	59/763/0.1406
	5000	609/3056/3.4063	F	73/932/1.0000
Quadratic function	1000	343/2436/0.5313	360/3700/0.6719	321/3122/0.5938
	5000	808/7176/6.5781	F	746/8948/7.1875

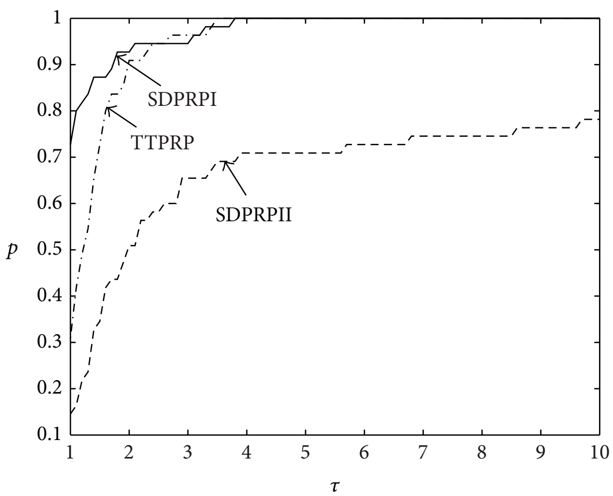

Figure 1: Performance profiles of three methods about the number of function evaluations.

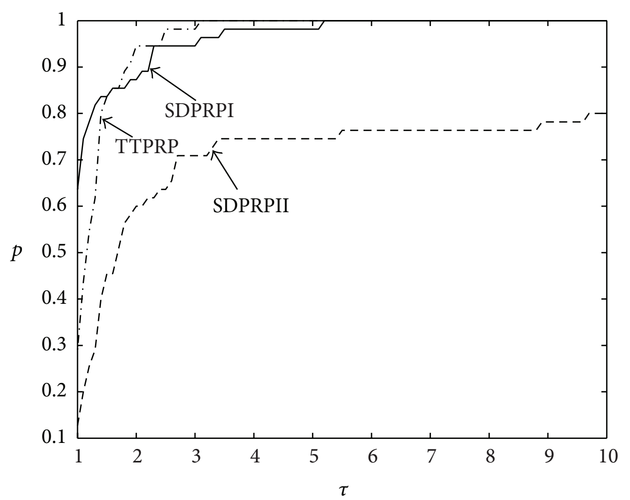

Figure 2: Performance profiles of three methods about CPU time.

CPU mean the number of iterations, the number of function evaluations, and the CPU time in seconds, respectively.

Figures 1 and 2 show the performance of these methods relative to the number of function evaluations and CPU time, respectively, which are evaluated using the profiles of Dolan and Moré [12]. That is, for each method, we plot the fraction P of problems for which the method is within a factor τ of the best time. The left side of the figure gives the percentage of the test problems for which a method is fastest; while the right side gives the percentage of the test problems that are successfully solved by each of the methods. The top curve is the method that solved most problems in a time that was within a factor τ of the best time. Figures 1 and 2 show that SDPRPI method performs a little better than TTPRP method and obviously better than SDPRPII method. It solves about 72% and 63% of the problems with the smallest number of function evaluations and CPU time, respectively. Obviously, the performance of SDPRPII method is not so good, and, in the future, we will further study the corresponding line search. Of course, more numerical experiments should be carried out to test our proposed methods.

5. Conclusion

In this paper, we have proposed two new Armijo-type line searches and proved that the sufficient descent PRP method proposed by Zhang et al. is strongly global convergent with the two new line searches. Numerical results show that the SDPRP method with the proposed line searches is efficient for the test problems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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