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Research Article

Spatial Advertisement Competition: Based on Game Theory

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Since advertisement is an important strategy of firms to improve market share, this paper highlights duopoly advertisement under the Hotelling model. A model of advertisement under spatial duopoly is established, and corresponding effects of brand values and transportation costs are all captured. This study presents the proportion of sales revenue spending on advertisement. The condition for free-rider in advertisement investment is discussed. Under firms with the identical brand values, if firms' advertisement points to corresponding consumers, price and advertisement investment are all reduced. Therefore, advertisement is discussed under spatial competition in this work.

1. Introduction

Hotelling [1] initially established a model to offer a rational outlet for spatial competition. Furthermore, there exists extensive research on the Hotelling model in many aspects (see [2–5]). In a recent paper, Vogel [6] derived interesting results regarding product differentiation in the Hotelling model. Vogel found that a firm's price, market share, and profit were all independent of its neighbors' marginal costs, conditional on the average marginal cost in the market. Vogel also proved that more productive firms were more isolated, all else being equal. Nie [7, 8] addressed maintenance commitment under spatial competitions and characterized the relationship between competitions and guarantee commitment under the Hotelling model. Recently, Nie [9] addressed the effects of spatial competitions on the innovation.

In extant literature, the effects of transportation costs on economic activities are extensively captured. Rare papers discuss the effects of spatial competitions on advertisement. This study tries to fill in this gap. This paper aims to capture the effects of spatial competition on the advertisement and to discuss the relationship between the advertisement investment and the spatial competition by industrial organization theory.

Here the literature on advertisement is briefly introduced. Bagwell [10] surveyed the literature about advertisement. Bagwell [10] divided modern works about advertisement in three groups. The first group employs data sources and evaluates the empirical findings of the earlier empirical work. The second group focuses on new data and reflects the influence of the intervening theoretical work. The third group culls from the intervening theoretical work, which follows Sutton's interesting and significant work [11]. Baye and Morgan [12] recently developed exogenous advertisement theory. Bagwell and Lee [13] recently discussed the nonprice advertisement competitions in retail and argued that under free entry, social surplus is higher when advertising is allowed.

This work discusses duopoly advertisement under spatial competitions. The effects of brand values and transportation costs on advertisement investment are characterized. The proportion of sales revenue spending on advertisement is achieved. Since the effects of advertisement investment exist in the market, conditions for free-rider are discussed. The advertisement pointing to its consumers is also addressed.

This paper is organized as follows. The model of spatial duopoly with advertisement is established in the next section. The model is analyzed in Section 3. The equilibrium price and the equilibrium output are all outlined and discussed. Furthermore, the relationships between transportation costs and advertisement investment are considered in this section. Some remarks are given and some further research is discussed in the final section.

2. Model

The advertisement model under spatial duopoly is formally established here. Consumers are uniformly distributed in the linear city: $z \in [0,1]$. Two producers in the linear city with locations $z_1 = 0$ and $z_2 = 1$, producing products with quality differentiation, are introduced in this industry.

Consumers. Transportation costs are fully undertaken by consumers. Based on the prices p_1 and p_2 along with locations $z_1 = 0$ and $z_2 = 1$, the utility of consumer in z buying quantity q_i from firm i is

$$u(q_i, a_i) = [A + \theta_i + a_1 + a_2 - tD(z, z_i) - p_i]\, q_i - \frac{1}{2} q_i^2, \quad (1)$$

where $t > 0$ represents the transportation cost for a unit product and $D(z, z_i)$ denotes the distance between the firm i and this consumer, for $i = 1, 2$. A is assumed to be large enough such that the market is fully covered. The variable t heavily depends on transport technologies and other factors, such as management. The distance may be geographic, related to differences in beliefs, cultures, and so on. In some papers, Larralde et al. [14] used quadratic transportation costs. In general, the distance function $D(z, z_i)$ is convex. In this paper, we always use the distance function $D(z, z_1) = |z - z_1|$.

The consumer in z is inclined to buying the product of the first producer if and only if the following inequality holds:

$$\theta_1 - [p_1 + tD(z, z_1)] > \theta_2 - [p_2 + tD(z, z_2)]. \quad (2)$$

Otherwise, this consumer is likely to buy the second producer's product. We assume that marginal transportation costs are identical for the two firms, while Armstrong and Vickers (2009) employed the different marginal transportation costs.

The demand based on (2) is outlined. The solution of the following equation is denoted by z^*:

$$\theta_1 - p_1 - tz = \theta_2 - p_2 - t(1 - z). \quad (3)$$

The explicit solution to (3) is

$$z^* = \frac{\theta_1 - p_1 - (\theta_2 - p_2) + t}{2t}. \quad (4)$$

The demand is given as follows:

$$D_1(p_1, a_1) = \int_0^{z^*} (A + \theta_1 + a_1 + a_2 - p_1 - tz)\, dz,$$
$$D_2(p_2, a_2) = \int_{z^*}^{1} [A + \theta_2 + a_1 + a_2 - p_2 - t(1 - z)]\, dz. \quad (5)$$

According to (5), advertisement has strong externality. When there are many firms, free-riders may exist.

Firms. The constant marginal costs incurred by the two firms are all c_0. Given price p_1, brand value $\theta_1 > 0$, advertisement investment a_1, and quantity q_1, the first firm's net profit is outlined by the following:

$$\pi_1 = (p_1 - c_0)\, q_1 - a_1^2. \quad (6)$$

a_1^2 denotes the advertisement costs of the first firm. The first firm maximizes the above profit function by selecting price p_1 and advertisement investment a_1.

Similarly, a_2^2 denotes the advertisement costs of the second firm. Given price p_2, brand value $\theta_2 > 0$, advertisement investment a_2, and quantity q_2, the second firm maximizes the following profit function:

$$\pi_2 = (p_2 - c_0)\, q_2 - a_2^2. \quad (7)$$

The second firm maximizes the above profit function by adjusting price and advertisement investment.

In this paper, we always consider the linear city, and the corresponding conclusions can be extended to the Hotelling model with multiple dimensions. Furthermore, the distance function $D(z, z_1) = |z - z_1|$ is employed, and it can be extended to general cases. The linear transportation cost, which can be easily extended, is utilized to simplify the model.

3. Main Results

The above model is considered in this section. We first consider demand in the spatial duopoly and then the equilibrium.

3.1. Equilibrium. Based on market clearing conditions, the profits of two firms are given as follows

$$\begin{aligned}\pi_1 &= (p_1 - c_0) \int_0^{z^*} (A + \theta_1 + a_1 + a_2 - p_1 - tz)\, dz - a_1^2,\\ \pi_2 &= (p_2 - c_0) \int_{z^*}^{1} [A + \theta_2 + a_1 + a_2 - p_2 - t(1 - z)]\, dz \\ &\quad - a_2^2,\end{aligned} \quad (8)$$

where $a_1 \geq 0$ and $a_2 \geq 0$. If the equilibrium is strictly interior, the first order optimal conditions of (8) are

$$\begin{aligned}\frac{\partial \pi_1}{\partial p_1} = &\int_0^{z^*} (A + \theta_1 + a_1 + a_2 - p_1 - tz)\, dz \\ &- (p_1 - c_0) \int_0^{z^*} dz + (p_1 - c_0) \\ &\times (A + \theta_1 + a_1 + a_2 - p_1 - tz^*) \frac{\partial z^*}{\partial p_1}\end{aligned}$$

$$
\begin{aligned}
&= \int_0^{z^*} (A+\theta_1+a_1+a_2-p_1-tz)\,dz \\
&\quad - (p_1-c_0)\int_0^{z^*} dz \\
&\quad - \frac{(p_1-c_0)}{2t}(A+\theta_1+a_1+a_2-p_1-tz^*) \\
&= \int_0^{z^*} (A+\theta_1+a_1+a_2-p_1-tz)\,dz \\
&\quad - \frac{(p_1-c_0)}{2t}(A+\theta_1+a_1+a_2-p_1+tz^*) \\
&= \int_0^{z^*} (A+\theta_1+a_1+a_2-p_1-tz)\,dz \\
&\quad - \frac{(p_1-c_0)}{2t}(A+\theta_1+a_1+a_2-p_1+tz^*) \\
&= (A+\theta_1+a_1+a_2-p_1)z^* - \frac{t}{2}(z^*)^2 \\
&\quad - \frac{(p_1-c_0)}{2t}(A+\theta_1+a_1+a_2-p_1+tz^*) \\
&= \frac{1}{2t}(A+\theta_1+a_1+a_2-p_1) \\
&\quad \times(\theta_1-p_1-\theta_2+p_2+t) \\
&\quad - \frac{1}{8t}(\theta_1-p_1-\theta_2+p_2+t)^2 \\
&\quad - \frac{(p_1-c_0)}{2t}[A+1.5\theta_1+a_1+a_2-1.5p_1 \\
&\qquad\qquad -0.5-\theta_2+0.5p_2+0.5t) = 0,
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
\frac{\partial\pi_1}{\partial a_1} &= (p_1-c_0)\int_0^{z^*} dz + (p_1-c_0) \\
&\quad \times (A+\theta_1+a_1+a_2-p_1-tz^*)\frac{\partial z^*}{\partial a_1} - 2a_1 \\
&= (p_1-c_0)\int_0^{z^*} dz - 2a_1 \\
&= (p_1-c_0)z^* - 2a_1 \\
&= \frac{1}{2t}(p_1-c_0)(\theta_1-p_1-\theta_2+p_2+t) - 2a_1 = 0,
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
\frac{\partial\pi_2}{\partial p_2} &= \int_{z^*}^{1} [A+\theta_2+a_1+a_2-p_2-t(1-z)]\,dz \\
&\quad - (p_2-c_0)\int_{z^*}^{1} dz - (p_2-c_0) \\
&\quad \times [A+\theta_2+a_1+a_2-p_2-t(1-z^*)]\frac{\partial z}{\partial p_2} \\
&= \int_{z^*}^{1} [A+\theta_2+a_1+a_2-p_2-t(1-z)]\,dz \\
&\quad - \frac{(p_2-c_0)}{2t}[A+\theta_2+a_1+a_2-p_2+t(1-z^*)] \\
&= (A+\theta_2+a_1+a_2-p_2-t)(1-z^*) \\
&\quad + \frac{t}{2}\left[1-(z^*)^2\right] \\
&\quad - \frac{(p_2-c_0)}{2t}[A+\theta_2+a_1+a_2-p_2+t(1-z^*)] \\
&= \frac{1}{2t}(A+\theta_2+a_1+a_2-p_2-t) \\
&\quad \times(-\theta_1+p_1+\theta_2-p_2+t) \\
&\quad + \frac{t}{2}\left[1-\frac{1}{4t^2}(\theta_1-p_1-\theta_2+p_2+t)^2\right] \\
&\quad - \frac{(p_2-c_0)}{2t}(A+1.5\theta_2+a_1+a_2-1.5p_2 \\
&\qquad\qquad -0.5\theta_1+0.5p_1+0.5t) = 0,
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
\frac{\partial\pi_2}{\partial a_2} &= (p_2-c_0)\int_{z^*}^{1} dz - (p_2-c_0) \\
&\quad \times [A+\theta_2+a_1+a_2-p_2-t(1-z^*)]\frac{\partial z}{\partial a_2} - 2a_2 \\
&= (p_2-c_0)(1-z^*) - 2a_2 \\
&= \frac{1}{2t}(p_2-c_0)(\theta_1-p_1-\theta_2+p_2+t) - 2a_2 = 0.
\end{aligned}
\tag{12}
$$

The equilibrium is determined by (9)–(12). Firstly, the existence of (6) and (7) is addressed. By second order differentiations, we have the following.

Lemma 1. *Equation* (6) *is concave in* p_1*, and* (7) *is concave in* p_2*. Moreover,* (6) *is concave in* a_1*, and* (7) *is concave in* a_2*.*

Proof. See the proof in Appendix. □

Remark 2. Lemma 1 indicates the existence and uniqueness of the above model.

Denote the equilibrium price to be $p^* = (p_1^*, p_2^*)$, the equilibrium advertisement investment to be $a^* = (a_1^*, a_2^*)$, and the corresponding profits to be $\pi^* = (\pi_1^*, \pi_2^*)$. The properties at the equilibrium are further discussed. By comparative static analysis to brand value, we have the relation $\partial p_i^*/\partial\theta_i > 0$, $\partial a_i^*/\partial\theta_i > 0$, $\partial p_i^*/\partial\theta_j < 0$, and $\partial a_i^*/\partial\theta_j < 0$, for $i, j = 1, 2$ and $i \neq j$. Based on implicit function theorem, (9) and (11) indicate $\partial p_i^*/\partial\theta_i > 0$ and $\partial p_i^*/\partial\theta_j < 0$. Equations (10) and (12) yield $\partial a_i^*/\partial\theta_i > 0$ and $\partial a_i^*/\partial\theta_j < 0$. This is summarized as follows.

Proposition 3. *Bigger brand yields both higher price and more advertisement investment of the corresponding firms. Bigger brand causes rival's both less advertisement investment and lower price.*

Remark 4. A firm with bigger brand owns more market power, which reduces the competition in this industry.

Equation (10) yields $a_1 = (1/4t)(p_1-c_0)(\theta_1-p_1-\theta_2+p_2)+(1/4)(p_1-c_0)$. Equation (12) manifests $a_2 = (1/4t)(p_2-c_0)(\theta_1-p_1-\theta_2+p_2)+(1/4t)(p_2-c_0)$. Therefore, for transportation costs, we have $\partial a_i^*/\partial t < 0$ if $\theta_i-p_i-(\theta_j-p_j) > 0$ and $\partial a_i^*/\partial t > 0$ if $\theta_i - p_i - (\theta_j - p_j) < 0$, for $i, j = 1, 2$ and $i \neq j$. This is summarized as follows.

Proposition 5. *Advertisement investment is increased in transportation costs for firms sharing less market size and decreased in transportation costs for firms sharing more market size.*

Remark 6. Transportation costs reduce the competitions of advertisement. Actually, transportation costs have deterring effects on firm's competition and these deterring effects cause the above conclusion.

By envelop theorem, for profit functions, we further have $\partial\pi_i/\partial\theta_i > 0$ and $\partial\pi_i/\partial\theta_j < 0$, for $i, j = 1, 2$ and $i \neq j$.

Proposition 7. *A firm benefits from its brand. Improved brand value damages rival's benefits.*

Remark 8. By brand promotion, firms improve the market share and earn more profits. Brand maintains the monopolistic power and damages rival's profits.

We further discuss the first order optimal conditions. For $i = 1, 2$, we further define price elasticity of demand and advertisement elasticity of demand at the equilibrium as follows:

$$\varepsilon_{1,p} = \frac{-p_1(\partial D_1/\partial p_1)}{D_1} = \frac{p_1\left[-(1/2t)\left(A+\theta_1+a_1+a_2-p_1+tz^*\right)\right]}{\int_0^{z^*}\left(A+\theta_1+a_1+a_2-p_1-tz\right)dz}, \tag{13}$$

$$\varepsilon_{2,p} = \frac{-p_2(\partial D_2/\partial p_2)}{D_2} = \frac{p_2\left[-(1/2t)\left(A+\theta_2+a_1+a_2-p_2+t-tz^*\right)\right]}{\int_{z^*}^{1}\left[A+\theta_2+a_1+a_2-p_2-t(1-z)\right]dz}, \tag{14}$$

$$\varepsilon_{1,a} = \frac{a_1(\partial D_1/\partial a_1)}{D_1} = \frac{a_1\int_0^{z^*}dz}{\int_0^{z^*}\left(A+\theta_1+a_1+a_2-p_1-tz\right)dz}, \tag{15}$$

$$\varepsilon_{2,a} = \frac{a_2(\partial D_2/\partial a_2)}{D_2} = \frac{a_2\int_{z^*}^{1}dz}{\int_{z^*}^{1}\left[A+\theta_2+a_1+a_2-p_2-t(1-z)\right]dz}. \tag{16}$$

Equations (9), (10), (13), and (15) jointly yield

$$\frac{a_1^2}{p_1D_1} = \frac{1}{2}\frac{\varepsilon_{1,a}}{\varepsilon_{1,p}}. \tag{17}$$

Equations (11), (12), (14), and (16) manifest

$$\frac{a_2^2}{p_2D_2} = \frac{1}{2}\frac{\varepsilon_{2,a}}{\varepsilon_{2,p}}. \tag{18}$$

Equations (17) and (18) mean that the proportion of sales revenue spending on advertisement is determined by a simple elasticity ratio under equilibrium.

Proposition 9. *The proportion of sales revenue spending on advertisement is half of a simple elasticity ratio under equilibrium.*

Remark 10. This interesting conclusion is consistent with advertisement investment, such as that of Bagwell [10]. Because of the quadratic cost function of advertisement, compared with that of Bagwell [10], there is a constant $1/2$.

Free-rider is here discussed. From (10) and (12), we have $\partial\pi_i/\partial a_i < 0$, for $a_i \leq 0$. Moreover, $\lim_{q_i^*\to 0} a_i^* = 0$. This is summarized as follows.

Proposition 11. *A firm with little market size has intention of free-riding in advertisement.*

Remark 12. The above conclusion illustrates that little firms may act as free-riders, which is consistent with reality. Under positive externality of advertisement, little firms have intention of free-riding.

3.2. Further Discussion. Consider that two firms have the same brand values or $\theta_1 = \theta_2$. Here we extend the model to cases in which advertisement has no effects on rival's demand. The profit functions are correspondingly outlined:

$$\begin{aligned}\tilde{\pi}_1 &= (p_1-c_0)\int_0^{z^*}\left(A+\theta_1+a_1-p_1-tz\right)dz - a_1^2,\\ \tilde{\pi}_2 &= (p_2-c_0)\\ &\quad\times\int_{z^*}^{1}\left[A+\theta_2+a_2-p_2-t(1-z)\right]dz - a_2^2.\end{aligned} \tag{19}$$

Denote the equilibrium price to be $\overline{p} = (\overline{p}_1, \overline{p}_2)$, the equilibrium advertisement investment to be $\overline{a} = (\overline{a}_1, \overline{a}_2)$, and the corresponding profits to be $\overline{\pi} = (\overline{\pi}_1, \overline{\pi}_2)$. By symmetry, we have $p_1^* = p_2^*$ and $\overline{p}_1 = \overline{p}_2$. Equations (9) and (11) indicate $p_1^* < c_0 + t$ and $p_2^* < c_0 + t$. By virtue of (9) and (11), we have $(\partial\tilde{\pi}_1/\partial p_1)|_{p_1=p_1^*} < 0$ and $(\partial\tilde{\pi}_2/\partial p_2)|_{p_2=p_2^*} < 0$. From the concavity of profit functions, we therefore have $\overline{p}_1 < p_1^*$ and $\overline{p}_2 < p_2^*$. Moreover, according to (10) and (12), we immediately have $\overline{a}_1 < a_1^*$ and $\overline{a}_2 < a_2^*$. This is summarized as follows.

Proposition 13. *If the advertisement investment of each firm points to the corresponding customers, the advertisement investment and the price are all lower than those in Section 3.1.*

Remark 14. If the advertisement investment points to the corresponding customers, firms lower price and launch less advertisement investment. The competitions both in price and advertisement are all fallen off.

4. Concluding Remarks

Duopoly advertisement in a spatial situation in this work is addressed. Under spatial competitions, this study characterizes the relationship between advertisement and transportation costs and brand values. Transportation costs reduce the advertisement investment for firms sharing larger market size, while improve the advertisement investment for firms sharing less market size. By virtue of spillover effects of advertisement, condition for free-rider is addressed. We also argue that the advertisement pointing to customers yields lower price and lower advertisement.

The economic implication of the above conclusions lies in the following. Firstly, reduction of the transportation can efficiently improve the competition. Without spillover effects, the advertisement is reduced. It is interesting to encourage joint advertising.

This is just the beginning of research on spatial advertisement competitions. Since there are many types of advertisements, it is interesting to consider other types of advertisements.

Appendix

Proof of Lemma 1. Equation (9) yields

$$\begin{aligned}\frac{\partial^2\pi_1}{\partial(p_1)^2} &= -z^* - \frac{1}{2t}\left(A+\theta_1+a_1+a_2-p_1\right) \\ &\quad + z^* - \frac{1}{2t}\left(A+\theta_1+a_1+a_2-p_1+tz^*\right) \\ &\quad + \frac{3\left(p_1-c_0\right)}{4t} \\ &= -\frac{1}{2t}\left(A+\theta_1+a_1+a_2-p_1\right) \\ &\quad - \frac{1}{2t}\left(A+\theta_1+a_1+a_2-p_1+tz^*\right) \\ &\quad + \frac{3\left(p_1-c_0\right)}{4t} < 0.\end{aligned} \tag{A.1}$$

Therefore, (1) is concave in p_1 because A is large enough. Equation (10) indicates $\partial^2\pi_1/\partial(a_1)^2 = -2 < 0$. Equation (1) is concave in p_1. Moreover, $\partial^2\pi_1/\partial a_1\partial p_1 = z^* - (p_1 - c_0)/2t$. Equation (1) is concave because the parameter A is large enough. Similar conclusion holds for (2). Conclusion is achieved and the proof is complete. □

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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