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Research Article

Some New Fixed Point Theorems in Complex Valued G-Metric Spaces

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Some new fixed point theorems are established in the setting of complex valued G-metric spaces. These new results improve and generalize Kang et al.'s results, the Banach contraction principle, and some well-known results in the literature.

1. Introduction and Preliminaries

It is well known that Banach contraction principle [1] plays an important role in various fields of applied mathematical analysis and scientific applications and has been generalized and improved in many various different directions; see [2–16] and references therein. In 2011, Azam et al. [2] introduced so-called complex valued metric spaces and proved the existence of fixed points under some contraction conditions. In 2006, Mustafa and Sims [3] introduced the concept of G-metric spaces to extend and generalize the notion of metric spaces. In 2013, Kang et al. [8] introduced the concept of complex valued G-metric spaces to generalize and improve the notion of G-metric spaces. In [8], the authors gave a complex valued G-metric version of Banach contraction principle.

In what follows we will give some definitions and known results which will be needed in the sequel. Throughout the present paper, the symbols $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{C}$ are used to denote the sets of positive integers, real numbers, and complex numbers, respectively.

In 2006, Mustafa and Sims [3] introduced a new class of metric spaces called generalized metric spaces or G-metric spaces as follows.

Definition 1 (see [3]). Let X be a nonempty set and let $G : X \times X \times X \to [0,\infty)$ be a function satisfying the following:

(G1) $G(x, y, z) = 0$ if $x = y = z$,

(G2) $0 < G(x, y, z)$ for all $x, y \in X$ with $x \neq y$,

(G3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$,

(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$ (symmetry in all three variables),

(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).

Then the function G is called a generalized metric or a G-metric on X and the pair (X, G) is called a G-metric space.

Example 2 (see [3]). Let (X, d) be a usual metric space. Then (X, G_1) and (X, G_2) are all G-metric spaces, where

$$G_1(x, y, z) = d(x, y) + d(y, z) + d(z, x),$$
$$G_2(x, y, z) = \max\{d(x, y), d(y, z), d(z, x)\} \quad (1)$$

for all $x, y, z \in X$.

For any $z_1, z_2 \in \mathbb{C}$, we can define a partial order $\lesssim$ on $\mathbb{C}$ as follows:

$$z_1 \lesssim z_2 \Longleftrightarrow \operatorname{Re}(z_1) \leqq \operatorname{Re}(z_2), \quad \operatorname{Im}(z_1) \leqq \operatorname{Im}(z_2). \quad (2)$$

So, it is easy to see that $z_1 \lesssim z_2$ holds if one of the following conditions is satisfied:

(C1) $\operatorname{Re}(z_1) = \operatorname{Re}(z_2)$ and $\operatorname{Im}(z_1) = \operatorname{Im}(z_2)$,

(C2) $\operatorname{Re}(z_1) < \operatorname{Re}(z_2)$ and $\operatorname{Im}(z_1) = \operatorname{Im}(z_2)$, (C3) $\operatorname{Re}(z_1) = \operatorname{Re}(z_2)$ and $\operatorname{Im}(z_1) < \operatorname{Im}(z_2)$,

(C4) $\operatorname{Re}(z_1) < \operatorname{Re}(z_2)$ and $\operatorname{Im}(z_1) < \operatorname{Im}(z_2)$.

In particular, we will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (C2), (C3), and (C4) is satisfied and we will write $z_1 \prec z_2$ if only (C4) is satisfied.

Remark 3. It is obvious that the following statements hold.

(1) If $0 \preceq z_1 \precnsim z_2$, then $|z_1| < |z_2|$.

(2) If $z_1 \preceq z_2$ and $z_2 \prec z_3$, then $z_1 \prec z_3$.

The idea of complex metric space was initialed by Azam et al. [2].

Definition 4. Let X be a nonempty set. Suppose that the mapping $d : X \times X \to \mathbb{C}$ satisfies

(C1) $0 \preceq d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$,

(C2) $d(x, y) = d(y, x)$ for all $x, y \in X$,

(C3) $d(x, y) \preceq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a complex valued metric on X and the pair (X, d) is called a complex valued metric space.

Example 5 (see [6, Example 2]). Let $X = X_1 \bigcup X_2$ where

$$X_1 = \{z \in \mathbb{C} : \operatorname{Re}(z) \geq 0,\ \operatorname{Im}(z) = 0\},$$
$$X_2 = \{z \in \mathbb{C} : \operatorname{Re}(z) = 0,\ \operatorname{Im}(z) \geq 0\}. \tag{3}$$

Define $d : X \times X \to \mathbb{C}$ as follows:

$$d(z_1, z_2) = \begin{cases} \dfrac{2}{3}|x_1 - x_2| + \dfrac{i}{2}|x_1 - x_2|, & \text{if } z_1, z_2 \in X_1, \\ \dfrac{1}{2}|y_1 - y_2| + \dfrac{i}{3}|y_1 - y_2|, & \text{if } z_1, z_2 \in X_2, \\ \left(\dfrac{2}{3}x_1 + \dfrac{1}{2}y_2\right) + i\left(\dfrac{1}{2}x_1 + \dfrac{1}{3}y_2\right), & \text{if } z_1 \in X_1,\quad z_2 \in X_2, \\ \left(\dfrac{1}{2}y_1 + \dfrac{2}{3}x_2\right) + i\left(\dfrac{1}{3}y_1 + \dfrac{1}{2}x_2\right), & \text{if } z_1 \in X_2,\quad z_2 \in X_1, \end{cases} \tag{4}$$

where $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2 \in X$. Then (X, d) is a complete complex valued metric space.

The notion of complex valued G-metric space was introduced by Kang et al. [8] to generalize the notion of complex valued metric space and G-metric space as follows.

Definition 6 (see [8]). Let X be a nonempty set and let $G_c : X \times X \times X \to \mathbb{C}$ be a function satisfying the following:

(CG1) $G_c(x, y, z) = 0$ if $x = y = z$,

(CG2) $0 \precnsim G_c(x, x, y)$ for all $x, y \in X$ with $x \neq y$,

(CG3) $G_c(x, x, y) \preceq G_c(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$,

(CG4) $G_c(x, y, z) = G_c(x, z, y) = G_c(y, z, x) = \cdots$ (symmetry in all three variables),

(CG5) $G_c(x, y, z) \preceq G_c(x, a, a) + G_c(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).

Then the function G_c is called a complex valued generalized metric or a complex valued G-metric on X. We call the pair (X, G_c) a complex valued G-metric space.

Remark 7. In fact, condition (CG2) defined in [8] was stated as follows:

(CG2) $0 \prec G_c(x, x, y)$ for all $x, y \in X$ with $x \neq y$.

In this paper, we use the weak version of (CG2) as in Definition 6.

Example 8. Let $X = \mathbb{C}$ and $G_c : X \times X \times X \to \mathbb{C}$ be defined by

$$G_c(z_1, z_2, z_3) = (|x_1 - x_2| + |x_2 - x_3| + |x_3 - x_1|) + i(|y_1 - y_2| + |y_2 - y_3| + |y_3 - y_1|), \tag{5}$$

where $z_i = x_i + iy_i \in \mathbb{C}$ for any $i \in \{1, 2, 3\}$. Then (X, G_c) is a complex valued G-metric space.

Definition 9. Let (X, G_c) be a complex valued G-metric space. A point v in X is a fixed point of a mapping $T : X \to X$ if $v = Tv$. The set of fixed points of T is denoted by $\mathcal{F}(T)$.

Definition 10 (see [8]). Let (X, G_c) be a complex valued G-metric space and let $\{x_n\}$ be a sequence in X. We say that $\{x_n\}$ is *complex value G-convergent* to $x \in X$ if, for every $c \in \mathbb{C}$ with $0 \prec c$, there exists $k \in \mathbb{N}$ such that $G_c(x, x_n, x_m) \prec c$ for all $n, m \geq k$. We refer to x as the limit of the sequence $\{x_n\}$ and we write $x_n \to x$ as $n \to \infty$.

Definition 11 (see [8]). Let (X, G_c) be a complex valued G-metric space.

(i) A sequence $\{x_n\}$ in X is said to be *complex valued G-Cauchy* if, for every $c \in \mathbb{C}$ with $0 \prec c$, there exists $k \in \mathbb{N}$ such that $G_c(x_n, x_m, x_l) \prec c$ for all $n, m, l \geq k$.

(ii) (X, G_c) is said to be *complete* if every complex valued G-Cauchy sequence in X is complex valued G-convergent in X.

Some crucial facts in complex valued G_c-metric spaces are listed as follows. First, the following proposition follows easily due to (CG5).

Proposition 12 (see [8]). *Let (X, G_c) be a complex valued G-metric space. Then, for any $x, y, z \in X$, the following hold:*

(1) $G_c(x, y, z) \preceq G_c(x, x, y) + G_c(x, x, z)$,

(2) $G_c(x, y, y) \preceq 2G_c(y, x, y)$.

Proposition 13 (see [8]). *Let (X, G) be a complex valued G-metric space. Then, for a sequence $\{x_n\}$ in X and point $x \in X$, the following are equivalent.*

(1) $\{x_n\}$ *is complex valued G-convergent to* x.

(2) $|G_c(x_n, x_n, x)| \to 0$ *as* $n \to \infty$.

(3) $|G_c(x_n, x, x)| \to 0$ *as* $n \to \infty$.

(4) $|G_c(x_m, x_n, x)| \to 0$ *as* $m, n \to \infty$.

Proposition 14 (see [8]). *Let (X, G) be a complex valued G-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is complex valued G-Cauchy sequence if and only if $|G(x_n, x_m, x_l)| \to 0$ as $n, m, l \to \infty$.*

Proposition 15 (see [8]). *Let (X, G) be a complex valued G-metric space. Then the function $G(x, y, z)$ is jointly continuous in all three of its variables.*

The main aim of this paper is to establish some new fixed point theorems which extend and generalize Kang et al.'s results in [8], the Banach contraction principle, and some well-known results in the literature.

2. Main Results

Recall that a function $\varphi : [0, \infty) \to [0, 1)$ is said to be an $\mathcal{MT}$*-function* (or $\mathcal{R}$*-function*) [11–16] if

$$\limsup_{s \to t^+} \varphi(s) < 1 \quad \forall t \in [0, \infty). \tag{6}$$

It is obvious that if $\varphi : [0, \infty) \to [0, 1)$ is a nondecreasing function or a nonincreasing function, then φ is an $\mathcal{MT}$-function. So the set of $\mathcal{MT}$-functions is a rich class.

Recently, Du [13] first proved the following characterizations of $\mathcal{MT}$-functions.

Theorem 16 (see [13]). *Let $\varphi : [0, \infty) \to [0, 1)$ be a function. Then the following statements are equivalent.*

(a) *φ is an $\mathcal{MT}$-function.*

(b) *For each $t \in [0, \infty)$, there exist $r_t^{(1)} \in [0, 1)$ and $\varepsilon_t^{(1)} > 0$ such that $\varphi(s) \leq r_t^{(1)}$ for all $s \in (t, t + \varepsilon_t^{(1)})$.*

(c) *For each $t \in [0, \infty)$, there exist $r_t^{(2)} \in [0, 1)$ and $\varepsilon_t^{(2)} > 0$ such that $\varphi(s) \leq r_t^{(2)}$ for all $s \in [t, t + \varepsilon_t^{(2)}]$.*

(d) *For each $t \in [0, \infty)$, there exist $r_t^{(3)} \in [0, 1)$ and $\varepsilon_t^{(3)} > 0$ such that $\varphi(s) \leq r_t^{(3)}$ for all $s \in (t, t + \varepsilon_t^{(3)}]$.*

(e) *For each $t \in [0, \infty)$, there exist $r_t^{(4)} \in [0, 1)$ and $\varepsilon_t^{(4)} > 0$ such that $\varphi(s) \leq r_t^{(4)}$ for all $s \in [t, t + \varepsilon_t^{(4)})$.*

(f) *For any nonincreasing sequence $\{x_n\}_{n \in \mathbb{N}}$ in $[0, \infty)$, we have $0 \leq \sup_{n \in \mathbb{N}} \varphi(x_n) < 1$.*

(g) *φ is a function of contractive factor; for any strictly decreasing sequence $\{x_n\}_{n \in \mathbb{N}}$ in $[0, \infty)$, we have $0 \leq \sup_{n \in \mathbb{N}} \varphi(x_n) < 1$.*

The following new fixed point theorem is one of the main results of this paper. It can be considered as a complex valued G-metric version of Banach contraction principle and will generalize and improve [8, Theorem 2.5] and some well-known results in the literature.

Theorem 17. *Let (X, G_c) be a complete complex valued G-metric space and let $T : X \to X$ be a mapping on X. Suppose that there exists a $\mathcal{MT}$-function $\varphi : [0, \infty) \to [0, 1)$ such that*

$$G_c(Tx, Ty, Tz) \precsim \varphi(|G_c(x, y, z)|)\, G_c(x, y, z) \quad \forall x, y, z \in X. \tag{7}$$

Then T has a unique fixed point on X.

Proof. Let $x_0 \in X$ be given. Define the sequence $\{x_n\}$ by

$$x_n = T^n x_0 = T x_{n-1} \quad \text{for each } n \in \mathbb{N}. \tag{8}$$

For each $n \in \mathbb{N}$, by (7), we have

$$G_c(x_n, x_{n+1}, x_{n+1}) \precsim \varphi(|G_c(x_{n-1}, x_n, x_n)|)\, G_c(x_{n-1}, x_n, x_n) \tag{9}$$

which implies

$$|G_c(x_n, x_{n+1}, x_{n+1})| \leq \varphi(|G_c(x_{n-1}, x_n, x_n)|)\, |G_c(x_{n-1}, x_n, x_n)|. \tag{10}$$

Let $\alpha_n = |G_c(x_{n-1}, x_n, x_n)|$ for $n \in \mathbb{N}$. Then, by (10), we have

$$\alpha_{n+1} \leq \varphi(\alpha_n)\, \alpha_n < \alpha_n \ \forall n \in \mathbb{N}. \tag{11}$$

So we know that $\{\alpha_n\}$ is a strictly decreasing sequence in $[0, \infty)$. Applying (g) of Theorem 16, we obtain

$$0 \leq \sup_{n \in \mathbb{N}} \varphi(\alpha_n) < 1. \tag{12}$$

That is,

$$0 \leq \sup_{n \in \mathbb{N}} \varphi(|G_c(x_{n-1}, x_n, x_n)|) < 1. \tag{13}$$

Let

$$\lambda = \sup_{n \in \mathbb{N}} \varphi(|G_c(x_{n-1}, x_n, x_n)|). \tag{14}$$

Then $\lambda \in [0, 1)$. For each $n \in \mathbb{N}$, by (10) again, we have

$$\begin{aligned} &|G_c(x_n, x_{n+1}, x_{n+1})| \\ &\quad \leq \varphi(|G_c(x_{n-1}, x_n, x_n)|)\, |G_c(x_{n-1}, x_n, x_n)| \\ &\quad \leq \lambda\, |G_c(x_{n-1}, x_n, x_n)| \leq \lambda^2\, |G_c(x_{n-2}, x_{n-1}, x_{n-1})| \\ &\quad \leq \cdots \leq \lambda^n\, |G_c(x_0, x_1, x_1)|. \end{aligned} \tag{15}$$

For any $n, m \in \mathbb{N}$ with $m > n$, by the last inequality and repeated use of (CG5), we get

$$\begin{aligned} &\left|G_c(x_n, x_m, x_m)\right| \\ &\quad \leq \left|G_c(x_n, x_{n+1}, x_{n+1})\right| + \left|G_c(x_{n+1}, x_{n+2}, x_{n+2})\right| \\ &\qquad + \cdots + \left|G_c(x_{m-1}, x_m, x_m)\right| \\ &\quad \leq \left(\lambda^n + \lambda^{n+1} + \cdots + \lambda^{m-1}\right)\left|G_c(x_0, x_1, x_1)\right| \\ &\quad < \frac{\lambda^n}{1-\lambda}\left|G_c(x_0, x_1, x_1)\right|. \end{aligned} \tag{16}$$

Since $\lambda \in [0, 1)$, $\lim_{n\to\infty}(\lambda^n/(1-\lambda))|G_c(x_0, x_1, x_1)| = 0$. Hence, by the last inequality, we obtain

$$\left|G_c(x_n, x_m, x_m)\right| \longrightarrow 0 \quad \text{as } m, n \longrightarrow \infty. \tag{17}$$

For any $n, m, l \in \mathbb{N}$, by Proposition 12, we obtain

$$G_c(x_n, x_m, x_l) \precsim G_c(x_n, x_m, x_m) + G_c(x_l, x_m, x_m), \tag{18}$$

which implies

$$\left|G_c(x_n, x_m, x_l)\right| \leq \left|G_c(x_n, x_m, x_m)\right| + \left|G_c(x_l, x_m, x_m)\right|. \tag{19}$$

From (17) and (19), we get $|G_c(x_n, x_m, x_l)| \to 0$ as $n, m, l \to \infty$. Applying Proposition 14, $\{x_n\}$ is a complex valued G-Cauchy sequence in (X, G_c). By the completeness of (X, G_c), there exists $v \in X$ such that $\{x_n\}$ is complex valued G-convergent to v.

Next, we prove that $Tv = v$. Assume that $Tv \neq v$. For each $n \in \mathbb{N}$, by (7), we have

$$G_c(x_{n+1}, Tv, Tv) \precsim \varphi\left(\left|G_c(x_n, v, v)\right|\right) G_c(x_n, v, v) \tag{20}$$

which deduces

$$\begin{aligned} \left|G_c(x_{n+1}, Tv, Tv)\right| &\leq \varphi\left(\left|G_c(x_n, v, v)\right|\right)\left|G_c(x_n, v, v)\right| \\ &< \left|G_c(x_n, v, v)\right|. \end{aligned} \tag{21}$$

Since $x_n \to v$ as $n \to \infty$ and G is continuous in all three of its variables, from Proposition 15 and by taking limit from both sides of (21), we get

$$\left|G_c(v, Tv, Tv)\right| \leq \left|G_c(v, v, v)\right| = 0. \tag{22}$$

Since $0 \precnsim G_c(v, Tv, Tv)$, by Remark 3, we know

$$0 < \left|G_c(v, Tv, Tv)\right|. \tag{23}$$

Hence, taking into account (22) and (23), we have

$$0 < \left|G_c(v, Tv, Tv)\right| \leq 0 \tag{24}$$

which is a contradiction. Therefore $Tv = v$ or $v \in \mathcal{F}(T)$.

Finally, we want to show the uniqueness of fixed point of T (i.e., $\mathcal{F}(T)$ is a singleton set). We have shown $v \in \mathcal{F}(T)$, so it suffices to show that $\mathcal{F}(T) = \{v\}$. Let $w \in \mathcal{F}(T)$. Suppose $w \neq v$. By (7), we obtain

$$G_c(v, v, w) = G_c(Tv, Tv, Tw) \precsim \varphi\left(\left|G_c(v, v, w)\right|\right) G_c(v, v, w) \tag{25}$$

which implies

$$\left|G_c(v, v, w)\right| \leq \varphi\left(\left|G_c(v, v, w)\right|\right)\left|G_c(v, v, w)\right|. \tag{26}$$

By (26), we have

$$\left(1 - \varphi\left(\left|G_c(v, v, w)\right|\right)\right)\left|G_c(v, v, w)\right| \leq 0. \tag{27}$$

Since $\varphi(|G_c(v, v, w)|) \in [0, 1)$, we have

$$\left|G_c(v, v, w)\right| \leq 0 \tag{28}$$

which deduces

$$\left|G_c(v, v, w)\right| = 0 \tag{29}$$

and hence $G_c(v, v, w) = 0$. This contradicts (CG2). Therefore, it must be $w = v$ and so $\mathcal{F}(T) = \{v\}$. The proof is completed. □

Here, we give a simple example illustrating Theorem 17.

Example 18. Let $X = \mathbb{C}$ and $G_c : X \times X \times X \to \mathbb{C}$ be defined by

$$\begin{aligned} G_c(z_1, z_2, z_3) = &\left(\left|x_1 - x_2\right| + \left|x_2 - x_3\right| + \left|x_3 - x_1\right|\right) \\ &+ i\left(\left|y_1 - y_2\right| + \left|y_2 - y_3\right| + \left|y_3 - y_1\right|\right), \end{aligned} \tag{30}$$

where $z_i = x_i + iy_i \in \mathbb{C}$ for any $i \in \{1, 2, 3\}$. Then (X, G_c) is a complex valued G-metric space. Define $T : X \to X$ and $\varphi : [0, \infty) \to [0, 1)$ by

$$\begin{aligned} Tz &= \frac{1}{10}z \quad \text{for } z \in X, \\ \varphi(t) &:= \begin{cases} \dfrac{4}{5}, & \text{if } t = 0, \\ \dfrac{1}{3}, & \text{if } t > 0. \end{cases} \end{aligned} \tag{31}$$

Then φ is a $\mathcal{MT}$-function. For any $z_1, z_2, z_3 \in \mathbb{C}$, where $z_i = x_i + iy_i$, we have

$$Tz_i = \frac{1}{10}z_i = \frac{x_i}{10} + i\frac{y_i}{10} \quad \text{for any } i \in \{1, 2, 3\}. \tag{32}$$

By a routine calculation, one can verify that

$$G_c(Tz_1, Tz_2, Tz_3) \precsim \phi\left(\left|G_c(z_1, z_2, z_3)\right|\right) G_c(z_1, z_2, z_3). \tag{33}$$

So all the hypotheses of Theorem 17 are fulfilled. It is therefore possible to apply Theorem 17 to get the fact that T has a unique fixed point on X (precisely speaking, 0 is the unique fixed point of T).

The following fixed point theorem established in G-metric space is immediate from Theorem 17.

Theorem 19. *Let (X, G) be a complete G-metric space and let $T: X \to X$ be a mapping on X. Suppose that there exists a $\mathcal{MT}$-function $\varphi : [0, \infty) \to [0, 1)$ such that*

$$G(Tx, Ty, Tz) \le \varphi(G(x, y, z))\, G(x, y, z) \quad \forall x, y, z \in X. \tag{34}$$

Then T has a unique fixed point on X.

Since any nondecreasing function or any nonincreasing function $\varphi : [0, \infty) \to [0, 1)$ is an $\mathcal{MT}$-function, by applying Theorem 17, we have the following results.

Corollary 20. *Let (X, G_c) be a complete complex valued G-metric space and let $T : X \to X$ be a mapping on X. Suppose that there exists a nondecreasing function $\varphi : [0, \infty) \to [0, 1)$ such that*

$$G_c(Tx, Ty, Tz) \precsim \varphi(|G_c(x, y, z)|)\, G_c(x, y, z) \quad \forall x, y, z \in X. \tag{35}$$

Then T has a unique fixed point on X.

Corollary 21. *Let (X, G) be a complete G-metric space and let $T : X \to X$ be a mapping on X. Suppose that there exists a nondecreasing function $\varphi : [0, \infty) \to [0, 1)$ such that*

$$G(Tx, Ty, Tz) \le \varphi(G(x, y, z))\, G(x, y, z) \quad \forall x, y, z \in X. \tag{36}$$

Then T has a unique fixed point on X.

Corollary 22. *Let (X, G_c) be a complete complex valued G-metric space and let $T : X \to X$ be a mapping on X. Suppose that there exists a nonincreasing function $\varphi : [0, \infty) \to [0, 1)$ such that*

$$G_c(Tx, Ty, Tz) \precsim \varphi(|G_c(x, y, z)|)\, G_c(x, y, z) \quad \forall x, y, z \in X. \tag{37}$$

Then T has a unique fixed point on X.

Corollary 23. *Let (X, G) be a complete G-metric space and let $T : X \to X$ be a mapping on X. Suppose that there exists a nonincreasing function $\varphi : [0, \infty) \to [0, 1)$ such that*

$$G(Tx, Ty, Tz) \le \varphi(G(x, y, z))\, G(x, y, z) \quad \forall x, y, z \in X. \tag{38}$$

Then T has a unique fixed point on X.

Corollary 24 (see [8, Theorem 2.5]). *Let (X, G_c) be a complete complex valued G-metric space and let $T : X \to X$ be a contraction mapping on X; that is,*

$$G_c(Tx, Ty, Tz) \precsim k G_c(x, y, z) \tag{39}$$

for all $x, y, z \in X$, where $k \in [0, 1)$. Then T has a unique fixed point.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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