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Research Article

An Improvement of the Differential Transformation Method and Its Application for Boundary Layer Flow of a Nanofluid

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The main feature of the boundary layer flow problems of nanofluids or classical fluids is the inclusion of the boundary conditions at infinity. Such boundary conditions cause difficulties for any of the series methods when applied to solve such a kind of problems. In order to solve these difficulties, the authors usually resort to either Padé approximants or the commercial numerical codes. However, an intensive work is needed to perform the calculations using Padé technique. Due to the importance of the nanofluids flow as a growing field of research and the difficulties caused by using Padé approximants to solve such problems, a suggestion is proposed in this paper to map the semi-infinite domain into a finite one by the help of a transformation. Accordingly, the differential equations governing the fluid flow are transformed into singular differential equations with classical boundary conditions which can be directly solved by using the differential transformation method. The numerical results obtained by using the proposed technique are compared with the available exact solutions, where excellent accuracy is found. The main advantage of the present technique is the complete avoidance of using Padé approximants to treat the infinity boundary conditions.

1. Introduction

Nanotechnology is an advanced technology, which deals with the synthesis of nanoparticles, processing of the nano materials and their applications. It is well known that 1 nm (nanometer) = 10^{-9} meter. Normally, if the particle sizes are in the 1–100 nm range, they are generally called nanoparticles. Nanotechnology has been widely used in industry since materials with sizes of nanometers possess unique physical and chemical properties. Nanoscale particle added fluids are called as nanofluid. The term "nanofluid" was first used by Choi [1] to describe a fluid in which nanometer-sized particles are suspended in conventional heat transfer basic fluids. Fluids such as oil, water, and ethylene glycol mixture are poor heat transfer fluids, since the thermal conductivity of these fluids plays important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface. Numerous methods have been taken to improve the thermal conductivity of these fluids by suspending nano/micro or larger-sized particle materials in liquids. An innovative technique to improve heat transfer is by using nanoscale particles in the base fluid [1]. Therefore, the effective thermal conductivity of nanofluids is expected to enhance heat transfer compared with conventional heat transfer liquids (Masuda et al. [2]). This phenomenon suggests the possibility of using nanofluids in advanced nuclear systems (Buongiorno and Hu [3]). Choi et al. [4] showed that the addition of a small amount (less than 1% by volume) of nanoparticles to conventional heat transfer liquids increased the thermal conductivity of the fluid up to approximately two times. A comprehensive survey of convective transport in nanofluids was made by Buongiorno and Hu [3] and very recently by Kakaç and Pramuanjaroenkij [5]. It may be also important to mention that a valuable book in nanofluids is published recently by Das et al. [6]. In addition, various interesting results in this regard can be found in [7–17].

Khan and Pop [18] were the first to investigate the boundary-layer flow of a nanofluid past a stretching sheet. The main feature of the boundary layer flow of nanofluids or classical fluid is the inclusion of the boundary conditions at infinity. Such conditions cause difficulties for any of the series methods when applied to solve this kind of problems. This because the infinity boundary condition cannot be applied directly to the series solution, where Padé approximants should be established before applying the boundary condition at infinity. Many authors [19–32] have been resorted to either Padé technique or some numerical commercial codes to solve the boundary value problems in unbounded domain. However, Padé technique requires a massive computational work to obtain accurate approximate solutions. Searching for a direct method to treat the boundary condition at infinity has been the main goal of many researchers for a long time to solve boundary value problems in unbounded domain. Such a direct method is proposed in this paper. The main idea is to transform the physical domain from unbounded into bounded through a transformation. Accordingly, a new system arises which is now subject to classical boundary conditions, where the boundary conditions at infinity disappeared as a result of the new transformation. The transformed system can be directly solved by the differential transformation method (DTM) [33–45] without any need to Padé approximants. In this paper, the governing system of ordinary differential equations describing the boundary-layer flow of a nanofluid past a stretching sheet is analyzed through the proposed improved version of the DTM. The main advantage of the present method is that not only it avoids the use of Padé approximants, but also gives the series solution in a straightforward manner.

2. Basic Equations

The basic equations of the steady two-dimensional boundary layer flow of a nanofluid past a stretching surface with the linear velocity $u_w(x) = ax$, where a is a constant and x is the coordinate measured along the stretching surface, as given by Kuznetsov and Nield [15] and Nield and Kuznetsov [16] and later by Khan and Pop [18], are as follows:

$$\begin{gathered}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \\
u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \\
u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \\
u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tau\left[D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right)\right], \\
u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right),
\end{gathered} \tag{1}$$

subject to the boundary conditions:

$$\begin{gathered}
v = 0, \quad u = u_w(x) = ax, \quad T = T_w, \quad C = C_w, \quad \text{at } y = 0, \\
u = v = 0, \quad T = T_\infty, \quad C = C_\infty, \quad \text{as } y \longrightarrow \infty.
\end{gathered} \tag{2}$$

A complete physical description of the present problem was well presented by Khan and Pop [18] as follows. The flow takes place at $y \geq 0$, where y is the coordinate measured normal to the stretching surface. A steady uniform stress leading to equal and opposite forces is applied along the x-axis so that the sheet is stretched keeping the origin fixed. It is assumed that at the stretching surface, the temperature T and the nanoparticle fraction C take constant values T_w and C_w, respectively. The ambient values, attained as y tends to infinity, of T and C, are denoted by T_∞ and C_∞, respectively.

Here u and v are the velocity components along the axes x and y, respectively, p is the fluid pressure, ρ_f is the density of the base fluid, α is the thermal diffusivity, ν is the kinematic viscosity, a is a positive constant, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, $\tau = (\rho c)_p/(\rho c)_f$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid with ρ being the density, c is the volumetric volume expansion coefficient, and ρ_p is the density of the particles. Khan and Pop [18] have looked for a similarity solution of (1) with the boundary conditions (2) by assuming that

$$\begin{gathered}
\psi = (a\nu)^{1/2} x f(\eta), \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \\
\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \qquad \eta = \left(\frac{a}{\nu}\right)^{1/2} y,
\end{gathered} \tag{3}$$

where the stream function ψ is defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Hence, a set of ordinary differential equations were obtained by [18] as

$$f''' + ff'' - \left(f'\right)^2 = 0, \tag{4}$$

$$\frac{1}{\mathrm{Pr}}\theta'' + f\theta' + \mathrm{Nb}\,\phi'\theta' + \mathrm{Nt}\left(\theta'\right)^2 = 0, \tag{5}$$

$$\phi'' + \mathrm{Le} f\phi' + \frac{\mathrm{Nt}}{\mathrm{Nb}}\theta'' = 0, \tag{6}$$

subject to the boundary conditions:

$$\begin{gathered}
f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1, \\
f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0,
\end{gathered} \tag{7}$$

where primes denote differentiation with respect to η and the four parameters are defined by

$$\begin{gathered}
\mathrm{Pr} = \frac{\nu}{\alpha}, \qquad \mathrm{Le} = \frac{\nu}{D_B}, \qquad \mathrm{Nb} = \frac{(\rho c)_p D_B(\phi_w - \phi_\infty)}{(\rho c)_f \nu}, \\
\mathrm{Nt} = \frac{(\rho c)_p D_T (T_w - T_\infty)}{(\rho c)_f \nu T_\infty},
\end{gathered} \tag{8}$$

where Pr, Le, Nb, and Nt denote the Prandtl number, the Lewis number, the Brownian motion parameter, and the thermophoresis parameter, respectively. The quantities of practical interest are the Nusselt number Nu and the Sherwood number Sh which are defined as

$$\mathrm{Nu} = \frac{x q_w}{k\left(T_w - T_\infty\right)}, \qquad \mathrm{Sh} = \frac{x q_m}{D_B\left(C_w - C_\infty\right)}, \tag{9}$$

where q_w and q_m are the wall heat and mass fluxes, respectively. According to Kuznetsov and Nield [15], $\mathrm{Re}_x^{-1/2}\mathrm{Nu}$ and $\mathrm{Re}_x^{-1/2}\mathrm{Sh}$ are known as the reduced Nusselt number Nur and reduced Sherwood number Shr, respectively,

$$\mathrm{Nur} = \mathrm{Re}_x^{-1/2}\mathrm{Nu} = -\theta'(0), \qquad \mathrm{Shr} = \mathrm{Re}_x^{-1/2}\mathrm{Sh} = -\phi'(0), \tag{10}$$

where $\mathrm{Re}_x = x u_w(x)/\nu$ is the local Reynolds number based on the stretching velocity $u_w(x)$. It should be noted that the exact solution of (4) with the boundary conditions given in (7) was first obtained by Crane [46] and given as

$$f(\eta) = 1 - e^{-\eta}. \tag{11}$$

Substituting $f(\eta)$ into (5)-(6), the given system reduces to a system of two coupled differential equations as

$$\theta'' + \mathrm{Pr}\left(1 - e^{-\eta} + \mathrm{Nb}\phi'\right)\theta' + \mathrm{Nt}\left(\theta'\right)^2 = 0, \qquad \phi'' + \mathrm{Le}\left(1 - e^{-\eta}\right)\phi' + \frac{\mathrm{Nt}}{\mathrm{Nb}}\theta'' = 0, \tag{12}$$

subject to the boundary conditions:

$$\theta(0) = 1, \quad \theta(\infty) = 0, \qquad \phi(0) = 1, \quad \phi(\infty) = 0. \tag{13}$$

3. Transformed Equations

In order to solve boundary value problems in unbounded domain by using the DTM, authors are usually resort to Padé approximant due to the boundary condition at infinity, and an approximate solution is only available in this case. As a well-known fact, Padé approximant requires a huge amount of computational work to find out the approximate solution. In this regard, we think that if it is possible to transform the unbounded domain into a bounded one then the BVPs may be easily solved without any need to Padé approximant. A first step in this direction is to transform the unbounded domain of the independent variable $\eta \in [0, \infty)$ into a bounded one $t \in [0, 1)$. Such a transformation is found as $t = 1 - e^{-\eta}$, accordingly the governing equations should be changed to be in terms of the new variable t. The effectiveness of this procedure shall be discussed in the next subsection to show the possibility of obtaining very accurate numerical solutions. In view of the mentioned transformation, the system (1)–(3) with the boundary conditions (7) is transformed into a new system in bounded domain given by

$$\begin{aligned} &(1-t)^2\theta'' - (1-t)(1-\mathrm{Pr}t)\theta' + \mathrm{Pr}(1-t)^2\left[\mathrm{Nb}\theta'\phi' + \mathrm{Nt}\left(\theta'\right)^2\right] = 0, \\ &(1-t)^2\phi'' - (1-t)(1-\mathrm{Le}t)\phi' + \frac{\mathrm{Nt}}{\mathrm{Nb}}\left[(1-t)^2\theta'' - (1-t)\theta'\right] = 0, \end{aligned} \tag{14}$$

with the boundary conditions:

$$\theta(0) = 1, \quad \theta(1) = 0, \tag{15}$$

$$\phi(0) = 1, \quad \phi(1) = 0, \tag{16}$$

where primes denote differentiation with respect to t.

4. Analysis and Results

In this section, the application of the DTM is discussed without resorting to Padé approximants. Applying the DTM to the previously mentioned system yielded the following recurrence scheme:

$$\begin{aligned} &\sum_{m=0}^{k}(k-m+1)(k-m+2)\left[\delta(m) - \delta(m-1)\right]\Theta(k-m+2) \\ &- \sum_{m=0}^{k}(k-m+1)\times\left[\delta(m) - \mathrm{Pr}\delta(m-1)\right]\Theta(k-m+1) \\ &+ \mathrm{Nb}\,\mathrm{Pr}\sum_{r=0}^{k}\sum_{m=0}^{r}(m+1)(r-m+1) \times\left[\delta(k-r) - \delta(k-r-1)\right]\Theta(m+1) \times\Phi(r-m+1) \\ &+ \mathrm{Nt}\,\mathrm{Pr}\sum_{r=0}^{k}\sum_{m=0}^{r}(m+1)(r-m+1)\left[\delta(k-r) - \delta(k-r-1)\right] \times\Theta(m+1)\Theta(r-m+1) = 0, \end{aligned} \tag{17a}$$

$$\begin{aligned} &\sum_{m=0}^{k}(k-m+1)(k-m+2)\left[\delta(m) - \delta(m-1)\right]\Phi(k-m+2) \\ &- \sum_{m=0}^{k}(k-m+1)\times\left[\delta(m) - \mathrm{Le}\,\delta(m-1)\right]\Phi(k-m+1) \\ &+ \frac{\mathrm{Nt}}{\mathrm{Nb}}\left(\sum_{m=0}^{k}(k-m+1)(k-m+2)\left[\delta(m) - \delta(m-1)\right] \times\Theta(k-m+2) - (k+1)\Theta(k+1)\right) = 0, \end{aligned} \tag{17b}$$

with the transformed boundary conditions

$$\Theta(0) = 1, \qquad \sum_{k=0}^{N} \Theta(k) = 0, \tag{18}$$

$$\Phi(0) = 1, \qquad \sum_{k=0}^{N} \Phi(k) = 0. \tag{19}$$

Equations (17a), (17b), (18), and (19) are used to obtain very accurate approximate numerical solutions, where two different cases are derived and discussed in the next two subsections.

4.1. Case 1: At Nt=0 and Nb=0. At Nt = 0 and Nb = 0, the boundary value problem for ϕ becomes ill-posed and consequently the boundary value problem for θ becomes

$$(1-t)^2\theta'' - (1-t)(1-\mathrm{Pr}\,t)\,\theta' = 0, \tag{20}$$

subject to the boundary conditions in (15). Accordingly, a simple recurrence scheme is obtained form (17a) and (17b), where Nt = Nb = 0 are inserted, hence

$$\begin{aligned} &\sum_{m=0}^{k} (k-m+1) \\ &\quad \times ((k-m+2)[\delta(m) - \delta(m-1)]\,\Theta(k-m+2) \\ &\quad - [\delta(m) - \mathrm{Pr}\delta(m-1)]\,\Theta(k-m+1)). \end{aligned} \tag{21}$$

Using the recurrence scheme (21) with the transformed initial conditions (18) for $k = 0, 1, 2, \ldots, 8$, a system of algebraic equations is obtained in $\Theta(1), \Theta(2), \ldots$, and $\Theta(10)$. Solving this system, the 10-term approximate solution at any Prandtl number is given in terms of the original similarity variable η as

$$\begin{aligned} &\Theta_{10}(\eta) \\ &= 1 + \Delta\Big[-3628800\left(1-e^{-\eta}\right) - 1814400\left(1-e^{-\eta}\right)^2 \\ &\quad + 604800\,(\mathrm{Pr}-2)\left(1-e^{-\eta}\right)^3 \\ &\quad + 151200\,(5\mathrm{Pr}-6)\left(1-e^{-\eta}\right)^4 \\ &\quad - 30240\left(3\mathrm{Pr}^2 - 26\mathrm{Pr} + 24\right)\left(1-e^{-\eta}\right)^5 \\ &\quad - 5040\left(35\mathrm{Pr}^2 - 154\mathrm{Pr} + 120\right)\left(1-e^{-\eta}\right)^6 \\ &\quad + 720\left(15\mathrm{Pr}^3 - 340\mathrm{Pr}^2 + 1044\mathrm{Pr} - 720\right) \\ &\quad \times \left(1-e^{-\eta}\right)^7 \\ &\quad + 90\left(315\mathrm{Pr}^3 - 3304\mathrm{Pr}^2 + 8028\mathrm{Pr} - 5040\right) \\ &\quad \times \left(1-e^{-\eta}\right)^8 \\ &\quad - 10\left(105\mathrm{Pr}^4 - 4900\mathrm{Pr}^3 + 33740\mathrm{Pr}^2 \right. \\ &\quad \left. -69264\mathrm{Pr} + 40320\right)\left(1-e^{-\eta}\right)^9 \\ &\quad - \left(3465\mathrm{Pr}^4 - 70532\mathrm{Pr}^3 + 367884\mathrm{Pr}^2 \right. \\ &\quad \left. -663696\mathrm{Pr} + 362880\right)\left(1-e^{-\eta}\right)^{10}\Big], \end{aligned} \tag{22}$$

where

$$\Delta = \frac{1}{4515\mathrm{Pr}^4 - 158682\mathrm{Pr}^3 + 1514564\mathrm{Pr}^2 - 5753736\mathrm{Pr} + 10628640}. \tag{23}$$

Here, it may be useful to mention that an exact solution for the current case is obtained very recently by the first author in [47] and given by

$$\theta(\eta) = \frac{\Gamma\left(\mathrm{Pr}, 0, \mathrm{Pr}\,e^{-\eta}\right)}{\Gamma(\mathrm{Pr}, 0, \mathrm{Pr})}, \tag{24}$$

where $\Gamma(a, z_0, z_1) = \int_{z_0}^{z_1} \mu^{a-1} e^{-\mu} d\mu$ is the generalized Gamma function which can be expressed in terms of the incomplete Gamma function as $\Gamma(a, z_0, z_1) = \Gamma(a, z_0) - \Gamma(a, z_1)$, where $\Gamma(a, z) = \int_{z}^{\infty} \mu^{a-1} e^{-\mu} d\mu$. There is no doubt that the availability of the exact solution gives the opportunity to validate the accuracy of the suggested approach. For the purpose of illustration, the approximate solutions are compared in Figures 1, 2, and 3 with the exact one given by (24) at different values of Prandtl number. These primary results reveal that the 10-term approximate solution is sufficient to obtain numerical solutions of high accuracy for certain range of Prandtl number, mainly Pr = 1, 2, 3. However, at Pr = 4 the 10-term approximate solution is not accurate as can be seen in Figure 4. This observation leads to the conclusion that with increasing Pr more terms in the approximate series solution are in fact needed. For example, at Pr = 5 the 20-term approximate solution is found sufficient in Figure 5, while 30-term approximate solution is found identical to the exact one at Pr = 10 in Figure 6. However, the main advantage of the suggested approach is the avoidance of Padé-approximants which has been used a long time to treat the boundary condition at infinity.

4.2. Case 2: At Nt=0, Pr=Le=1, and Nb ≠ 0. Substituting Nt = 0, Pr = 1, and Le = 1 into (17a) and (17b) yields

$$\begin{aligned} &\sum_{m=0}^{k} (k-m+1)(k-m+2)[\delta(m) - \delta(m-1)]\,\Theta(k-m+2) \\ &\quad - \sum_{m=0}^{k} (k-m+1) \times [\delta(m) - \mathrm{Pr}\,\delta(m-1)]\,\Theta(k-m+1) \\ &\quad + \mathrm{Nb}\,\mathrm{Pr} \sum_{r=0}^{k} \sum_{m=0}^{r} (m+1)(r-m+1) \\ &\qquad \times [\delta(k-r) - \delta(k-r-1)]\,\Theta(m+1) \\ &\qquad \times \Phi(r-m+1) = 0, \end{aligned}$$

$$\sum_{m=0}^{k}(k-m+1)(k-m+2)[\delta(m)-\delta(m-1)]\Phi(k-m+2)$$
$$-\sum_{m=0}^{k}(k-m+1)\times[\delta(m)-\mathrm{Le}\,\delta(m-1)]$$
$$\times\Phi(k-m+1)=0. \tag{25}$$

Using the recurrence scheme (25) with the transformed initial conditions (18) and (19) for $k = 0,1,2,\ldots,4$, we get a system of algebraic equations in $\Theta(1),\Theta(2),\ldots,\Theta(6)$ and $\Phi(1),\Phi(2),\ldots,\Phi(6)$. The solution of the required system leads to the following 6-term approximate solution for the θ equation:

$$\begin{aligned}\Theta_6(\eta) = {} & 1+\Theta(1)(1-e^{-\eta})+\Theta(2)(1-e^{-\eta})^2\\ & +\Theta(3)(1-e^{-\eta})^3+\Theta(4)(1-e^{-\eta})^4\\ & +\Theta(5)(1-e^{-\eta})^5+\Theta(6)(1-e^{-\eta})^6,\end{aligned} \tag{26}$$

where $\Theta(1),\Theta(2),\ldots$, and $\Theta(6)$ are expressed in terms of Nb but ignored here for lengthy results. However, the 6-term series solution for the ϕ equation is given explicitly as

$$\begin{aligned}\Phi_6(\eta) & \\ = 1 - & \frac{1}{1237}\\ \times & \big[720(1-e^{-\eta})+360(1-e^{-\eta})^2+120(1-e^{-\eta})^3\\ & +30(1-e^{-\eta})^4+6(1-e^{-\eta})^5+(1-e^{-\eta})^6\big].\end{aligned} \tag{27}$$

The exact solutions are obtained in [47] as

$$\begin{aligned}\phi(\eta) &= \frac{1-e^{-e^{-\eta}}}{1-e^{-1}},\\ \theta(\eta) &= \frac{1-e^{-\alpha\mathrm{Nb}(1-e^{-e^{-\eta}})}}{1-e^{-\mathrm{Nb}}}.\end{aligned} \tag{28}$$

The obtained truncated series solution $\Theta_6(\eta)$ is compared with the exact one in Figures 7–9 at several values of Nb. As observed from Figures 7 and 8, the approximate solution is coincided with the exact one at certain values, Nb = 0.1 and Nb = 0.3. However, it approaches the exact curve at Nb = 0.5, where more terms are needed in this case. In addition, the approximate solution $\Phi_6(\eta)$ is found identical to the exact curve as shown in Figure 10.

5. Conclusions

A system of ordinary differential equations describing the boundary layer flow of a nanofluid past a stretching sheet is investigated in this paper via a new approach. The suggested approach is based on transforming the boundary conditions

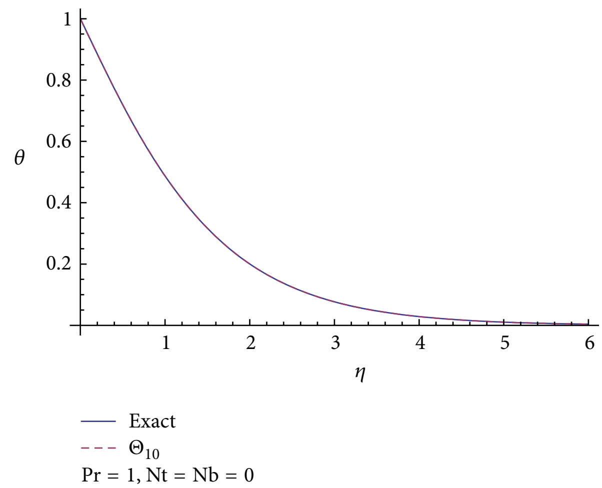

FIGURE 1

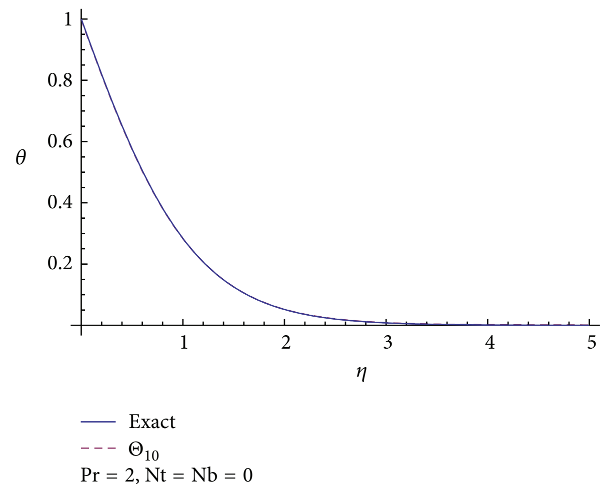

FIGURE 2

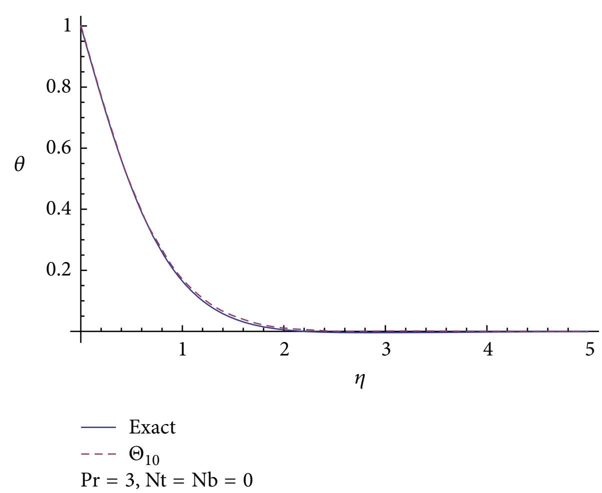

FIGURE 3

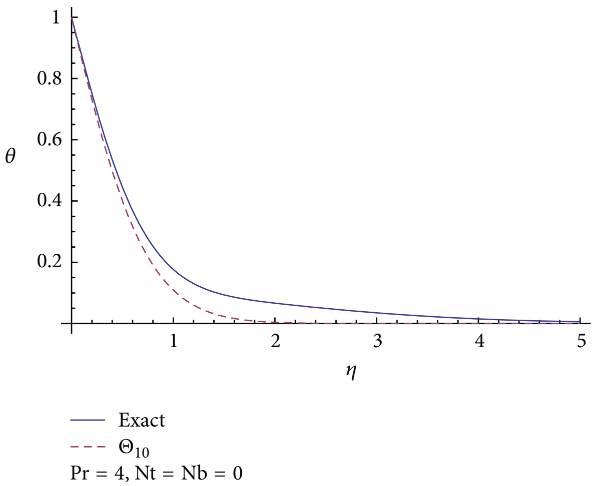

Figure 4

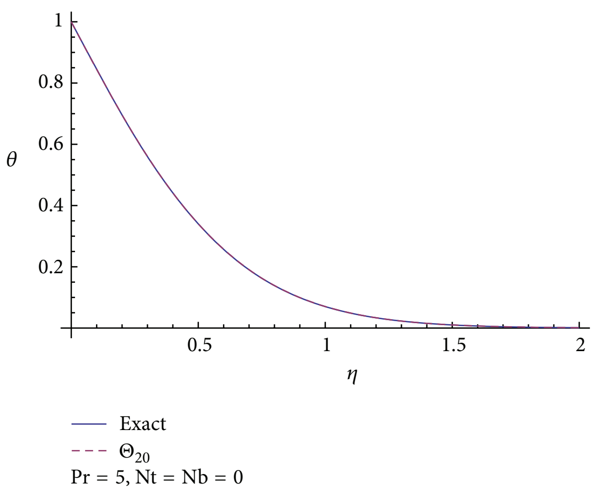

Figure 5

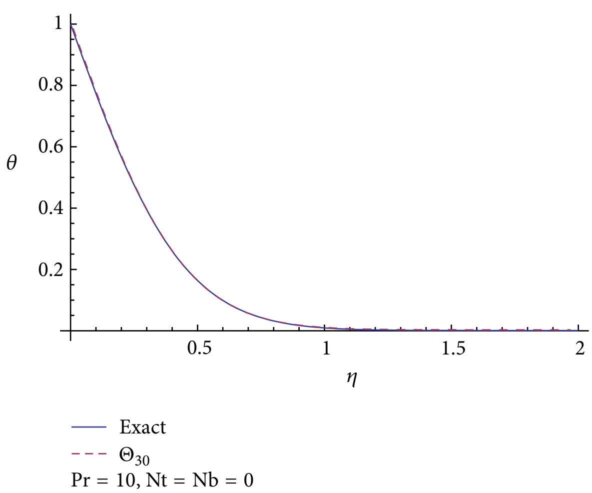

Figure 6

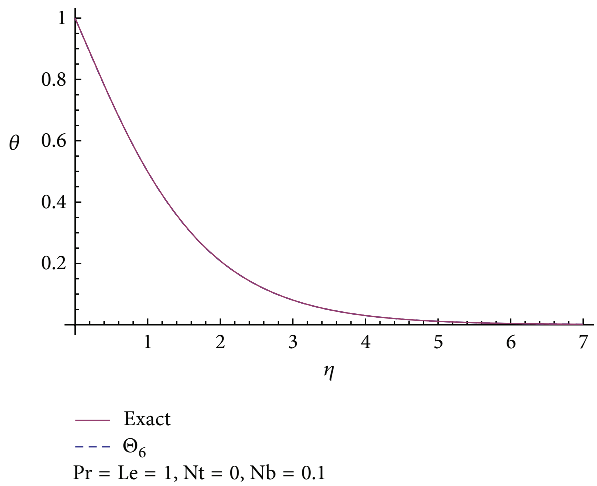

Figure 7

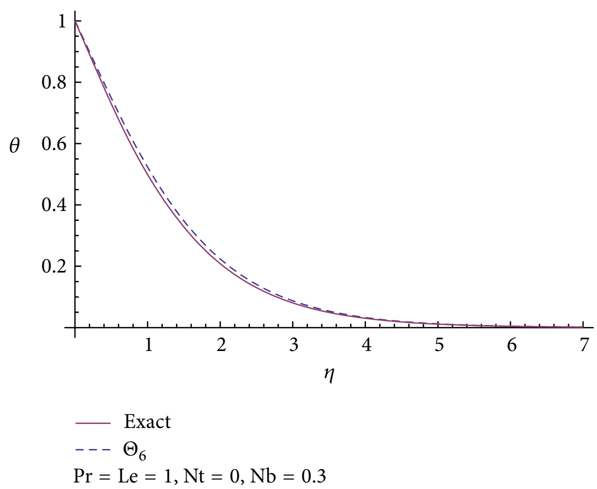

Figure 8

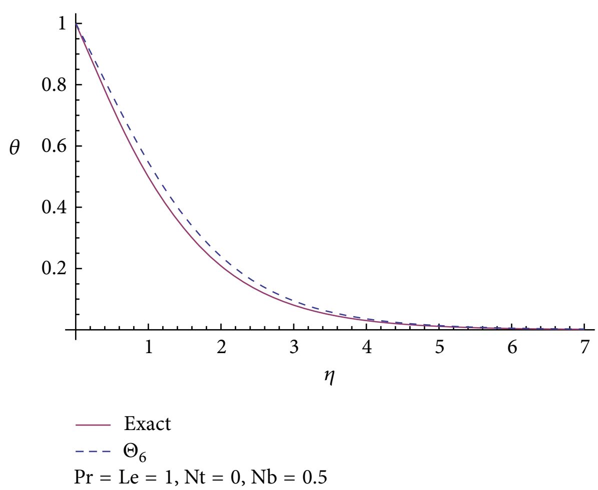

Figure 9

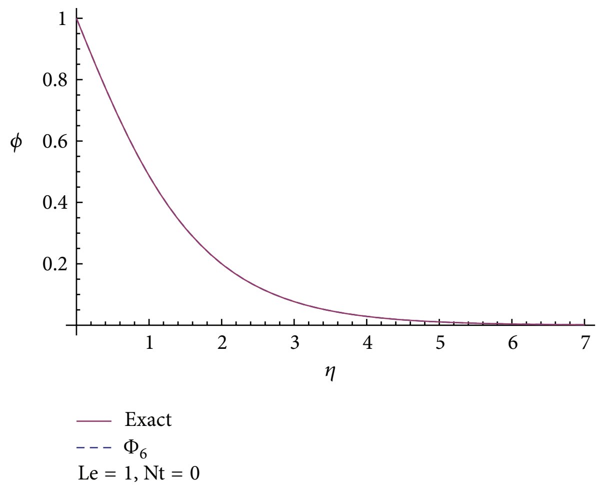

Figure 10

at infinity into classical conditions prior to the application of the differential transformation method. A transformation is successfully used to map the unbounded physical domain into a bounded one. In addition, the current results are validated through various comparisons with the available exact solutions. In comparison with Padé technique, the new method of solution is found not only straightforward but also effective in obtaining accurate numerical solutions, where Padé approximant was completely avoided.

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