

## Research Article

# Resilient $L_2$ - $L_\infty$ Filtering of Uncertain Markovian Jumping Systems within the Finite-Time Interval

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This paper studies the resilient  $L_2$ - $L_\infty$  filtering problem for a class of uncertain Markovian jumping systems within the finite-time interval. The objective is to design such a resilient filter that the finite-time  $L_2$ - $L_\infty$  gain from the unknown input to an estimation error is minimized or guaranteed to be less than or equal to a prescribed value. Based on the selected Lyapunov-Krasovskii functional, sufficient conditions are obtained for the existence of the desired resilient  $L_2$ - $L_\infty$  filter which also guarantees the stochastic finite-time boundedness of the filtering error dynamic systems. In terms of linear matrix inequalities (LMIs) techniques, the sufficient condition on the existence of finite-time resilient  $L_2$ - $L_\infty$  filter is presented and proved. The filter matrices can be solved directly by using the existing LMIs optimization techniques. A numerical example is given at last to illustrate the effectiveness of the proposed approach.

## 1. Introduction

More recently, the finite-time stability and control problems have received great attention in the literature; see [1–6]. Compared with the Lyapunov stable dynamical systems, a finite-time stable dynamical system does not require the steady-state behavior of control dynamics over an infinite-time interval and the asymptotic pattern of system trajectories. The main attention may be related to the transient characteristics of the dynamical systems over a fixed finite-time interval, for instance, keeping the acceptable values in a prescribed bound in the presence of saturations. However, more details are related to the stability and control problems of various dynamic systems, and very few reports in the literature consider the filtering problems.

Since the Kalman filtering theory [7] has been introduced in the early 1960s, the filtering problem has been extensively investigated. In the filtering scheme, its objective is to estimate the unavailable state variables (or a linear combination of the states) of a given system. During the past decades, many filtering schemes have been developed, such as Kalman filtering [8],  $H_\infty$  filtering [9], reduced-order  $H_\infty$  filtering [10], and  $L_2$ - $L_\infty$  filtering [11]. Then, extension of this effort to the problem of resilient Kalman filtering with respect to

estimator gain perturbations was considered in [12]. And the resilient  $H_\infty$  filtering [13] was also raised. Among the filtering schemes, the resilient  $L_2$ - $L_\infty$  filtering was not considered. In practical engineering applications, the peak values of filtering error should always be considered. Compared with the  $H_\infty$  filtering scheme, the external disturbances are both assumed to be energy bounded; but  $L_2$ - $L_\infty$  filtering setting requires the mapping from the external disturbances to the filtering error is minimized or no larger than some prescribed level in terms of the  $L_2$ - $L_\infty$  performance norm.

In this paper, we have studied the resilient finite-time  $L_2$ - $L_\infty$  filtering problem for uncertain Markovian Jumping Systems (MJSs). Firstly, the augmented filtering error dynamic system is constructed based on the state estimated filter with resilient filtering parameters. Secondly, a sufficient condition is established on the existence of the robust finite-time filter such that the filtering error dynamic MJSs are finite-time bounded and satisfy a prescribed level of  $L_2$ - $L_\infty$  disturbance attenuation with the finite-time interval. And the design criterion is presented by means of LMIs techniques. Subsequently, the robust finite-time  $L_2$ - $L_\infty$  filter matrices can be solved directly by using the existing LMIs optimization algorithms. In order to illustrate the proposed result, a numerical example is given at last.

Let us introduce some notations. The symbols  $\mathfrak{R}^n$  and  $\mathfrak{R}^{n \times m}$  stand for an  $n$ -dimensional Euclidean space and the set of all  $n \times m$  real matrices, respectively,  $A^T$  and  $A^{-1}$  denote the matrix transpose and matrix inverse,  $\text{diag}\{A \ B\}$  represents the block-diagonal matrix of  $A$  and  $B$ ,  $\sigma_{\max}(A)$  and  $\sigma_{\min}(A)$ , respectively, denote the maximal and minimal eigenvalues of a real matrix  $A$ ,  $\|*\|$  denotes the Euclidean norm of vectors,  $\mathbf{E}\{*\}$  denotes the mathematics statistical expectation of the stochastic process or vector,  $P > 0$  stands for a positive-definite matrix,  $I$  is the unit matrix with appropriate dimensions,  $0$  is the zero matrix with appropriate dimensions, and  $*$  means the symmetric terms in a symmetric matrix.

## 2. Problem Formulation

Given a probability space  $(\Omega, F, P_r)$  where  $\Omega$  is the sample space,  $F$  is the algebra of events, and  $P_r$  is the probability measure defined on  $F$ . Let us consider a class of linear uncertain MJSs defined in the probability space  $(\Omega, F, P_r)$  and described by the following differential equations:

$$\begin{aligned} \dot{x}(t) &= [A(r_t) + \Delta A(r_t)] x(t) + B(r_t) w(t), \\ y(t) &= [C(r_t) + \Delta C(r_t)] x(t) + D(r_t) w(t), \\ z(t) &= E(r_t) x(t), \\ x(t) &= x_0, \quad r_t = r_0, \quad t = 0, \end{aligned} \quad (1)$$

where  $x(t) \in \mathfrak{R}^n$  is the state,  $y(t) \in \mathfrak{R}^l$  is the measured output,  $w(t) \in L_2^m[0, +\infty)$  is the unknown input,  $z(t) \in \mathfrak{R}^q$  is the controlled output, and  $x_0$  and  $r_0$  are, respectively, the initial states and mode.  $A(r_t)$ ,  $B(r_t)$ ,  $C(r_t)$ ,  $D(r_t)$ , and  $E(r_t)$  are known mode-dependent constant matrices with appropriate dimensions. The jump parameter  $r_t$  represents a continuous-time discrete-state Markov stochastic process taking values on a finite set  $\mathbf{M} = \{1, 2, \dots, N\}$  with transition rate matrix  $\Pi = \{\pi_{ij}\}$ ,  $i, j \in \mathbf{M}$ , and has the following transition probability from mode  $i$  at time  $t$  to mode  $j$  at time  $t + \Delta t$  as

$$P_{ij} = P_r \{r_{t+\Delta t} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & i = j, \end{cases} \quad (2)$$

where  $\Delta t > 0$  and  $\lim_{\Delta t \rightarrow 0} (o(\Delta t)/\Delta t) \rightarrow 0$ .

In this relation,  $\pi_{ij} \geq 0$  is the transition probability rates from mode  $i$  at time  $t$  to mode  $j$  ( $i \neq j$ ) at time  $t + \Delta t$ , and

$$\sum_{j=1, j \neq i}^N \pi_{ij} = -\pi_{ii} \quad \text{for } i, j \in \mathbf{M}, \quad i \neq j. \quad (3)$$

For presentation convenience, we denote  $A(r_t)$ ,  $B(r_t)$ ,  $C(r_t)$ ,  $D(r_t)$ ,  $E(r_t)$ ,  $\Delta A(r_t)$ , and  $\Delta C(r_t)$  as  $A_i$ ,  $B_i$ ,  $C_i$ ,  $D_i$ ,  $E_i$ ,  $\Delta A_i$ , and  $\Delta C_i$ , respectively.

And the matrices with the symbol  $\Delta(*)$  are considered as the uncertain matrices satisfying the following conditions:

$$\begin{bmatrix} \Delta A_i \\ \Delta C_i \end{bmatrix} = \begin{bmatrix} M_{1i} \\ M_{2i} \end{bmatrix} \Gamma_i(t) N_{1i}, \quad (4)$$

where  $M_{1i}$ ,  $M_{2i}$ , and  $N_{1i}$  are known mode-dependent matrices with appropriate dimensions, and  $\Gamma_i(t)$  is the time-varying unknown matrix function with Lebesgue norm measurable elements satisfying  $\Gamma_i^T(t)\Gamma_i(t) \leq I$ .

*Remark 1.* It is always impossible to obtain the exact mathematical model of practical dynamics due to the complexity process, the environmental noises, and the difficulties of measuring various and uncertain parameters, and so forth; thus, the model of practical dynamics to be controlled almost contains some types of uncertainties. In general, the uncertainties  $\Delta(*)$  in (1) satisfying the restraining conditions (4) and  $\Gamma_i^T(t)\Gamma_i(t) \leq I$  are said to be admissible. The unknown mode-dependent matrix  $\Gamma_i(t)$  can also be allowed to be state dependent; that is,  $\Gamma_i(t) = \Gamma_i(t, x(t))$ , as long as  $\|\Gamma_i(t, x(t))\| \leq 1$  is satisfied.

We now consider the following resilient filter:

$$\begin{aligned} \dot{\hat{x}}(t) &= (A_{fi} + \Delta A_{fi}) \hat{x}(t) + B_{fi} y(t), \\ \hat{z}(t) &= (C_{fi} + \Delta C_{fi}) \hat{x}(t), \\ \hat{x}(t) &= \hat{x}_0, \quad r_t = r_0, \quad t = 0, \end{aligned} \quad (5)$$

where  $\hat{x}(t) \in \mathfrak{R}^n$  is the filter state,  $\hat{z}(t) \in \mathfrak{R}^q$  is the filter output,  $\hat{x}_0$  is the initial estimation states, and the mode-dependent matrices  $A_{fi}$ ,  $B_{fi}$ , and  $C_{fi}$  are unknown filter parameters to be designed for each value  $i \in \mathbf{M}$ .  $\Delta A_{fi}$  and  $\Delta C_{fi}$  are uncertain filter parameter matrices satisfying the following conditions:

$$\begin{bmatrix} \Delta A_{fi} \\ \Delta C_{fi} \end{bmatrix} = \begin{bmatrix} M_{3i} \\ M_{4i} \end{bmatrix} \Gamma_{fi}(t) N_{2i}, \quad (6)$$

where  $M_{3i}$ ,  $M_{4i}$ ,  $N_{2i}$ , and  $\Gamma_{fi}(t)$  are defined similarly as (4).

The objective of this paper is to design the resilient  $L_2$ - $L_\infty$  filter of uncertain MJSs in (1) and obtain an estimate  $\hat{z}(t)$  of the signal  $z(t)$  such that the defined guaranteed performance criteria are minimized in an  $L_2$ - $L_\infty$  estimation error sense. Define  $e(t) = x(t) - \hat{x}(t)$  and  $r(t) = z(t) - \hat{z}(t)$ , such that the filtering error dynamic MJSs are given by

$$\begin{aligned} \dot{e}(t) &= (A_{fi} + \Delta A_{fi}) e(t) \\ &\quad + [A_i + \Delta A_i - (A_{fi} + \Delta A_{fi}) - B_{fi}(C_i + \Delta C_i)] x(t) \\ &\quad + (B_i - B_{fi}D_i) w(t), \\ r(t) &= (C_{fi} + \Delta C_{fi}) e(t) + [E_i - (C_{fi} + \Delta C_{fi})] x(t). \end{aligned} \quad (7)$$

Let  $\xi(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}$ , and we have

$$\begin{aligned} \dot{\xi}(t) &= \bar{A}_i \xi(t) + \bar{B}_i w(t), \\ r(t) &= \bar{C}_i \xi(t), \end{aligned} \quad (8)$$

where

$$\begin{aligned} \bar{A}_i &= \begin{bmatrix} A_i + \Delta A_i - (A_{fi} + \Delta A_{fi}) - B_{fi}(C_i + \Delta C_i) & 0 \\ A_{fi} + \Delta A_{fi} & \end{bmatrix}, \\ \bar{B}_i &= \begin{bmatrix} B_i \\ B_i - B_{fi}D_i \end{bmatrix}, \\ \bar{C}_i &= [E_i - (C_{fi} + \Delta C_{fi}) \quad C_{fi} + \Delta C_{fi}]. \end{aligned} \quad (9)$$

The external disturbance  $w(t)$  is varying and satisfies the constraint condition with respect to the finite-time interval  $[0 \ T]$  as follows:

$$\int_0^T w^T(t) w(t) dt \leq W, \quad (10)$$

where  $W$  is a positive scalar.

*Definition 2.* Given a time-constant  $T > 0$ , the filtering error dynamic MJSs (8) (setting  $w(t) = 0$ ) are said to be stochastically finite-time stable (FTS) with respect to  $(c_1 \ c_2 \ T \ \bar{R}_i)$ , if

$$\mathbf{E} \{ \xi^T(0) \bar{R}_i \xi(0) \} \leq c_1 \implies \mathbf{E} \{ \xi^T(t) \bar{R}_i \xi(t) \} < c_2, \quad (11)$$

$$t \in [0 \ T],$$

where  $c_1 > 0, c_2 > 0, \bar{R}_i = \text{diag}[R_i \ R_i] > 0$ , and  $R_i$  is the weight coefficient matrix.

*Definition 3.* Given a time-constant  $T > 0$ , the filtering error dynamic MJSs (8) are stochastically finite-time bounded (FTB) with respect to  $(c_1 \ c_2 \ T \ \bar{R}_i \ W)$ , wherein  $c_1 > 0, c_2 > 0, \bar{R}_i > 0$ , if condition (10) holds.

*Definition 4.* For the filtering error dynamic MJSs (8), if there exist filter parameters  $A_{fi}, B_{fi}$ , and  $C_{fi}$ , as well as a positive scalar  $\gamma$ , such that the filtering error dynamic MJSs (8) are stochastically FTB and under the zero-valued initial condition, the system output error satisfies the following cost function inequality for  $T > 0$  with attenuation  $\gamma > 0$  and for all admissible  $w(t)$  with the constraint condition (10):

$$J = \|r(t)\|_{E_{\infty}}^T - \gamma \|w(t)\|_2^T < 0, \quad (12)$$

where  $\|r(t)\|_{E_{\infty}}^T = \sup_{t \in [0 \ T]} \mathbf{E}[\|r(t)\|]$ ,  $\|w(t)\|_2^T = \sqrt{\int_0^T w^T(t)w(t) dt}$ .

Then, the resilient filter (5) is called the stochastic finite-time  $L_2$ - $L_{\infty}$  filter of the uncertain dynamic MJSs (1) with  $\gamma$ -disturbance attenuation.

### 3. Main Results

In this section, we will study the robust stochastic finite-time resilient filtering problem for the filtering error dynamic MJSs (8) in an  $L_2$ - $L_{\infty}$  estimation error sense. Before proceeding with the study, the following lemma is needed.

**Lemma 5** (see [14]). Let  $T, M$ , and  $N$  be real matrices with appropriate dimensions. Then, for all time-varying unknown matrix function  $F(t)$  satisfying  $F^T(t)F(t) \leq I$ , the following relation holds:

$$T + MF(t)N + N^T F^T(t)M^T > 0, \quad (13)$$

if and only if there exists a positive scalar  $\alpha > 0$ , such that

$$T + \alpha^{-1}MM^T\alpha N^T N < 0. \quad (14)$$

**Theorem 6.** For given  $T > 0, \eta > 0, c_1 > 0$ , and  $\bar{R}_i > 0$ , the filtering error dynamic MJSs (8) are stochastically FTB with respect to  $(c_1 \ c_2 \ T \ \bar{R}_i \ W)$  and have a  $L_2$ - $L_{\infty}$  performance level  $\gamma > 0$ , if there exist positive constants,  $\gamma > 0, c_2 > 0$ , and mode-dependent symmetric positive-definite matrices  $\tilde{P}_i$ , such that

$$\begin{bmatrix} \tilde{P}_i \bar{A}_i + \bar{A}_i^T \tilde{P}_i + \sum_{j=1}^N \pi_{ij} \tilde{P}_j - \eta \tilde{P}_i & \tilde{P}_i \bar{B}_i \\ B_{fi}^T \tilde{P}_i & -e^{-\eta T} I \end{bmatrix} < 0, \quad (15)$$

$$\bar{C}_i^T \bar{C}_i < \gamma^2 \tilde{P}_i, \quad (16)$$

$$e^{\eta T} c_1 \bar{\sigma}_{\tilde{P}} + \frac{W}{\eta} (1 - e^{-\eta T}) < c_2 \underline{\sigma}_{\tilde{P}}, \quad (17)$$

where  $\hat{P}_i = \bar{R}_i^{-1/2} \tilde{P}_i \bar{R}_i^{-1/2}$ ,  $\bar{\sigma}_{\tilde{P}} = \max_{i \in \mathbf{M}} \sigma_{\max}(\hat{P}_i)$ , and  $\underline{\sigma}_{\tilde{P}} = \min_{i \in \mathbf{M}} \sigma_{\min}(\hat{P}_i)$ .

*Proof.* Let the mode at time  $t$  be  $i$ ; that is,  $r_t = r \in \mathbf{M}$ . Take the stochastic Lyapunov-Krasovskii functional  $V(\xi(t), r_t, t > 0) : \mathfrak{R}^n \times \mathbf{M} \times \mathfrak{R}_+ \rightarrow \mathfrak{R}_+$  as

$$V(\xi(t), i) = \xi^T(t) \tilde{P}_i \xi(t). \quad (18)$$

Then, we introduce a weak infinitesimal generator  $\mathfrak{F}[*]$  (see [15, 16]), acting on  $V(\xi(t), i)$ , for all  $i \in \mathbf{M}$ , which is defined as

$$\begin{aligned} \mathfrak{F}V(\xi(t), i) &= \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} [\mathbf{E} \{ V(\xi(t + \Delta t), r_{t+\Delta t}, t + \Delta t) | \\ &\quad \xi(t), r_t = i \} - V(\xi(t), i, t)]. \end{aligned} \quad (19)$$

The time derivative of  $V(\xi(t), i)$  along the trajectories of the filtering error dynamic MJSs (8) is given by

$$\begin{aligned} \mathfrak{F}V(\xi(t), i) &= 2\xi^T(t) \tilde{P}_i \dot{\xi}(t) + \xi^T(t) \sum_{j=1}^N \pi_{ij} \tilde{P}_j \xi(t) \\ &= \xi^T(t) \left( \tilde{P}_i \bar{A}_i + \bar{A}_i^T \tilde{P}_i + \sum_{j=1}^N \pi_{ij} \tilde{P}_j \right) \xi(t) \\ &\quad + 2\xi^T(t) \tilde{P}_i \bar{B}_i w(t). \end{aligned} \quad (20)$$

Considering the  $L_2$ - $L_{\infty}$  filtering performance for the dynamic filtering error system (8), we introduce the following cost function by Definition 4 with  $t \geq 0$ :

$$J(t) = \mathbf{E} [\mathfrak{F}V(\xi(t), i)] - \eta \mathbf{E} [V(\xi(t), i)] - e^{-\eta T} w^T(t) w(t). \quad (21)$$

It follows from relation (16) that  $J(t) < 0$ ; that is,

$$\mathbf{E} \{ \mathfrak{S}V(\xi(t), i) \} < \eta \mathbf{E} \{ V(\xi(t), i) \} + e^{-\eta t} w^T(t) w(t). \quad (22)$$

Then, multiplying the previous inequality by  $e^{-\eta t}$ , we have

$$\mathbf{E} \left\{ \mathfrak{S} \left[ e^{-\eta t} V(\xi(t), i) \right] \right\} < e^{-\eta(T+t)} w^T(t) w(t). \quad (23)$$

Integrating the above inequality from 0 to  $T$ , we have

$$\begin{aligned} & e^{-\eta T} \mathbf{E} \{ V(\xi(T), i) \} - \mathbf{E} \{ V(\xi(0), r_0) \} \\ & < e^{-\eta T} \int_0^T e^{-\eta s} w^T(s) w(s) ds. \end{aligned} \quad (24)$$

Considering  $V(\xi(0), r_0) \geq 0$ , as well as the zero initial condition; that is,  $\xi(0) = 0$ , for  $t > 0$ , then it follows that

$$\mathbf{E} \{ V(\xi(T), i) \} < \int_0^T e^{-\eta s} w^T(s) w(s) ds. \quad (25)$$

Then, it can be verified from the defined Lyapunov-Krasovskii functional that

$$\mathbf{E} \{ \xi^T(T) \tilde{P}_i \xi(T) \} = \mathbf{E} \{ V(\xi(T), i) \} < \int_0^T e^{-\eta s} w^T(s) w(s) ds. \quad (26)$$

By (15) and within the finite-time interval  $[0 \ T]$ , we can also get

$$\begin{aligned} \mathbf{E} \{ r^T(T) r(T) \} &= \mathbf{E} \{ \xi^T(T) \bar{C}_i^T \bar{C}_i \xi(T) \} \\ &< \gamma^2 \mathbf{E} \{ \xi^T(T) \tilde{P}_i \xi(T) \} \\ &= \gamma^2 \mathbf{E} \{ V(\xi(T), i) \} \\ &< \gamma^2 \int_0^T e^{-\eta s} w^T(s) w(s) ds \\ &< \gamma^2 \int_0^T w^T(s) w(s) ds. \end{aligned} \quad (27)$$

Since the previous inequality is always true for any  $T > 0$ , the following relation:

$$\sup_{t \in [0 \ T]} \mathbf{E} \{ \|r(t)\| \} < \gamma \sqrt{\int_0^T w^T(s) w(s) ds}. \quad (28)$$

It is easy to get the following result:

$$\begin{aligned} \|r(t)\|_{E\infty}^T &= \gamma \sum_{i=1}^N \pi_i \left\{ \sup_{t \in [0 \ T]} \mathbf{E} \{ \|r(t)\| \} \right\} \\ &< \gamma \sum_{i=1}^N \pi_i \sqrt{\int_0^T w^T(s) w(s) ds}. \\ &= \gamma \sqrt{\int_0^T w^T(s) w(s) ds} = \gamma \|w(t)\|_2^T. \end{aligned} \quad (29)$$

Therefore, the cost function inequality (10) can be guaranteed, which implies  $J = \|r(t)\|_{E\infty}^T - \gamma \|w(t)\|_2^T < 0$ .

Denote that  $\tilde{P}_i = \tilde{R}_i^{-1/2} \tilde{P}_i \tilde{R}_i^{-1/2}$ ,  $\bar{\sigma}_{\tilde{P}} = \max_{i \in \mathbf{M}} \sigma_{\max}(\tilde{P}_i)$ , and  $\underline{\sigma}_{\tilde{P}} = \min_{i \in \mathbf{M}} \sigma_{\min}(\tilde{P}_i)$ . From equality (24), we have

$$\begin{aligned} \mathbf{E} \{ \xi^T(t) \tilde{P}_i \xi(t) \} &= \mathbf{E} \{ V(\xi(t), i) \} < \int_0^t e^{-\eta s} w^T(s) w(s) ds \\ &+ e^{\eta t} \mathbf{E} \{ V(\xi(0), r_0) \} < e^{\eta t} \mathbf{E} \{ V(\xi(0), r_0) \} \\ &+ W \int_0^t e^{-\eta s} ds < e^{\eta t} c_1 \bar{\sigma}_{\tilde{P}} \\ &+ \frac{W}{\eta} (1 - e^{-\eta t}) \leq e^{\eta T} c_1 \bar{\sigma}_{\tilde{P}} \\ &+ \frac{W}{\eta} (1 - e^{-\eta T}). \end{aligned} \quad (30)$$

On the other hand, it results from the stochastic Lyapunov-Krasovskii functional that

$$\mathbf{E} \{ \xi^T(t) \tilde{P}_i \xi(t) \} \geq \underline{\sigma}_{\tilde{P}} \mathbf{E} \{ \xi^T(t) \tilde{R}_i \xi(t) \}. \quad (31)$$

Then, we can get

$$\mathbf{E} \{ \xi^T(t) \tilde{R}_i \xi(t) \} < \frac{e^{\eta T} c_1 \bar{\sigma}_{\tilde{P}} + (W/\eta)(1 - e^{-\eta T})}{\underline{\sigma}_{\tilde{P}}}. \quad (32)$$

It implies that for all  $t \in [0 \ T]$ , we have  $\mathbf{E} \{ \xi^T(t) \tilde{R}_i \xi(t) \} < c_2$  by condition (17). This completes the proof.  $\square$

**Theorem 7.** For given  $T > 0$ ,  $\eta > 0$ ,  $c_1 > 0$ , and  $R_i > 0$ , the filtering error dynamic MJSs (8) are stochastically FTB with respect to  $(c_1 \ c_2 \ T \ R_i \ W)$  with  $R_i \in \mathfrak{R}^{n \times n} > 0$  and have a prescribed  $L_2$ - $L_\infty$  performance level  $\gamma > 0$ , if there exist a set of mode-dependent symmetric positive-definite matrices  $P_i$ , a set of mode-dependent matrices  $X_i$ ,  $Y_i$ , and a positive scalar  $\sigma_1$  and mode-dependent sequences  $\alpha_i$ ,  $\beta_i$ ,  $\lambda_i$ , satisfying the following matrix inequalities for all  $i \in \mathbf{M}$ :

$$\begin{bmatrix} \Lambda_{1i} & * & P_i B_i & P_i M_{1i} & 0 \\ \Lambda_{2i} & \Lambda_{3i} & P_i B_i - Y_i D_i & P_i M_{1i} - Y_i M_{2i} & P_i M_{3i} \\ * & * & -e^{-\eta T} I & 0 & 0 \\ * & * & * & -\alpha_i I & 0 \\ * & * & * & * & -\beta_i I \end{bmatrix} < 0, \quad (33)$$

$$\begin{bmatrix} -P_i & 0 & -E_i^T + C_{fi}^T & N_{2i}^T \\ 0 & -P_i & -C_{fi}^T & -N_{2i}^T \\ * & * & -\gamma^2 I + \lambda_i M_{4i} M_{4i}^T & 0 \\ * & * & * & -\lambda_i I \end{bmatrix} < 0, \quad (34)$$

$$R_i < P_i < \sigma_1 R_i, \quad (35)$$

$$e^{\eta T} c_1 \sigma_1 + \frac{W}{\eta} (1 - e^{-\eta T}) < c_2, \quad (36)$$

where  $\Lambda_{1i} = P_i A_i + A_i^T P_i + \sum_{j=1}^N \pi_{ij} P_j - \eta P_i + \alpha_i N_{1i}^T N_{1i} + \beta_i N_{2i}^T N_{2i}$ ,  $\Lambda_{2i} = P_i A_i - X_i - Y_i C_i - \beta_i N_{2i}^T N_{2i}$ ,  $\Lambda_{2i} = X_i + X_i^T + \sum_{j=1}^N \pi_{ij} P_j - \eta P_i + \beta_i N_{2i}^T N_{2i}$ .

Moreover, the suitable filter parameters can be given as

$$A_{fi} = P_i^{-1} X_i, \quad B_{fi} = P_i^{-1} Y_i, \quad C_{fi} = C_{fi}. \quad (37)$$

*Proof.* For convenience, we set  $\tilde{P}_i = \text{diag}\{P_i, P_i\}$ . Then, we can get the following relations according to matrix inequalities (15) and (16):

$$\Pi_i + \Delta\Pi_{1i} + \Delta\Pi_{2i} < 0, \quad (38)$$

$$\Sigma_i + \Delta\Sigma_i < 0, \quad (39)$$

where

$$\begin{aligned} \Pi_i &= \begin{bmatrix} P_i A_i + A_i^T P_i + \sum_{j=1}^N \pi_{ij} P_j - \eta P_i & * & P_i B_i \\ P_i A_i - P_i A_{fi} - P_i B_{fi} C_i & P_i A_{fi} + A_{fi}^T P_i + \sum_{j=1}^N \pi_{ij} P_j - \eta P_i & P_i B_i - P_i B_{fi} D_i \\ * & * & -e^{-\eta T} I \end{bmatrix}, \\ \Sigma_i &= \begin{bmatrix} -P_i & 0 & -E_i^T + C_{fi}^T \\ 0 & -P_i & -C_{fi}^T \\ * & * & -\gamma^2 I \end{bmatrix}, \\ \Delta\Pi_{1i} &= \begin{bmatrix} P_i \Delta A_i + \Delta A_i^T P_i & * & 0 \\ P_i \Delta A_i - P_i B_{fi} \Delta C_i & 0 & 0 \\ * & * & 0 \end{bmatrix}, \\ \Delta\Pi_{2i} &= \begin{bmatrix} 0 & * & 0 \\ -P_i \Delta A_{fi} & P_i \Delta A_{fi} + \Delta A_{fi}^T P_i & 0 \\ * & * & 0 \end{bmatrix}, \\ \Delta\Sigma_i &= \begin{bmatrix} 0 & 0 & \Delta C_{fi}^T \\ 0 & 0 & -\Delta C_{fi}^T \\ * & * & 0 \end{bmatrix}. \end{aligned} \quad (40)$$

Referring to Lemma 5,  $\Delta\Pi_{1i}$  and  $\Delta\Pi_{2i}$  can be presented as the following form:

$$\begin{aligned} \Delta\Pi_{1i} &= L_{1i} \Gamma_i(t) L_{2i} + L_{2i}^T \Gamma_i^T(t) L_{1i}^T < \alpha_i^{-1} L_{1i} L_{1i}^T + \alpha_i L_{2i}^T L_{2i}, \\ \Delta\Pi_{2i} &= L_{3i} \Gamma_{fi}(t) L_{4i} + L_{4i}^T \Gamma_{fi}^T(t) L_{3i}^T < \beta_i^{-1} L_{3i} L_{3i}^T + \beta_i L_{4i}^T L_{4i}, \\ \Delta\Sigma_i &= L_{5i} \Gamma_{fi}(t) L_{6i} + L_{6i}^T \Gamma_{fi}^T(t) L_{5i}^T < \lambda_i L_{5i} L_{5i}^T + \lambda_i^{-1} L_{6i}^T L_{6i}, \end{aligned} \quad (41)$$

where  $L_{1i} = \text{col}[P_i M_{1i} \quad P_i M_{1i} - P_i B_{fi} M_{2i} \quad 0]$ ,  $L_{2i} = [N_{1i} \quad 0 \quad 0]$ ,  $L_{3i} = \text{col}[0 \quad P_i M_{3i} \quad 0]$ ,  $L_{4i} = [-N_{2i} \quad N_{2i} \quad 0]$ ,  $L_{5i} = \text{col}[0 \quad 0 \quad M_{4i}]$ , and  $L_{6i} = [N_{2i} \quad -N_{2i} \quad 0]$ .

Then, inequalities (15) and (16) are equivalent to LMIs (38) and (39) by letting  $X_i = P_i A_{fi}$  and  $Y_i = P_i B_{fi}$ .

On the other hand, we set  $\tilde{R}_i = \text{diag}[R_i \quad R_i]$ , and LMI (35) implies that

$$1 < \underline{\sigma}_{\tilde{P}} = \min_{i \in \mathcal{M}} \sigma_{\min}(\tilde{P}_i), \quad \bar{\sigma}_{\tilde{P}} = \max_{i \in \mathcal{M}} \sigma_{\max}(\tilde{P}_i) < \sigma_1. \quad (42)$$

Then, recalling condition (17), we can get LMI (36). This completes the proof.  $\square$

To obtain an optimal finite-time  $L_2$ - $L_\infty$  filtering performance against unknown inputs, uncertainties, and model errors, the attenuation lever  $\gamma^2$  can be reduced to the minimum possible value such that LMIs (33)–(36) are satisfied. The optimization problem can be described as follows:

$$\min_{P_i, X_i, Y_i, C_{fi}, D_{fi}, \alpha_i, \beta_i, \lambda_i, \rho} \rho \quad (43)$$

s. t. LMIs (33)–(36) with  $\rho = \gamma^2$ .

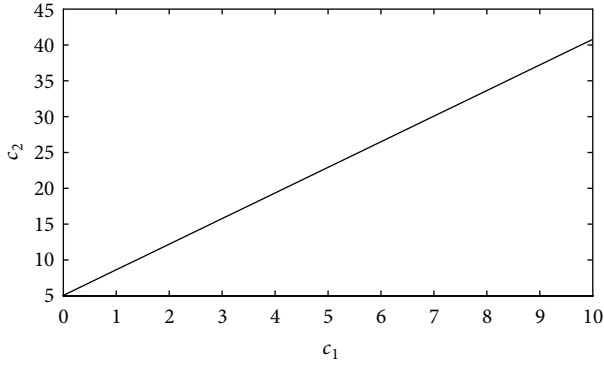


FIGURE 1: The optimal minimal upper bound  $c_2$  with different initial  $c_1$ .

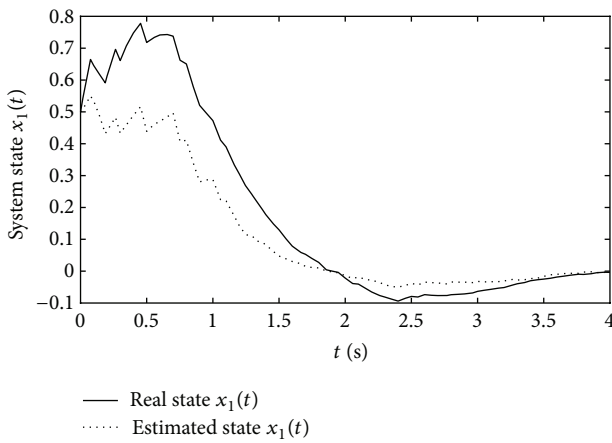


FIGURE 2: System response with state  $x_1(t)$ .

*Remark 8.* In Theorems 6 and 7, if  $c_2$  is a variable to be solved, then (17) and (36) can be always satisfied as long as  $c_2$  is sufficiently large. For these, we can also fix  $\gamma$  and look for the optimal admissible  $c_1$  or  $c_2$  guaranteeing the stochastic finite-time boundedness of desired filtering error dynamic properties.

### 4. Numeral Examples

*Example 9.* Consider a class of MJSSs with two operation modes described as follows.

Mode 1:

$$\begin{aligned}
 A_1 &= \begin{bmatrix} -2 & 1 \\ -1 & -1 \end{bmatrix}, & B_1 &= \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}, & C_1 &= [1 \ 0.5], \\
 D_1 &= 0.1, & E_1 &= [0.1 \ 0.2], & M_{11} &= \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}, \\
 M_{21} &= -0.1, & M_{31} &= \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, & M_{41} &= 0.2, \\
 N_{11} &= [0.2 \ 0.1], & N_{21} &= [0.1 \ -0.2];
 \end{aligned}
 \tag{44}$$

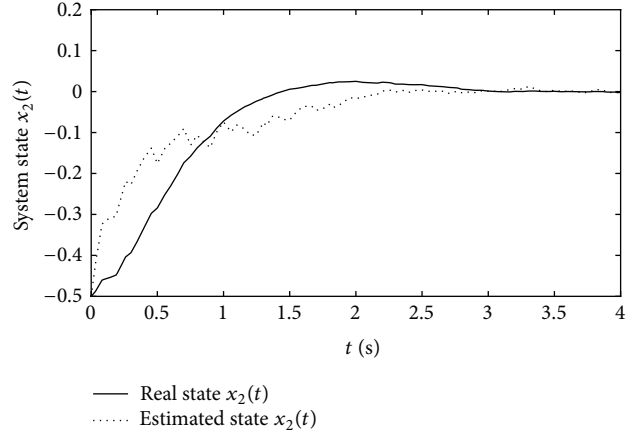


FIGURE 3: System response with state  $x_2(t)$ .

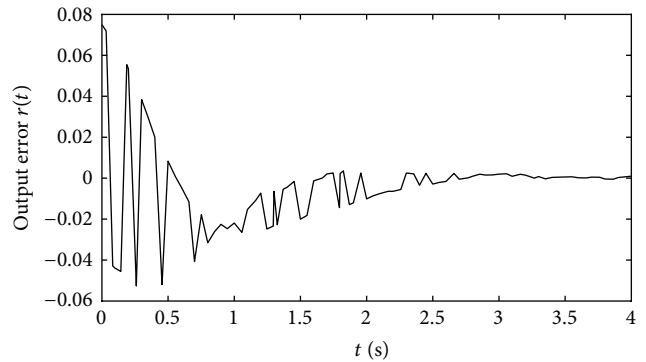


FIGURE 4: System response with output error  $r(t)$ .

Mode 2:

$$\begin{aligned}
 A_2 &= \begin{bmatrix} 0 & 3 \\ -1 & -2 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}, & C_2 &= [1 \ 1], \\
 D_2 &= -0.2, & E_2 &= [0.2 \ -0.1], & M_{12} &= \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix}, \\
 M_{22} &= 0.2, & M_{32} &= \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, & M_{42} &= -0.2, \\
 N_{12} &= [-0.1 \ 0.1], & N_{22} &= [0.1 \ 0.1].
 \end{aligned}
 \tag{45}$$

The mode switching is governed by a Markov chain that has the following transition rate matrix:

$$\Pi = \begin{bmatrix} -0.3 & 0.3 \\ 0.5 & -0.5 \end{bmatrix}.
 \tag{46}$$

In this paper, we choose the initial values for  $W = 2$ ,  $T = 4$ ,  $\eta = 0.25$ , and  $R_i = I_2$ . Then, we fix  $\gamma = 0.8$  and look for the optimal admissible  $c_2$  with different  $c_1$  which can guarantee the stochastic finite-time boundedness of desired filtering error dynamic properties. Figure 1 gives the optimal minimal admissible  $c_2$  with different initial upper bound  $c_1$ .

For  $c_1 = 1$ , we solve LMIs (33)–(36) by Theorem 7 and the optimization algorithm (43) and get the following optimal resilient finite-time  $L_2$ - $L_\infty$  filter:

$$\begin{aligned} A_{f1} &= \begin{bmatrix} -7.7193 & -3.3359 \\ -6.9662 & -6.6546 \end{bmatrix}, \\ B_{f1} &= \begin{bmatrix} 3.0189 \\ 6.7316 \end{bmatrix}, \quad C_{f1} = [0.05 \quad 0.1]; \\ A_{f2} &= \begin{bmatrix} -1.3004 & 0.3608 \\ -3.0043 & -9.2166 \end{bmatrix}, \\ B_{f2} &= \begin{bmatrix} 1.1415 \\ 2.0586 \end{bmatrix}, \quad C_{f2} = [0.1 \quad -0.05]. \end{aligned} \quad (47)$$

And then, we can also get the attenuation lever as  $\gamma = 0.0777$  and the relevant upper bound  $c_2 = 592.45$ .

To show the effectiveness of the designed optimal resilient finite-time  $L_2$ - $L_\infty$  filter, we assume that the unknown inputs are unknown white noise with noise power 0.05 over a finite-time interval  $t \in [0 \quad 4]$ . For the selected initial conditions  $x(0) = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}$  and  $r_0 = 2$ , the simulation results of the jumping modes and the response of system states are shown in Figures 2, 3, and 4. It is clear from the simulation figures that the estimated states can track the real states smoothly.

## 5. Conclusion

The resilient finite-time  $L_2$ - $L_\infty$  filtering problems for a class of stochastic MJSs with uncertain parameters have been studied. By using the Lyapunov-Krasovskii functional approach and LMIs optimization techniques, a sufficient condition is derived such that the filtering dynamic error MJSs are finite-time bounded and satisfy a prescribed level of  $L_2$ - $L_\infty$  disturbance attenuation in a finite time-interval. The robust resilient filter gains can be solved directly by using the existing LMIs. Simulation results illustrate the effectiveness of the proposed a pproach.

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