Research Article

MHD Flow of a Viscous Fluid over an Exponentially Stretching Sheet in a Porous Medium

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Radiation effects on magnetohydrodynamic (MHD) boundary-layer flow and heat transfer characteristic through a porous medium due to an exponentially stretching sheet have been studied. Formulation of the problem is based upon the variable thermal conductivity. The heat transfer analysis is carried out for both prescribed surface temperature (PST) and prescribed heat flux (PHF) cases. The developed system of nonlinear coupled partial differential equations is transformed to nonlinear coupled ordinary differential equations by using similarity transformations. The series solutions for the transformed of the transformed flow and heat transfer problem were constructed by homotopy analysis method (HAM). The obtained results are analyzed under the influence of various physical parameters.

1. Introduction

Since the revolutionary work of Sakiadis [1, 2] on boundary-layer flow past a moving surface a great deal of research work has been carried out for the two-dimensional boundary-layer flows. Crane [3] extended the Sakiadis [1, 2] flow problem by assuming a stretching boundary. Crane's problem is one of the flow problems in boundary layer theory that possesses an exact solution. The stretching velocity in Crane's problem is linearly proportional to the distance from origin. The heat and mass transfer on the flow past a porous stretching surface is discussed by P. S. Gupta and A. S. Gupta [4]. Brady and Acrivos [5] proved the existence and uniqueness of the solution for the stretching flow. Three-dimensional flow due to a stretching surface was analyzed by McLeod and Rajagopal [6]. In another paper Wang [7] extended Crane's problem for a stretching cylinder. Flow problems due to a stretching surface have important applications in technology, geothermal energy recovery, and manufacturing process such as oil recovery, artificial fibers, metal extrusion, and metal spinning. Extensive literature is available regarding the steady

flows in Newtonian and non-Newtonian fluid flows over a stretching sheet. The readers are referred to the studies [8–14] and references therein. The transportation of heat in a porous medium has applications in geothermal systems, rough oil mining, soil-water contamination, and biomechanical problems. Vajravelu [15] reported steady flow and heat transfer of viscous fluids by considering different heating process in a porous medium.

The above mentioned problems deal with linear stretching of the surface. Magyari and Keller [16] initiated the work by assuming an exponentially stretching surface. The heat transfer analysis for flow past an exponentially stretching surface was carried out by Elbashbeshy [17]. The same problem by considering the constitutive equation of a viscoelastic fluid was investigated by Khan [18]. Due to the applications in electrical power generator, astrophysical flows, solar power technology, space vehicle reentry, and so forth, the heat transfer analysis in the presence of radiation is another important area of research. Raptis [19] investigated the radiation effects for the flow past a semi-infinite flat plate. The influence of thermal radiation in a viscoelastic fluid past a stretching sheet

is illustrated by Chen [20]. Sajid and Hayat [21] discussed the homotopy series solution for the influence of radiation on the flow past an exponentially stretching sheet. Chiam [22] investigated the heat transfer analysis with variable thermal conductivity in a stagnation point flow towards a stretching sheet. In another paper, the analysis of effects of variable thermal conductivity was discussed by Chiam [23].

The purpose of the present paper is to demonstrate an analysis for the MHD flow and heat transfer analysis of a viscous fluid towards an exponentially stretching sheet in the presence of radiation effects and Darcy's resistance. The analytic series solution is presented by using homotopy analysis method [24–30].

2. Mathematical Formulation of the Problem

Consider a steady, incompressible, two-dimensional MHD flow of an electrically conducting viscous fluid towards an exponentially stretching sheet in a porous medium. The fluid occupies the space $y > 0$ and is flowing in the x direction and y-axis is normal to the flow. A constant magnetic field of strength B_0 is applied in the normal direction and the induced magnetic field is neglected which is a valid assumption when the magnetic Reynolds number is small. The boundary-layer equations that govern the present flow and heat transfer problem are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\frac{\partial^2 u}{\partial y^2} - \frac{v}{k'}u - \frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$\frac{\partial p}{\partial y} = 0, \tag{3}$$

$$\rho c_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y} \right) - \frac{\partial q_r}{\partial y}, \tag{4}$$

where u and v are the components of velocity in x and y directions, respectively, p is the pressure, ρ is the density, v represents kinematic viscosity, k' is the permeability of porous medium, σ is the electrical conductivity, T is the fluid temperature, c_p is the specific heat, q_r is radiative heat flux, and k is the variable thermal conductivity defined in the following way:

$$k = \begin{cases} k_\infty\left[1 + \epsilon\theta\left(\eta\right)\right] & \text{in PST case} \\ k_\infty\left[1 + \epsilon\phi\left(\eta\right)\right] & \text{in PHF case,} \end{cases} \tag{5}$$

in which ϵ is the small parameter ($\epsilon > 0$ for gases and $\epsilon < 0$ for solids and liquids) and $\theta(\eta)$ and $\phi(\eta)$ are dimensionless temperature distributions for PST and PHF

cases, respectively. The relevant boundary conditions for the present problem are

$$u = U_0 e^{x/l}, \qquad v = 0,$$

$$T = T_\infty + Ae^{ax/2l} \text{ (PST)}, \tag{6}$$

$$-k\frac{\partial T}{\partial y} = Be^{(b+l)x/2l} \text{ (PHF)}, \quad \text{at } y = 0,$$

$$u \longrightarrow 0, \qquad T \longrightarrow T_\infty \quad \text{as } y \longrightarrow \infty, \tag{7}$$

where U_0 is a characteristic velocity, l is the characteristic length scale, and A, a, B, and b are the parameters of temperature profiles depending on the properties of the liquid. Equation (7) suggests that $\partial p/\partial x = 0$ far away from the plate. Since there is no variation in the pressure in y direction as evident from (3), therefore, $\partial p/\partial x = 0$ is zero throughout the flow field and we have from (2)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - \frac{v}{k'}u - \frac{\sigma B_0^2}{\rho}u. \tag{8}$$

The radiative heat flux is defined as

$$q_r = -\frac{4\sigma_1}{3m}\frac{\partial T^4}{\partial y}, \tag{9}$$

in which σ_1 is the Stefan-Boltzmann constant and m is the coefficient of mean absorption. Assuming T^4 as a linear combination of temperature, then

$$T^4 \cong -3T_\infty^4 + 4T_\infty^3 T. \tag{10}$$

Substituting (9) and (10) in (4) one gets

$$\rho c_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y} \right) + \frac{16T_\infty^3 \sigma_1}{3m}\frac{\partial^2 T}{\partial y^2}. \tag{11}$$

Introducing the dimensionless variables

$$\eta = \sqrt{\frac{U_0}{2vl}}e^{x/2l}y, \qquad \psi(x, y) = \sqrt{2vlU_0}e^{x/2l}f(\eta). \tag{12}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \text{ (PST)}, \qquad \phi(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \text{ (PHF)}. \tag{13}$$

In terms of above variables flow and heat transfer problems take the following form:

$$f''' - 2f'^2 + ff'' - 2(R + M)f' = 0,$$

$$(1 + \text{Tr} + \epsilon\theta)\theta'' + \text{Pr}f\theta' - a\text{Pr}f'\theta + \epsilon\theta'^2 = 0 \quad \text{(PST)},$$

$$(1 + \text{Tr} + \epsilon\phi)\phi'' + \text{Pr}f\phi' - b\text{Pr}f'\phi + \epsilon\phi'^2 = 0 \quad \text{(PHF)},$$

$$f(0) = 0, \qquad f'(0) = 1,$$

$$\theta(0) = 1, \qquad \phi'(0) = -\frac{1}{1 + \epsilon},$$

$$f'(\infty) = \theta(\infty) = \phi(\infty) = 0,$$

$$\tag{14}$$

where the dimensionless parameters are given as

$$R = \frac{\nu\varphi l}{k'U_0}e^{-x/l}, \qquad \Pr = \frac{\mu c_p}{k_\infty},$$

$$M = \frac{l\sigma B_\circ^2}{\rho U_0}e^{-x/l}, \qquad \mathrm{Tr} = \frac{16\sigma_1 T_\infty^3}{3k_\infty K}. \tag{15}$$

3. HAM Solution

To find the series solutions for velocity and temperature profiles given in (14), we use homotopy analysis method (HAM). The velocity and temperature distributions can be expressed in terms of the following base functions:

$$\left\{\eta^j \exp(-n\eta) \mid j \geq 0, \ n \geq 0\right\} \tag{16}$$

in the form

$$f(\eta) = a_{0,0}^0 + \sum_{n=0}^\infty \sum_{k=0}^\infty a_{m,n}^k \eta^k \exp(-n\eta),$$

$$\theta(\eta) = \sum_{n=0}^\infty \sum_{k=0}^\infty b_{m,n}^k \eta^k \exp(-n\eta), \tag{17}$$

$$\phi(\eta) = \sum_{n=0}^\infty \sum_{k=0}^\infty c_{m,n}^k \eta^k \exp(-n\eta),$$

where $a_{m,n}^k$, $b_{m,n}^k$, and $c_{m,n}^k$ are the coefficients. The following initial guesses are selected for solution expressions of $f(\eta)$, $\theta(\eta)$, and $\phi(\eta)$:

$$f_0(\eta) = 1 - e^{-\eta}, \qquad \theta_0(\eta) = e^{-\eta},$$

$$\phi_0(\eta) = \frac{1}{1+\epsilon}e^{-\eta}, \tag{18}$$

and auxiliary linear operators are

$$\mathscr{L}_1(f) = f'' - f', \qquad \mathscr{L}_2(\theta) = \theta'' - \theta,$$

$$\mathscr{L}_3(\phi) = \phi'' - \phi, \tag{19}$$

which satisfy

$$\mathscr{L}_1\left[C_1 + C_2 e^\eta + C_3 e^{-\eta}\right] = 0, \qquad \mathscr{L}_2\left[C_4 e^\eta + C_5 e^{-\eta}\right] = 0,$$

$$\mathscr{L}_3\left[C_6 e^\eta + C_7 e^{-\eta}\right] = 0, \tag{20}$$

where C_i ($i = 1, 2, \ldots, 7$) are arbitrary constants. The zero-order deformation problems are

$$(1-p)\mathscr{L}_1\left[\hat{f}(\eta,p) - f_0(\eta)\right] = p\hbar_f \mathscr{N}_f\left[\hat{f}(\eta,p)\right],$$

$$(1-p)\mathscr{L}_2\left[\hat{\theta}(\eta,p) - \theta_0(\eta)\right] = p\hbar_\theta \mathscr{N}_\theta\left[\hat{f}(\eta,p)\right],$$

$$(1-p)\mathscr{L}_3\left[\hat{\phi}(\eta,p) - \phi_0(\eta)\right] = p\hbar_\phi \mathscr{N}_\phi\left[\hat{f}(\eta,p)\right],$$

$$\hat{f}(0,p) = 0, \qquad \hat{f}'(0,p) = 1,$$

$$\hat{\theta}(0,p) = 1, \qquad \frac{\partial\hat{\phi}(0,p)}{\partial\eta} = -\frac{1}{1+\epsilon},$$

$$\hat{f}'(\infty,p) = \hat{\theta}(\infty,p) = \hat{\phi}(\infty,p) = 0. \tag{21}$$

The nonlinear operators are given by

$$\mathscr{N}_f\left[\hat{f}(\eta,p)\right] = \frac{\partial^3 \hat{f}(\eta,p)}{\partial\eta^3} - 2\left(\frac{\partial\hat{f}(\eta,p)}{\partial\eta}\right)^2$$
$$+ \hat{f}(\eta,p)\frac{\partial^2 \hat{f}(\eta,p)}{\partial\eta^2} - 2(R+M)\frac{\partial\hat{f}(\eta,p)}{\partial\eta}, \tag{22}$$

$$\mathscr{N}_\theta\left[\hat{f}(\eta,p), \hat{\theta}(\eta,p)\right]$$
$$= \begin{bmatrix} \left\{1 + \mathrm{Tr} + \epsilon\hat{\theta}(\eta,p)\right\}\dfrac{\partial^2\hat{\theta}(\eta,p)}{\partial\eta^2} + \Pr\hat{f}(\eta,p)\dfrac{\partial\hat{\theta}(\eta,p)}{\partial\eta} \\ -a\Pr\dfrac{\partial\hat{f}(\eta,p)}{\partial\eta}\hat{\theta}(\eta,p) + \epsilon\left(\dfrac{\partial\hat{\theta}(\eta,p)}{\partial\eta}\right)^2 \end{bmatrix}, \tag{23}$$

$$\mathscr{N}_\phi\left[\hat{f}(\eta,p), \hat{\phi}(\eta,p)\right]$$
$$= \begin{bmatrix} \left\{1 + \mathrm{Tr} + \epsilon\hat{\phi}(\eta,p)\right\}\dfrac{\partial^2\hat{\phi}(\eta,p)}{\partial\eta^2} + \Pr\hat{f}(\eta,p)\dfrac{\partial\hat{\phi}(\eta,p)}{\partial\eta} \\ -b\Pr\dfrac{\partial\hat{f}(\eta,p)}{\partial\eta}\hat{\phi}(\eta,p) + \epsilon\left(\dfrac{\partial\hat{\phi}(\eta,p)}{\partial\eta}\right)^2 \end{bmatrix}, \tag{24}$$

in which $p \in [0,1]$ is an embedding parameter and $\hbar_f, \hbar_\theta$, and $\hbar_\phi$ are nonzero auxiliary parameters. For $p = 0$ and $p = 1$, we, respectively, have

$$\hat{f}(\eta,0) = f_0(\eta), \qquad \hat{f}(\eta,1) = f(\eta),$$

$$\hat{\theta}(\eta,0) = \theta_0(\eta), \qquad \hat{\theta}(\eta,1) = \theta(\eta), \tag{25}$$

$$\hat{\phi}(\eta,0) = \phi_0(\eta), \qquad \hat{\phi}(\eta,1) = \phi(\eta).$$

As p varies from 0 to 1, $\widehat{f}(\eta, p)$, $\widehat{\theta}(\eta, p)$, and $\widehat{\phi}(\eta, p)$ vary from $f_0(\eta)$, $\theta_0(\eta)$, and $\phi_0(\eta)$ to $f(\eta)$, $\theta(\eta)$, and $\phi(\eta)$, respectively. Using Taylor's series one can write

$$\widehat{f}(\eta, p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) p^m,$$

$$\widehat{\theta}(\eta, p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) p^m, \qquad (26)$$

$$\widehat{\phi}(\eta, p) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta) p^m,$$

where

$$f_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m f(\eta, p)}{\partial p^m} \right|_{p=0},$$

$$\theta_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \theta(\eta, p)}{\partial p^m} \right|_{p=0}, \qquad (27)$$

$$\phi_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \phi(\eta, p)}{\partial p^m} \right|_{p=0}.$$

Note that the zero-order deformation equations contain three auxiliary parameters $\hbar_f$, $\hbar_\theta$, and $\hbar_\phi$. The convergence of series solutions strongly depends upon these three parameters. Assuming $\hbar_f$, $\hbar_\theta$, and $\hbar_\phi$ are chosen in such a way that the above series are convergent at $p = 1$, then

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta),$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta), \qquad (28)$$

$$\phi(\eta) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta).$$

Taking the derivative of (21)–(24) m-times with respect to p, then selecting $p = 0$ and dividing by $m!$ the following expressions for mth order deformation problems are obtained:

$$\mathcal{L}_1 \left[f_m(\eta) - \chi_m f_{m-1}(\eta) \right] = \hbar_f \mathfrak{N}_m^f(\eta),$$

$$\mathcal{L}_2 \left[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta) \right] = \hbar_\theta \mathfrak{N}_m^\theta(\eta),$$

$$\mathcal{L}_3 \left[\phi_m(\eta) - \chi_m \phi_{m-1}(\eta) \right] = \hbar_\phi \mathfrak{N}_m^\phi(\eta), \qquad (29)$$

$$f_m(0) = f_m'(0) = f_m'(\infty) = \theta_m(0) = \theta_m(\infty)$$

$$= \phi_m'(0) = \phi_m(\infty) = 0,$$

in which

$$\mathfrak{N}_m^f(\eta) = f_{m-1}''' - 2(R + M) f_{m-1}'$$
$$+ \sum_{k=0}^{m-1} \left[f_{m-1-k} f_k'' - 2 f_{m-1-k}' f_k' \right], \qquad (30)$$

$$\mathfrak{N}_m^\theta(\eta) = (1 + \mathrm{Tr}) \theta_{m-1}'' + \sum_{k=0}^{m-1} \left[\epsilon \theta_{m-1-k} \theta_k'' + \mathrm{Pr} f_{m-1-k} \theta_k' \right.$$
$$\left. - a \mathrm{Pr} f_{m-1-k}' \theta_k + \epsilon \theta_{m-1-k}' \theta_k' \right], \qquad (31)$$

$$\mathfrak{N}_m^\phi(\eta) = (1 + \mathrm{Tr}) \phi_{m-1}'' + \sum_{k=0}^{m-1} \left[\epsilon \phi_{m-1-k} \phi_k'' + \mathrm{Pr} f_{m-1-k} \phi_k' \right.$$
$$\left. - b \mathrm{Pr} f_{m-1-k}' \phi_k + \epsilon \phi_{m-1-k}' \phi_k' \right], \qquad (32)$$

$$\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1. \end{cases} \qquad (33)$$

The system of linear nonhomogeneous equations (29)–(32) is solved using Mathematica in the order $m = 1, 2, 3, \ldots$.

4. Results and Discussion

The governing nonlinear equations for the MHD flow and heat transfer analysis for a viscous fluid in a porous medium with radiation effects and variable thermal conductivity are solved analytically using homotopy analysis method for both PST and PHF cases. The solutions are given in the form of analytic expressions in (28). These analytic expressions contain the auxiliary parameters $\hbar_f$, $\hbar_\theta$, and $\hbar_\phi$. The convergence and rate of approximation of the obtained series solutions strongly depend upon these parameters. To examine the range of acceptable values of these parameters, we draw the $\hbar$-curves in Figure 1 for the functions $f(\eta)$, $\theta(\eta)$, and $\phi(\eta)$. This figure depicts that the admissible values of $\hbar_f$, $\hbar_\theta$, and $\hbar_\phi$ are in the interval $[-0.6, -0.3]$.

To see the influence of porosity parameter R, Prandtl number Pr, thermal conductivity ϵ, temperature parameters a and b, and thermal radiation Tr on the temperature distributions for both PST and PHF cases, Figures 2–6 have been plotted. Figures (a) are for the PST case and Figures (b) are for PHF case. In Figures 2(a) and 2(b), variation of temperature distributions with variation in thermal conductivity parameter ϵ is shown. Figure 2(a) illustrates that temperature and thermal boundary-layer thickness in the PST case increases by increasing thermal conductivity. The results in the PHF case are opposite to that of PST case and are shown in Figure 2(b). Figure 3 elucidates that the temperature and thermal boundary-layer thickness increases by an increase in the porosity parameter. Since in the governing equation magnetic parameter and porosity parameter appear in the same way, therefore, the effects of magnetic parameter are similar to those of porosity parameter. The behavior of temperature with changing Prandtl number is quite opposite to that of
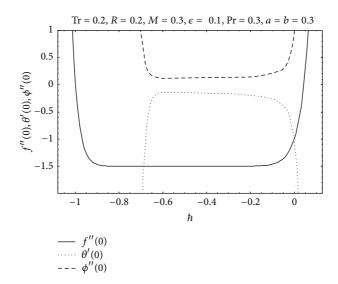
FIGURE 1: $\hbar$-curves for the functions $f(\eta)$, $\theta(\eta)$, and $\phi(\eta)$ at 20th-order of approximation.
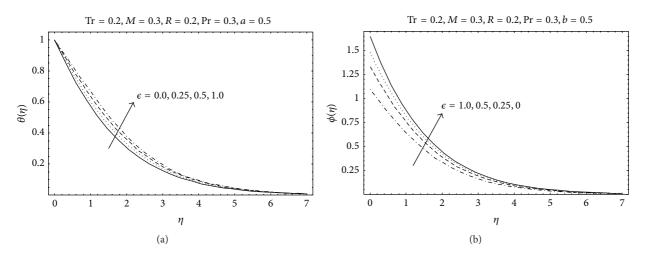

(a)

(b)

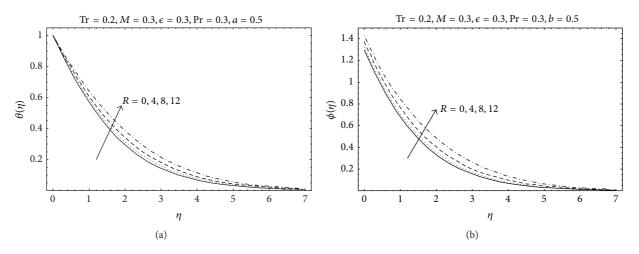FIGURE 2: Influence of thermal conductivity parameter ϵ on temperature profiles (a) for PST case and (b) for PHF case.

(a)

(b)

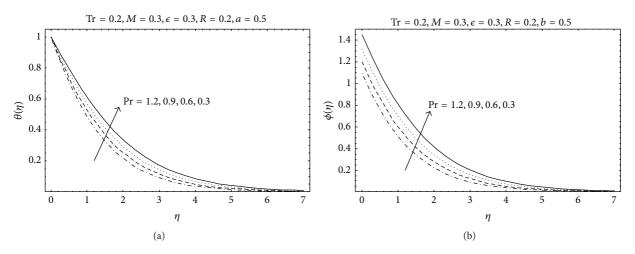FIGURE 3: Influence of porosity parameter R on temperature profiles (a) for PST case and (b) for PHF case.

$\mathrm{Tr} = 0.2, M = 0.3, \epsilon = 0.3, R = 0.2, a = 0.5$

$\mathrm{Tr} = 0.2, M = 0.3, \epsilon = 0.3, R = 0.2, b = 0.5$

Pr = 1.2, 0.9, 0.6, 0.3

Pr = 1.2, 0.9, 0.6, 0.3

(a)

(b)

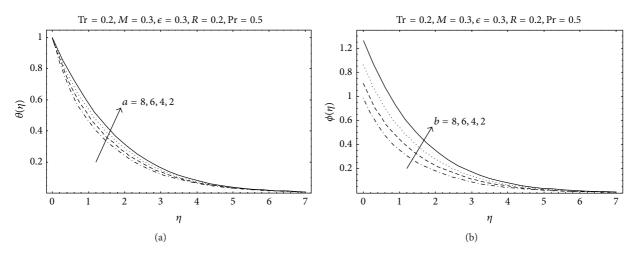FIGURE 4: Influence of Prandtl number Pr on temperature profiles (a) for PST case and (b) for PHF case.

$\mathrm{Tr} = 0.2, M = 0.3, \epsilon = 0.3, R = 0.2, \mathrm{Pr} = 0.5$

$\mathrm{Tr} = 0.2, M = 0.3, \epsilon = 0.3, R = 0.2, \mathrm{Pr} = 0.5$

$a = 8, 6, 4, 2$

$b = 8, 6, 4, 2$

(a)

(b)

FIGURE 5: Influence of parameters a and b on temperature profiles (a) for PST case and (b) for PHF case.

$a = 0.5, M = 0.3, \epsilon = 0.3, R = 0.2, \mathrm{Pr} = 0.5$

$b = 0.5, M = 0.3, \epsilon = 0.3, R = 0.2, \mathrm{Pr} = 0.5$

Tr = 0.5, 1.0, 1.5, 2.0
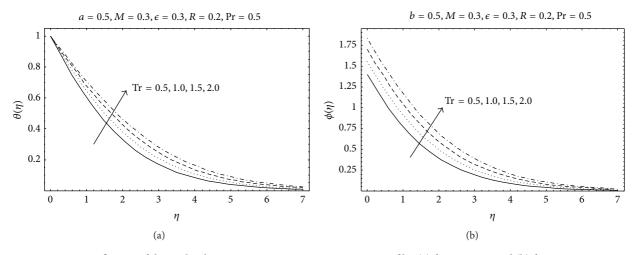
Tr = 0.5, 1.0, 1.5, 2.0

(a)

(b)

FIGURE 6: Influence of thermal radiation parameter Tr on temperature profiles (a) for PST case and (b) for PHF case.

porosity parameter and is given in Figure 4. Figure 5 is made to show the influence of parameter a in the PST case and parameter b in the PHF case on the temperature distributions. It is evident from Figure 5 that effects of these parameters are the same as that of Prandtl number. The influence of thermal radiation on the temperature profiles for both PST and PHF cases is depicted in Figure 6. The plotted profiles show that the temperature and thermal boundary-layer thickness is an increasing function of the thermal radiation.

5. Conclusions

Magnetohydrodynamic boundary-layer flow and heat transfer characteristic due to exponentially stretching sheet in the presence of thermal radiation, Darcy resistance, and variable thermal conductivity have been studied in the paper. The similarity solutions are developed using homotopy analysis method. Following are the findings of current study.

(1) The variable thermal conductivity has an impact in enhancing the temperatures profile in PST case and reduction in temperature in PHF case.

(2) The effect of porosity parameter R increases the temperature in both PST and PHF cases.

(3) The effect of increasing the values of Prandtl number is to decrease the thermal boundary-layer thickness in both of the cases.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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