Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 278907, 4 pages
http://dx.doi.org/10.1155/2014/278907

Research Article

A New Difference Sequence Set of Order α and Its Geometrical Properties

Mikail Et[1] and Vatan Karakaya[2]

[1] *Department of Mathematics, Firat University, 23119, Elazğ, Turkey*
[2] *Department of Mathematical Engineering, Yildiz Technical University, Davutpasa Campus, Esenler, 34750 Istanbul, Turkey*

Correspondence should be addressed to Vatan Karakaya; vkkaya@yildiz.edu.tr

Received 15 June 2014; Accepted 19 August 2014; Published 11 September 2014

Academic Editor: Cristina Pignotti

We introduce a new class of sequences named as $m_\alpha(\Delta^r, \phi, p)$ and, for this space, we study some inclusion relations, topological properties, and geometrical properties such as order continuous, the Fatou property, and the Banach-Saks property of type p.

1. Introduction, Definitions, and Preliminaries

By w we denote the space of all complex (or real) sequences. If $x \in w$, then we simply write $x = (x_k)$ instead of $x = (x_k)_{k=0}^{\infty}$. We will write ℓ_∞, c, and c_0 for the sequence spaces of all bounded, convergent, and null sequences, respectively. Also by ℓ_1 and ℓ_p we denote the spaces of all absolutely summable and p-absolutely summable series, respectively.

The notion of difference sequence spaces was generalized by Et and Çolak [1] such as $X(\Delta^r) = \{x = (x_k) : \Delta^r x \in X\}$, for $X = \ell_\infty$, c, and c_0. They showed that these sequence spaces are BK-spaces with the norm

$$\|x\|_\Delta = \sum_{i=1}^{r} |x_i| + \|\Delta^r x\|_\infty, \tag{1}$$

where $r \in \mathbb{N}$, $\Delta^0 x = x$, $\Delta x = (x_k - x_{k+1})$, $\Delta^r x = (\Delta^r x_k) = (\Delta^{r-1} x_k - \Delta^{r-1} x_{k+1})$, and $\Delta^r x_k = \sum_{v=0}^{r} (-1)^v \binom{r}{v} x_{k+v}$. Recently difference sequences and related concepts have been studied in ([2–13]) and by many others.

Let E be a sequence space. Then E is called

(i) *solid* (or *normal*) if $(\alpha_k x_k) \in E$ for all sequences (α_k) of scalars with $|\alpha_k| \leq 1$ for all $k \in \mathbb{N}$, whenever $(x_k) \in E$,

(ii) *symmetric* if $(x_k) \in E$ implies $(x_{\pi(k)}) \in E$, where π is a permutation of $\mathbb{N}$,

(iii) *monotone* provided E contains the canaonical preimages of all its step spaces,

(iv) *sequence algebra* if $x \cdot y \in E$, whenever $x, y \in E$.

It is well known that if E is normal then E is monotone.

Throughout this paper φ_s denotes the class of all subsets of $\mathbb{N}$; those do not contain more than s elements. Let (ϕ_n) be a nondecreasing sequence of positive numbers such that $n\phi_{n+1} \leq (n+1)\phi_n$ for all $n \in \mathbb{N}$. The class of all sequences (ϕ_n) is denoted by Φ. The sequence space $m(\phi)$ was introduced by Sargent [14] and he studied some of its properties and obtained some relations with the space ℓ_p. Later on it was investigated by Tripathy and Sen [15] and Tripathy and Mahanta [16].

Let us recall that a sequence $\{v(i)\}_{i=1}^{\infty}$ in a Banach space X is called *Schauder basis* of X (or *basis* for short) if for each $x \in X$ there exists a unique sequence $\{\lambda(i)\}_{i=1}^{\infty}$ of scalars such that $x = \sum_{i=1}^{\infty} \lambda(i) v(i)$; that is, $\lim_{n\to\infty} \sum_{i=1}^{n} \lambda(i) v(i) = x$.

A sequence space X with a linear topology is called a *K-space* if each of the projection maps $P_i : X \to \mathbb{C}$ defined by $P_i(x) = x(i)$ for $x = (x(i))_{i=1}^{\infty} \in X$ is continuous for each natural i. A *Fréchet space* is a complete metric linear space and the metric is generated by an F-norm and a Fréchet space which is a K-space is called an *FK-space;* that is, a K-space X is called an FK-space if X is a complete linear metric space. In other words, X is an FK-space if X is a Fréchet space with continuous coordinate projections. All the sequence spaces mentioned above are FK spaces except the space c_{00}.

An FK-space X which contains the space c_{00} is said to have the *property AK* if for every sequence $\{x(i)\} \in X$, $x = \sum_{i=1}^{\infty} x(i)e(i)$ where $e(i) = (0, 0, \ldots, 1(i\text{th-place}), 0, 0, \ldots)$.

A Banach space X is said to be a *Köthe sequence space* (see [17, 18]) if X is a subspace of w such that

(i) if $x \in w$, $y \in X$ and $|x(i)| \leq |y(i)|$ for all $i \in \mathbb{N}$, then $x \in X$ and $\|x\| \leq \|y\|$;

(ii) there exists an element $x \in X$ such that $x(i) > 0$ for all $i \in \mathbb{N}$.

We say that $x \in X$ is order continuous if for any sequence (x_n) in X such that $x_n(i) \leq |x(i)|$ for each $i \in \mathbb{N}$ and $x_n(i) \to 0$ $(n \to \infty)$ we have that $\|x_n\| \to 0$ holds.

A Köthe sequence space X is said to be order continuous if all sequences in X are order continuous. It is easy to see that $x \in X$ is order continuous if and only if $\|(0, 0, \ldots, 0, x(n+1), x(n+2), \ldots)\| \to 0$ as $n \to \infty$.

A Köthe sequence space X is said to have the *Fatou property* if, for any real sequence x and any $\{x_n\}$ in X such that $x_n \uparrow x$ coordinatewisely and $\sup_n \|x_n\| < \infty$, we have that $x \in X$ and $\|x_n\| \to \|x\|$.

A Banach space X is said to have the *Banach-Saks property* if every bounded sequence $\{x_n\}$ in X admits a subsequence $\{z_n\}$ such that the sequence $\{t_k(z)\}$ is convergent in X with respect to the norm, where

$$t_k(z) = \frac{1}{k}(z_1 + z_2 + \cdots + z_k), \quad \forall k \in \mathbb{N}. \tag{2}$$

Some of recent works on geometric properties of sequence space can be found in the following list ([19–21]).

2. Inclusion and Topological Properties of the Space $m_\alpha(\Delta^r, \phi, p)$

In this section we introduce a new class of sequences and establish some inclusion relations. Also we show that this space is not perfect and normal.

Let r be a fixed positive integer, $\alpha \in (0, 1]$ any real number, and p a positive real number such that $1 \leq p < \infty$. Now we define the sequence space $m_\alpha(\Delta^r, \phi, p)$ as

$$m_\alpha(\Delta^r, \phi, p) = \left\{x \in w : \sup_{s\geq 1, \sigma \in \varphi_s} \frac{1}{\phi_s^\alpha} \sum_{n\in\sigma} |\Delta^r x_n|^p < \infty\right\}. \tag{3}$$

In the case $p = 1$, we will write $m_\alpha(\Delta^r, \phi)$ instead of $m_\alpha(\Delta^r, \phi, p)$ and in the special case $m = 0$ and $p = 1$ we will write $m_\alpha(\phi)$ instead of $m_\alpha(\Delta^r, \phi, p)$.

The proof of each of the following results is straightforward, so we choose to state these results without proof.

Theorem 1. *Let $\phi \in \Phi$, $\alpha \in (0, 1]$, and let p be a positive real number such that $1 \leq p < \infty$. Then the sequence space $m_\alpha(\Delta^r, \phi, p)$ is a BK-space normed by*

$$\|x\| = \sum_{i=1}^{r} |x_i| + \sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s^\alpha} \left(\sum_{n\in\sigma} |\Delta^r x_n|^p\right)^{1/p}. \tag{4}$$

Theorem 2. *Let $\phi \in \Phi$, $\alpha \in (0, 1]$, and let p be a positive real number such that $1 \leq p < \infty$; then $m_\alpha(\Delta^r, \phi) \subset m_\alpha(\Delta^r, \phi, p)$.*

Theorem 3. *Let α and β be fixed real numbers such that $0 < \alpha \leq \beta \leq 1$ and p a positive real number such that $1 \leq p < \infty$; then $m_\alpha(\Delta^r, \phi, p) \subset m_\beta(\Delta^r, \phi, p)$.*

Theorem 4. *Let α and β be fixed real numbers such that $0 < \alpha \leq \beta \leq 1$ and p a positive real number such that $1 \leq p < \infty$. For any two sequences (ϕ_s) and (ψ_s) of real numbers such that $\phi, \psi \in \Phi$. Then $m_\alpha(\Delta^r, \phi, p) \subset m_\beta(\Delta^r, \psi, p)$ if and only if $\sup_{s\geq 1}(\phi_s^\alpha / \psi_s^\beta) < \infty$.*

Proof. Let $x \in m_\alpha(\Delta^r, \phi, p)$ and $\sup_{s\geq 1}(\phi_s^\alpha/\psi_s^\beta) < \infty$. Then

$$\sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s^\alpha} \sum_{n\in\sigma} |\Delta^r x_n|^p < \infty \tag{5}$$

and there exists a positive number K such that $\phi_s^\alpha \leq K\psi_s^\beta$ and so that $1/\psi_s^\beta \leq K/\phi_s^\alpha$ for all s. Therefore for all s we have

$$\frac{1}{\psi_s^\beta} \sum_{n\in\sigma} |\Delta^r x_n|^p \leq \frac{K}{\phi_s^\alpha} \sum_{n\in\sigma} |\Delta^r x_n|^p. \tag{6}$$

Now taking supremum over $s \geq 1$ and $\sigma \in \varphi_s$ we get

$$\sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\psi_s^\beta} \sum_{n\in\sigma} |\Delta^r x_n|^p \leq \sup_{s\geq 1, \sigma\in\varphi_s} \frac{K}{\phi_s^\alpha} \sum_{n\in\sigma} |\Delta^r x_n|^p \tag{7}$$

and so $x \in m_\beta(\Delta^r, \psi, p)$.

Conversely let $m_\alpha(\Delta^r, \phi, p) \subset m_\beta(\Delta^r, \psi, p)$ and suppose that $\sup_{s\geq 1}(\phi_s^\alpha/\psi_s^\beta) = \infty$. Then there exists an increasing sequence (s_i) of naturals numbers such that $\lim_i(\phi_{s_i}^\alpha/\psi_{s_i}^\beta) = \infty$. Let $B \in \mathbb{R}^+$, where $\mathbb{R}^+$ is the set of positive real numbers; then there exists $i_0 \in \mathbb{N}$ such that $\phi_{s_i}^\alpha/\psi_{s_i}^\beta > B$ for all $s_i \geq i_0$. Hence $\phi_{s_i}^\alpha > B\psi_{s_i}^\beta$ and so $1/\psi_{s_i}^\beta > B/\phi_{s_i}^\alpha$. Then we can write

$$\frac{1}{\psi_{s_i}^\beta} \sum_{n\in\sigma} |\Delta^r x_n|^p > \frac{B}{\phi_{s_i}^\alpha} \sum_{n\in\sigma} |\Delta^r x_n|^p \tag{8}$$

for all $s_i \geq i_0$. Now taking supremum over $s_i \geq i_0$ and $\sigma \in \varphi_s$ we get

$$\sup_{s_i\geq i_0, \sigma\in\varphi_s} \frac{1}{\psi_{s_i}^\beta} \sum_{n\in\sigma} |\Delta^r x_n|^p > \sup_{s_i\geq i_0, \sigma\in\varphi_s} \frac{B}{\phi_{s_i}^\alpha} \sum_{n\in\sigma} |\Delta^r x_n|^p. \tag{9}$$

Since (9) holds for all $B \in \mathbb{R}^+$ (we may take the number B sufficiently large), we have

$$\sup_{s_i\geq i_0, \sigma\in\varphi_s} \frac{1}{\psi_{s_i}^\beta} \sum_{n\in\sigma} |\Delta^r x_n|^p = \infty \tag{10}$$

when $x \in m_\alpha(\Delta^r, \phi, p)$ with

$$0 < \sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s^\alpha} \sum_{n\in\sigma} |\Delta^r x_n|^p < \infty. \tag{11}$$

Hence $x \notin m_\beta(\Delta^r, \psi, p)$. This contradicts to $m_\alpha(\Delta^r, \phi, p) \subset m_\beta(\Delta^r, \psi, p)$. Hence $\sup_{s\geq 1}(\phi_s^\alpha/\psi_s^\beta) < \infty$. □

The following results are derivable easily from Theorem 4.

Corollary 5. *Let α and β be fixed real numbers such that $0 < \alpha \le \beta \le 1$ and p a positive real number such that $1 \le p < \infty$. For any two sequences (ϕ_s) and (ψ_s) of real numbers such that $\phi, \psi \in \Phi$. Then one has*

(i) $m_\alpha(\Delta^r, \phi, p) = m_\beta(\Delta^r, \psi, p)$ *if and only if* $0 < \inf_{s\ge1}(\phi_s^\alpha/\psi_s^\beta) < \sup_{s\ge1}(\phi_s^\alpha/\psi_s^\beta) < \infty$,

(ii) $m_\alpha(\Delta^r, \phi, p) = m_\alpha(\Delta^r, \psi, p)$ *if and only if* $0 < \inf_{s\ge1}(\phi_s^\alpha/\psi_s^\alpha) < \sup_{s\ge1}(\phi_s^\alpha/\psi_s^\alpha) < \infty$,

(iii) $m_\alpha(\Delta^r, \phi, p) = m_\beta(\Delta^r, \phi, p)$ *if and only if* $0 < \inf_{s\ge1}(\phi_s^\alpha/\phi_s^\beta) < \sup_{s\ge1}(\phi_s^\alpha/\phi_s^\beta) < \infty$.

Theorem 6. *Consider $m_\alpha(\Delta^{r-1}, \phi, p) \subset m_\alpha(\Delta^r, \phi, p)$ and the inclusion is strict.*

Proof. It follows from Minkowski's inequality. To show the inclusion is strict, let $\phi_n = 1$ for all $n \in \mathbb{N}$, $\alpha = 1$, $p = 1$, and $x = (k^{r-1})$; then $x \in m_\alpha(\Delta^r, \phi, p) \setminus m_\alpha(\Delta^{r-1}, \phi, p)$. □

Theorem 7. *The sequence space $m_\alpha(\phi)$ is solid and hence monotone, but the sequence space $m_\alpha(\Delta^r, \phi, p)$ is neither solid nor symmetric and sequence algebra for $m \ge 1$.*

Proof. Let $x \in m_\alpha(\phi)$ and $y = (y_n)$ be sequences such that $|x_n| \le |y_n|$ for each $n \in \mathbb{N}$. Then we get

$$\sup_{s\ge1,\sigma\in\varphi_s} \frac{1}{\phi_s^\alpha} \sum_{n\in\sigma} |x_n| \le \sup_{s\ge1,\sigma\in\varphi_s} \frac{1}{\phi_s^\alpha} \sum_{n\in\sigma} |y_n|. \tag{12}$$

Hence $m_\alpha(\phi)$ is solid and hence monotone. To show the space $m_\alpha(\Delta^r, \phi, p)$ is normal, let $\phi_n = 1$, for all $n \in \mathbb{N}$, $\alpha = 1$, $p = 1$, and $x = (k^{r-1})$, then $x \in m_\alpha(\Delta^r, \phi, p)$; but $(\alpha_k x_k) \notin m_\alpha(\Delta^r, \phi, p)$ when $\alpha_k = (-1)^k$ for all $k \in \mathbb{N}$. Hence $m_\alpha(\Delta^r, \phi, p)$ is not solid. The other cases can be proved on considering similar examples. □

Theorem 8. *Consider*

$$\ell_p(\Delta^r) \subset m_\alpha(\Delta^r, \phi, p) \subset \ell_\infty(\Delta^r). \tag{13}$$

Proof. It is omitted. □

Theorem 9. *If $0 < p < q$, then $m_\alpha(\Delta^r, \phi, p) \subset m_\alpha(\Delta^r, \phi, q)$.*

Proof. Proof follows from the following inequality:

$$\left(\sum_{k=1}^{n} |x_k|^q\right)^{1/q} < \left(\sum_{k=1}^{n} |x_k|^q\right)^{1/q}, \quad (0 < p < q). \tag{14}$$

□

3. Geometrical Properties of the Space $m_\alpha(\Delta^r,\phi,p)$

In this section, we study some geometrical properties of the space $m_\alpha(\Delta^r, \phi, p)$. Some of these geometrical properties are the order continuous, the Fatou property, and the Banach-Saks property of type p. Let us start with the following theorem.

Theorem 10. *The space $m_\alpha(\Delta^r, \phi, p)$ is order continuous.*

Proof. To prove this theorem, we have to show that $m_\alpha(\Delta^r, \phi, p)$ is an AK-space. It is easy to see that $m_\alpha(\Delta^r, \phi, p)$ contains c_{00} which is the space of real sequences which have only a finite number of nonzero coordinates. By using definition of AK-properties, we have that $x = \{x(i)\} \in m_\alpha(\Delta^r, \phi, p)$ has a unique representation $x = \sum_{i=1}^{\infty} x(i)e(i)$; that is, $\|x - x^{[j]}\|_{m_\alpha(\Delta^r,\phi,p)} = \|(0, 0, \dots, x(j), x(j+1), \dots)\|_{m_\alpha(\Delta^r,\phi,p)} \to 0$ as $j \to \infty$, which means that $m_\alpha(\Delta^r, \phi, p)$ has AK. Hence, since BK-space $m_\alpha(\Delta^r, \phi, p)$ containing c_{00} has AK-property, the space $m_\alpha(\Delta^r, \phi, p)$ is order continuous. □

Theorem 11. *The space $m_\alpha(\Delta^r, \phi, p)$ has the Fatou property.*

Proof. Let x be any real sequence from $(w)_+$ and $\{x_n\}$ any nondecreasing sequence of nonnegative elements from $m_\alpha(\Delta^r, \phi, p)$ such that $x_n(i) \to x(i)$ as $n \to \infty$ coordinatewisely and $\sup_n \|x_n\|_{m_\alpha(\Delta^r,\phi,p)} < \infty$.

Let us denote $\tau = \sup_n \|x_n\|_{m_\alpha(\Delta^r,\phi,p)}$. Then, since the supremum is homogeneous, we have

$$\begin{aligned}
&\frac{1}{\tau}\left(\sum_{i=1}^{r} |x_n(i)| + \sup_{s\ge1,\sigma\in\varphi_s} \frac{1}{\phi_s^\alpha}\left(\sum_{i\in\sigma} |\Delta^r x_n(i)|^p\right)^{1/p}\right) \\
&= \sum_{i=1}^{r} \left|\frac{x_n(i)}{\tau}\right| + \sup_{s\ge1,\sigma\in\varphi_s} \frac{1}{\phi_s^\alpha}\left(\sum_{n\in\sigma} \left|\frac{\Delta^r x_n(i)}{\tau}\right|^p\right)^{1/p} \\
&\le \sum_{i=1}^{r} \left|\frac{x_n(i)}{\|x_n\|_{m_\alpha(\Delta^r,\phi,p)}}\right| \\
&\quad + \sup_{s\ge1,\sigma\in\varphi_s} \frac{1}{\phi_s^\alpha}\left(\sum_{n\in\sigma} \left|\frac{\Delta^r x_n(i)}{\|x_n\|_{m_\alpha(\Delta^r,\phi,p)}}\right|^p\right)^{1/p} \\
&= \frac{1}{\|x_n\|_{m_\alpha(\Delta^r,\phi,p)}} \|x_n\|_{m_\alpha(\Delta^r,\phi,p)} = 1.
\end{aligned} \tag{15}$$

Moreover, by the assumptions that $\{x_n\}$ is nondecreasing and convergent to x coordinatewisely and by the Beppo-Levi theorem, we have

$$\begin{aligned}
&\frac{1}{\tau}\lim_{n\to\infty}\left(\sum_{i=1}^{r} |x_n(i)| + \sup_{s\ge1,\sigma\in\varphi_s} \frac{1}{\phi_s^\alpha}\left(\sum_{i\in\sigma} |\Delta^r x_n(i)|^p\right)^{1/p}\right) \\
&= \sum_{i=1}^{r} \left|\frac{x(i)}{\tau}\right| + \sup_{s\ge1,\sigma\in\varphi_s} \frac{1}{\phi_s^\alpha}\left(\sum_{i\in\sigma} \left|\frac{\Delta^r x(i)}{\tau}\right|^p\right)^{1/p} \\
&= \left\|\frac{x}{s}\right\|_{m_\alpha(\Delta^r,\phi,p))} \le 1,
\end{aligned} \tag{16}$$

whence

$$\|x\|_{m_\alpha(\Delta^r,\phi,p)))} \le \tau = \sup_n \|x_n\|_{m_\alpha(\Delta^r,\phi,p))} = \lim_{n\to\infty} \|x_n\|_{m_\alpha(\Delta^r,\phi,p))} < \infty. \quad (17)$$

Therefore, $x \in m_\alpha(\Delta^r,\phi,p))$. On the other hand, since $0 \le x_n \le x$ for any natural number n and the sequence $\{x_n\}$ is nondecreasing, we obtain that the sequence $\{\|x_n\|_{m_\alpha(\Delta^r,\phi,p))}\}$ is bounded from above by $\|x\|_{m_\alpha(\Delta^r,\phi,p))}$. As a result, $\lim_{n\to\infty}\|x_n\|_{m_\alpha(\Delta^r,\phi,p))} \le \|x\|_{m_\alpha(\Delta^r,\phi,p))}$, which together with the opposite inequality proved already yields that $\|x\|_{m_\alpha(\Delta^r,\phi,p))} = \lim_n \|x_n\|_{m_\alpha(\Delta^r,\phi,p))}$. □

Theorem 12. *The space $m_\alpha(\Delta^r,\phi,p))$ has the Banach-Saks property of the type p.*

Proof. It can be proved with standard technic. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] M. Et and R. Çolak, “On some generalized difference sequence spaces,” *Soochow Journal of Mathematics*, vol. 21, no. 4, pp. 377–386, 1995.

[2] B. Altay and F. Basar, “On the fine spectrum of the difference operator Δ on c_0 and and c,” *Information Science*, vol. 168, no. 1–4, pp. 217–224, 2004.

[3] Ç. A. Bektas, M. Et, and R. Çolak, “Generalized difference sequence spaces and their dual spaces,” *Journal of Mathematical Analysis and Applications*, vol. 292, no. 2, pp. 423–432, 2004.

[4] N. L. Braha, “On asymptotically Δ^m-lacunary statistical equivalent sequences,” *Applied Mathematics and Computation*, vol. 219, no. 1, pp. 280–288, 2012.

[5] R. Çolak and M. Et, “On some difference sequence sets and their topological properties,” *Bulletin of the Malaysian Mathematical Sciences Society*, vol. 28, no. 2, pp. 125–130, 2005.

[6] M. Et, “Strongly almost summable difference sequences of order m defined by a modulus,” *Studia Scientiarum Mathematicarum Hungarica*, vol. 40, no. 4, pp. 463–476, 2003.

[7] M. Güngor and M. Et, “Δ^r-strongly almost summable sequences defined by Orlicz functions,” *Indian Journal of Pure and Applied Mathematics*, vol. 34, no. 8, pp. 1141–1151, 2003.

[8] M. Karakas, M. Et, and V. Karakaya, “Some geometric properties of a new difference sequence space involving lacunary sequences,” *Acta Mathematica Scientia*, vol. 33, no. 6, pp. 1711–1720, 2013.

[9] M. Mursaleen, R. Çolak, and M. Et, “Some geometric inequalities in a new banach sequence space,” *Journal of Inequalities and Applications*, vol. 2007, Article ID 86757, 6 pages, 2007.

[10] H. Polat, V. Karakaya, and N. Şimşek, “Difference sequence spaces derived by using a generalized weighted mean,” *Applied Mathematics Letters*, vol. 24, no. 5, pp. 608–614, 2011.

[11] B. C. Tripathy, “On a class of difference sequences related to the p-normed space l_p,” *Demonstratio Mathematica*, vol. 36, no. 4, pp. 867–872, 2003.

[12] S. A. Mohiuddine, K. Raj, and A. Alotaibi, “Some paranormed double difference sequence spaces for Orlicz functions and bounded-regular matrices,” *Abstract and Applied Analysis*, vol. 2014, Article ID 419064, 10 pages, 2014.

[13] S. A. Mohiuddine, K. Raj, and A. Alotaibi, “On some classes of double difference sequences of interval numbers,” *Abstract and Applied Analysis*, vol. 2014, Article ID 516956, 8 pages, 2014.

[14] W. L. C. Sargent, “Some sequence spaces related to the l_p spaces,” *Journal of the London Mathematical Society. Second Series*, vol. 35, pp. 161–171, 1960.

[15] B. C. Tripathy and M. Sen, “On a new class of sequences related to the space L_p,” *Tamkang Journal of Mathematics*, vol. 33, no. 2, pp. 167–171, 2002.

[16] B. C. Tripathy and S. Mahanta, “On a class of sequences related to the l_p space defined by Orlicz functions,” *Soochow Journal of Mathematics*, vol. 29, no. 4, pp. 379–391, 2003.

[17] Y. A. Cui and H. Hudzik, “On the Banach-Saks and weak Banach-Saks properties of some Banach sequence spaces,” *Acta Scientiarum Mathematicarum*, vol. 65, no. 1-2, pp. 179–187, 1999.

[18] J. Diestel, *Sequence and Series in Banach Spaces*, vol. 92 of *Graduate Texts in Mathematics*, Springer, Berlin, Germany, 1984.

[19] V. Karakaya and F. Altun, “On some geometric properties of a new paranormed sequence space,” *Journal of Function Spaces*, vol. 2014, Article ID 685382, 8 pages, 2014.

[20] H. Hudzik, V. Karakaya, and M. Mursaleen, “Banach-Saks type and GGurarii modulus of convexity of some Banach sequence spaces,” *Abstract and Applied Analysis*, vol. 2014, Article ID 427382, 9 pages, 2014.

[21] M. Et, M. Karakaş, and V. Karakaya, “Some geometric properties of a new difference sequence space defined by de la Vallée-Poussin mean,” *Applied Mathematics and Computation*, vol. 234, pp. 237–244, 2014.