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Research Article

Some Retarded Difference Inequalities of Product Form and Their Application

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A class of new nonlinear retarded difference inequalities is established. An application of the obtained inequalities to the estimation of finite difference equations is given.

1. Introduction

Difference inequalities which give explicit bounds on unknown functions provide a very useful and important tool in the study of many qualitative as well as quantitative properties of solutions of nonlinear difference equations. Various investigators have discovered many useful and new difference inequalities, mainly inspired by their applications in various branches of difference equations; see [1–25] and the references cited therein.

Sugiyama [2] established the most precise and complete discrete analogue of the Gronwall inequality (see [1]) in the following form.

Let $u(n)$ and $f(n)$ be nonnegative functions defined for $n \in \mathbf{N}$, and suppose that $f(n) \geq 0$ for every $n \in \mathbf{N}$. If

$$u(n) \leq u_0 + \sum_{s=n_0}^{n-1} f(s)\, u(s), \quad n \in \mathbf{N}, \tag{1}$$

where $\mathbf{N}$ is the set of points $n_0 + k$ $(k = 0, 1, 2, \ldots)$, $n_0 \geq 0$ is a given integer, and u_0 is a nonnegative constant, then

$$u(n) \leq u_0 \prod_{s=n_0}^{n-1} [1 + f(s)], \quad n \in \mathbf{N}. \tag{2}$$

Pachpatte [4] established a generalized discrete analogue of the Gronwall inequality in the following form.

Let $m(s)$ be a positive and monotone nondecreasing function on $\mathbf{N}$, and let $a(s)$, $b(s)$ be nonnegative functions on $\mathbf{N}$. If $u(n)$ satisfies

$$u(n) \leq m(s) + \sum_{s=n_0}^{n-1} a(s) \left(u(s) + \sum_{\tau=n_0}^{s-1} b(\tau)\, u(\tau) \right), \quad \forall n \in \mathbf{N}, \tag{3}$$

then

$$u(n) \leq P(n)\, m(s), \quad \forall n \in \mathbf{N}, \tag{4}$$

where

$$P(n) = 1 + \sum_{s=n_0}^{n-1} a(s) \prod_{\tau=n_0}^{s-1} [1 + a(\tau) + b(\tau)], \quad \forall n \in \mathbf{N}. \tag{5}$$

Besides the results mentioned above, the following results are closely related to the investigation of the present paper, and, particularly, they will be used as lemmas for the proofs of our main results in Theorems 3 and 4.

Lemma 1 (see [5]). *Let $u(n)$, $a(n)$, $b(n)$, $c(n)$, and $d(n)$ be nonnegative functions defined on $\mathbf{N}$, for which the inequality*

$$u(n) \leq u_0 + \sum_{s=n_0}^{n-1} a(s)\, u(s) + \sum_{s=n_0}^{n-1} b(s) \times \left(\sum_{t=n_0}^{s-1} c(t) \left(\sum_{\tau=n_0}^{t-1} d(\tau)\, u^{\alpha}(\tau) \right) \right), \quad \forall n \in \mathbf{N}, \tag{6}$$

holds, where u_0 is a nonnegative constant and $0 < \alpha < 1$. If $1 + a(n) - b(n) \geq 0$ and $1 + a(n) + b(n) - c(n) \geq 0$ for all $n \in \mathbf{N}$, then

$$u(n) \leq u_0 \prod_{s=n_0}^{n-1} [1 + a(s) - b(s)] + \sum_{s=n_0}^{n-1} b(s) \prod_{t=s+1}^{n-1} [1 + a(t) - b(t)] \times \left\{ u_0 \prod_{t=n_0}^{s-1} [1 + a(t) + b(t) - c(t)] + \sum_{t=n_0}^{s-1} c(t) \prod_{\tau=t+1}^{s-1} [1 + a(\tau) + b(\tau) - c(\tau)] \times \prod_{\tau=n_0}^{t-1} [1 + a(\tau) + b(\tau) + c(\tau)] \times \left[u_0^{1-\alpha} + (1-\alpha) \sum_{\tau=n_0}^{t-1} d(\tau) \times \prod_{\rho=n_0}^{\tau} [1 + a(\rho) + b(\rho) + c(\rho)]^{\alpha-1} \right]^{1/(1-\alpha)} \right\}, \quad \forall n \in \mathbf{N}. \tag{7}$$

Lemma 2 (see [3, 7]). *Let $w(n,r)$ be a real-valued function defined for $n \in \mathbf{N}, 0 \leq r < \infty$ and monotone nondecreasing with respect to r for any fixed $n \in \mathbf{N}$. Let $u(n)$ be a real-valued function defined for $n \in \mathbf{N}$ such that*

$$\Delta u(n) \leq w(n, u(n)), \quad \forall n \in \mathbf{N}. \tag{8}$$

Let $r(n)$ be a solution of

$$\Delta r(n) = w(n, r(n)), \quad r(n_0) = r_0, \quad \forall n \in \mathbf{N}, \tag{9}$$

such that $u(n_0) \leq r(n_0)$. Then

$$u(n) \leq r(n), \quad \forall n \in \mathbf{N}. \tag{10}$$

Pachpatte [7, 8] also established some difference inequalities of product form as follows.

Let u, a, and b be nonnegative functions defined on $\mathbf{N}$ and let c be a nonnegative constant. Let $w(n,r)$ be a nonnegative function defined for $n \in \mathbf{N}$, $0 \leq r < \infty$ and monotone nondecreasing with respect to r for any fixed $n \in \mathbf{N}$. If $u(n)$ satisfies

$$u^2(n) \leq c^2 + 2 \sum_{s=n_0}^{n-1} u(s) \times \left[a(s) \left(u(s) + \sum_{t=n_0}^{s-1} b(t) u(t) \right) + w(s, u(s)) \right], \quad \forall n \in \mathbf{N}, \tag{11}$$

then

$$u(n) \leq P(n) r(n), \quad \forall n \in \mathbf{N}, \tag{12}$$

where $P(n)$ is defined by (5) and $r(n)$ is a solution of

$$\Delta r(n) = w(n, P(n) r(n)), \quad r(n_0) = c, \quad \forall n \in \mathbf{N}. \tag{13}$$

Let u, a, and b be nonnegative functions defined for $n \in \mathbf{N}$ and let c be a nonnegative constant. Let $w(n,r)$ be a nonnegative function defined for $n \in \mathbf{N}$, $0 \leq r < \infty$ and monotone nondecreasing with respect to r for any fixed $n \in \mathbf{N}$. If $u(n)$ satisfies

$$u^2(n) \leq c^2 + \sum_{s=n_0}^{n-1} a(s)(u(s+1) + u(s)) \times \left[\left(u(s) + \sum_{\tau=n_0}^{s-1} b(\tau) u(\tau) \right) + w(s, u(s)) \right], \quad \forall n \in \mathbf{N}, \tag{14}$$

then

$$u(n) \leq P(n) r(n), \quad \forall n \in \mathbf{N}, \tag{15}$$

where $P(n)$ is defined by (5) and $r(n)$ is a solution of the difference equation

$$\Delta r(n) = a(n) w(n, P(n) r(n)), \quad r(n_0) = c, \quad \forall n \in \mathbf{N}. \tag{16}$$

Motivated by the results given in [5, 7, 8], in this paper, we discuss new nonlinear difference inequalities:

$$u^2(n) \leq c^2 + \sum_{s=n_0}^{n-1} f(s)(u(s+1) + u(s)) \times \left[\left(u(s) + \sum_{t=n_0}^{s-1} g(t) \sum_{\tau=n_0}^{t-1} h(\tau) u^{\alpha}(\tau) \right) + w(s, u(s)) \right], \quad 0 < \alpha < 1, \quad \forall n \in \mathbf{N}. \tag{17}$$

It is important to note that the inequality given above can be used as tools in the study of certain classes of finite difference equations. In Section 3 we provide an application of our results to the estimation of finite difference equations.

2. Main Results

Throughout this paper, let $\mathbf{N} := \{n_0, n_0+1, n_0+2, \ldots\}$ and $\mathbf{N}_T := \{n_0, n_0+1, n_0+2, \ldots, T\}$, $T \in \mathbf{N}$. For function $u(n)$, $n \in \mathbf{N}$, we define the operator Δ by $\Delta u(n) = u(n+1) - u(n)$. Obviously, the linear difference equation $\Delta u(n) = f(n)$ with the initial condition $u(n_0) = 0$ has the solution $u(n) = \sum_{s=n_0}^{n-1} f(s)$. For convenience, in the sequel we complementarily define that $\sum_{s=n_0}^{n_0-1} f(s) = 0$ and $\prod_{s=n_0}^{n_0-1} f(s) = 1$.

Theorem 3. *Let $m(s)$ be a nonnegative and monotone nondecreasing function defined on $\mathbf{N}$, and let $f(s)$, $g(s)$, and $h(s)$ be nonnegative functions defined on $\mathbf{N}_0$. Let α be a constant with $0 < \alpha < 1$. If $u(n)$ satisfies*

$$u(n) \le m(n) + \sum_{s=n_0}^{n-1} f(s) u(s) + \sum_{s=n_0}^{n-1} f(s) \times \sum_{t=n_0}^{s-1} g(t) \left(\sum_{\tau=n_0}^{t-1} h(\tau) u^{\alpha}(\tau) \right), \quad \forall n \in \mathbf{N}, \tag{18}$$

then

$$u(n) \le W_1(m(n), n), \quad \forall n \in \mathbf{N}, \tag{19}$$

where

$$\begin{aligned} W_1(m(n), n) := m(n) + \sum_{s=n_0}^{n-1} f(s) \times \Bigg\{ & m(n) \prod_{t=n_0}^{s-1} [1 + 2f(t) - g(t)] + \sum_{t=n_0}^{s-1} g(t) \prod_{\tau=t+1}^{s-1} [1 + 2f(\tau) - g(\tau)] \\ & \times \prod_{\tau=n_0}^{t-1} [1 + 2f(\tau) + g(\tau)] \times \Bigg[m^{1-\alpha}(n) + (1-\alpha) \sum_{\tau=n_0}^{t-1} h(\tau) \\ & \times \prod_{\rho=n_0}^{\tau} [1 + 2f(\rho) + g(\rho)]^{\alpha-1} \Bigg]^{1/(1-\alpha)} \Bigg\}, \quad \forall n \in \mathbf{N}. \end{aligned} \tag{20}$$

Proof. Fix $T \in \mathbf{N}$, where T is chosen arbitrarily; since $m(t)$ is a nonnegative and monotone nondecreasing function, from (18), we have

$$u(n) \le m(T) + \sum_{s=n_0}^{n-1} f(s) u(s) + \sum_{s=n_0}^{n-1} f(s) \sum_{t=n_0}^{s-1} g(t) \left(\sum_{\tau=n_0}^{t-1} h(\tau) u^{\alpha}(\tau) \right), \quad \forall n \in \mathbf{N}_T. \tag{21}$$

Now an application of Lemma 1 to (21) yields

$$\begin{aligned} u(n) \le m(T) + \sum_{s=n_0}^{n-1} f(s) \times \Bigg\{ & m(T) \prod_{t=n_0}^{s-1} [1 + 2f(t) - g(t)] + \sum_{t=n_0}^{s-1} g(t) \prod_{\tau=t+1}^{s-1} [1 + 2f(\tau) - g(\tau)] \\ & \times \prod_{\tau=n_0}^{t-1} [1 + 2f(\tau) + g(\tau)] \times \Bigg[m^{1-\alpha}(T) + (1-\alpha) \sum_{\tau=n_0}^{t-1} h(\tau) \\ & \times \prod_{\rho=n_0}^{\tau} [1 + 2f(\rho) + g(\rho)]^{\alpha-1} \Bigg]^{1/(1-\alpha)} \Bigg\}, \quad \forall n \in \mathbf{N}_T. \end{aligned} \tag{22}$$

Since $T \in \mathbf{N}$ is arbitrary, from (22), we get the required estimate (19). □

Theorem 4. *Let u, f, g, and h be nonnegative functions defined for $n \in \mathbf{N}$ and let c be a nonnegative constant. Let $w(n, r)$ be a real-valued function defined for $n \in \mathbf{N}$, $0 \le r < \infty$ and monotone nondecreasing with respect to r for any fixed $n \in \mathbf{N}$. Let α be a constant with $0 < \alpha < 1$. If $u(n)$ satisfies (17), then*

$$u(n) \le W_1(v(n), n), \quad \forall n \in \mathbf{N}, \tag{23}$$

where $W_1(v(n), n)$ is defined by (20) in Theorem 3 and $v(n)$ is a solution of the difference equation

$$\Delta r(n) = f(n) w(n, W_1(r(n), n)), \qquad r(n_0) = c, \quad \forall n \in \mathbf{N}. \tag{24}$$

Proof. We first assume that $c > 0$ and define a function $z(n)$ by the right-hand side of (17). Then $z(n)$ is a nonnegative and monotone nondecreasing function defined on $\mathbf{N}_0$. We have

$$z(n_0) = c^2, \quad u(n) \le \sqrt{z(n)}, \quad \forall n \in \mathbf{N}. \tag{25}$$

Using the definitions of the operator Δ and z, we obtain

$$\begin{aligned}\Delta z(n) &= f(n)(u(n+1)+u(n)) \\ &\quad \times\left[\left(u(n)+\sum_{t=n_0}^{n-1} g(t)\sum_{\tau=n_0}^{t-1} h(\tau)u^{\alpha}(\tau)\right)+w(n,u(n))\right] \\ &\le f(n)\left(\sqrt{z(n+1)}+\sqrt{z(n)}\right) \\ &\quad \times\left[\left(\sqrt{z(n)}+\sum_{t=n_0}^{n-1} g(t)\sum_{\tau=n_0}^{t-1} h(\tau)\left(\sqrt{z(\tau)}\right)^{\alpha}\right)\right. \\ &\quad \left.+w\left(n,\sqrt{z(n)}\right)\right], \quad \forall n\in\mathbf{N}.\end{aligned} \tag{26}$$

From (26), we have

$$\begin{aligned}\Delta\left(\sqrt{z(n)}\right) &= \frac{\Delta z(n)}{\sqrt{z(n+1)}+\sqrt{z(n)}} \\ &\le f(n)\left[\left(\sqrt{z(n)}+\sum_{t=n_0}^{n-1} g(t)\right.\right. \\ &\quad \left.\left.\times\sum_{\tau=n_0}^{t-1} h(\tau)\left(\sqrt{z(\tau)}\right)^{\alpha}\right)+w\left(n,\sqrt{z(n)}\right)\right], \\ &\quad \forall n\in\mathbf{N}.\end{aligned} \tag{27}$$

Setting $n = s$ in (27) and substituting $s = n_0, n_0+1, n_0+2, \ldots, n-1$, successively, we get

$$\begin{aligned}\sqrt{z(n)} &\le c+\sum_{s=n_0}^{n-1} f(s) \\ &\quad \times\left[\left(\sqrt{z(s)}+\sum_{t=n_0}^{s-1} g(t)\sum_{\tau=n_0}^{t-1} h(\tau)\left(\sqrt{z(\tau)}\right)^{\alpha}\right)\right. \\ &\quad \left.+w\left(s,\sqrt{z(s)}\right)\right], \quad \forall n\in\mathbf{N}.\end{aligned} \tag{28}$$

Define a function $z_1(n)$ by

$$z_1(n) = c+\sum_{s=n_0}^{n-1} f(s)\,w\left(s,\sqrt{z(s)}\right), \quad \forall n\in\mathbf{N}. \tag{29}$$

Then $z_1(n_0) = c$ and

$$\Delta z_1(n) = f(n)\,w\left(n,\sqrt{z(n)}\right), \quad \forall n\in\mathbf{N}. \tag{30}$$

Using (29), inequality (28) can be written as

$$\begin{aligned}\sqrt{z(n)} &\le z_1(n) \\ &\quad +\sum_{s=n_0}^{n-1} f(s)\left(\sqrt{z(s)}+\sum_{t=n_0}^{s-1} g(t)\sum_{\tau=n_0}^{t-1} h(\tau)\right. \\ &\quad \left.\times\left(\sqrt{z(\tau)}\right)^{\alpha}\right), \quad \forall n\in\mathbf{N},\end{aligned} \tag{31}$$

since $z_1(n)$ is positive and monotone nondecreasing for $n\in\mathbf{N}$ and $f(s)$, $g(s)$, and $h(s)$ satisfy the conditions in Theorem 3. Now an application of Theorem 3 to (31) yields

$$\sqrt{z(n)} \le W_1(z_1(n), n), \quad \forall n\in\mathbf{N}, \tag{32}$$

where $W_1(z_1(n), n)$ is defined by (20) in Theorem 3. Since $w(n,r)$ is monotone nondecreasing with respect to r for any fixed $n\in\mathbf{N}$, from (30) and (32), we have

$$\Delta z_1(n) \le f(n)\,w\left(n, W_1(z_1(n), n)\right), \quad \forall n\in\mathbf{N}. \tag{33}$$

Now with a suitable application of Lemma 2, we obtain

$$z_1(n) \le v(n), \quad \forall n\in\mathbf{N}, \tag{34}$$

where $v(n)$ is a solution of (24). Using (25), (32), and (34), we obtain our required estimation (23). □

If c is nonnegative, we can carry out the above procedure with $c+\epsilon$ instead of c, where ϵ is an arbitrary small number. Letting $\epsilon \to 0$, we obtain (23).

3. Application to Finite Difference Equations

In this section, we apply our result to the following difference equation:

$$\begin{aligned}\Delta x(n) &= f(n) \\ &\quad \times\left[F\left(n, x(n), \sum_{t=n_0}^{n-1} g(t)\sum_{\tau=n_0}^{t-1} H(t,\tau,x(\tau))\right)\right. \\ &\quad \left.+K(n,x(n))\right], \quad \forall n\in\mathbf{N},\end{aligned} \tag{35}$$

where K, H, and F are real-valued functions defined, respectively, on $\mathbf{N}\times\mathbf{R}$, $\mathbf{N}^2\times\mathbf{R}$, and $\mathbf{N}\times\mathbf{R}^2$ and f is as defined in Theorem 4. We assume that

$$\begin{aligned}|K(n,x(n))| &\le w(n,|x(n)|), \\ |H(n,t,x(t))| &\le \sum_{t=n_0}^{n-1} h(t)\,|x(t)|^{\alpha}, \\ |F(n,x(n),y(n))| &\le |x(n)|+|y(n)|,\end{aligned} \tag{36}$$

where g, h, w, and α are as defined in Theorem 4. From (35), we have

$$x^2(n+1) - x^2(n) = f(n)[x(n+1) + x(n)] \times \left[F\left(n, x(n), \sum_{t=n_0}^{n-1} g(t) \sum_{\tau=n_0}^{t-1} H(t,\tau,x(\tau))\right) + K(n,x(n))\right], \quad \forall n \in \mathbf{N}. \tag{37}$$

From (37), we have

$$x^2(n) = x^2(n_0) + \sum_{s=n_0}^{n-1} f(s)[x(s+1) + x(s)] \times \left[F\left(s, x(s), \sum_{t=n_0}^{s-1} g(t) \times \sum_{\tau=n_0}^{t-1} H(t,\tau,x(\tau))\right) + K(s,x(s))\right], \quad \forall n \in \mathbf{N}. \tag{38}$$

Using conditions (36), we obtain

$$|x(n)|^2 = x^2(n_0) + \sum_{s=n_0}^{n-1} f(s)[|x(s+1)| + |x(s)|] \times \left[|x(s)| + \sum_{t=n_0}^{s-1} g(t) \sum_{\tau=n_0}^{t-1} h(\tau)|x(\tau)|^{\alpha} + w(s,|x(s)|)\right], \quad \forall n \in \mathbf{N}. \tag{39}$$

Now an application of Theorem 4 to (39) yields the estimation of the difference equation (35) as follows:

$$|x(n)| \le v(n) + \sum_{s=n_0}^{n-1} f(s) \left\{ v(n) \prod_{t=n_0}^{s-1} [1 + 2f(t) - g(t)] + \sum_{t=n_0}^{s-1} g(t) \prod_{\tau=t+1}^{s-1} [1 + 2f(\tau) - g(\tau)] \times \prod_{\tau=n_0}^{t-1} [1 + 2f(\tau) + g(\tau)] \times \left[v^{1-\alpha}(n) + (1-\alpha) \sum_{\tau=n_0}^{t-1} h(\tau) \times \prod_{\rho=n_0}^{\tau} [1 + 2f(\rho) + g(\rho)]^{\alpha-1} \right]^{1/(1-\alpha)} \right\}, \quad \forall n \in \mathbf{N}, \tag{40}$$

where $v(n)$ is a solution of the difference equation

$$\Delta r(n) = f(n) w(n, W_1(r(n), n)), \qquad r(n_0) = |x(n_0)|, \quad \forall n \in \mathbf{N}. \tag{41}$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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