Research Article

BIBO Stability Analysis for Delay Switched Systems with Nonlinear Perturbation

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The problem of bounded-input bounded-output (BIBO) stability is investigated for a class of delay switched systems with mixed time-varying discrete and constant neutral delays and nonlinear perturbation. Based on the Lyapunov-Krasovskii functional theory, new BIBO stabilization criteria are established in terms of delay-dependent linear matrix inequalities. The numerical simulation is carried out to demonstrate the effectiveness of the results obtained in the paper.

1. Introduction

Time delay is a source of instability and poor performance and appears in many dynamic systems, for example, biological systems, chemical systems, metallurgical processing systems, nuclear reactor systems, and electrical networks [1]. Since the existence of time delays may lead to oscillation, divergence, or instability, considerable effort has been devoted to this area. As an important system performance index, BIBO stability means that any bounded input yields a bounded output and can be considered in many aspects, such as the free system dynamics, the basic single or double loop modulators, and the issues connected with bilinear input/output maps. Consequently, bounded-input bounded-output (BIBO) stability analysis of dynamical systems has attracted many scholars' attention. For instance, in [2], BIBO stability criterion is derived for a three-dimensional fuzzy two-term control system, in [3], the problem on BIBO stabilization for a system with nonlinear perturbations is studied by discussing the existence of the positive definite solution to an auxiliary algebraic Riccati matrix equation, in [4],

based on linear matrix inequality techniques, the stabilization criterion for uncertain time-delay system is presented to guarantee that bounded input can lead to bounded output, and in [5], BIBO stability for feedback control systems with time delay is studied through investigating the boundedness of the solutions for a class of nonlinear Volterra integral equations.

Recently, switched system becomes a research hotspot. Its motivation comes from the fact that many practical systems are inherently multimodal and the fact that some of intelligent control methods are based on the idea of switching between different controllers. Up till now, many investigations about stability of multiform switched systems have been carried out; see, for instance, [6–19] and references therein. Hence, it is our intention in this paper to tackle such an important yet challenging problem for BIBO stability analysis of delay switched systems. In addition, perturbations [20–26] and time delays [27–29] exist in many kinds of systems, and this makes the practical control problem complicated and has received much attention from

scholars. Hence, in this paper, the BIBO stability for delay switched system with mixed time-varying discrete and constant neutral delays and nonlinear perturbation is concerned, and some original BIBO stability criteria are established in terms of linear matrix inequalities (LMIs). Finally, some simulation results are given to illustrate the effectiveness of our results. The main contributions of the paper are of two folds: (1) a delay-dependent technique is applied successfully into the analysis results process; (2) a Lyapunov-Krasovskii functional is constructed to derive a new form of the bounded real lemma (BRL) for the system under consideration.

The remainder of this paper is organized as follows. The model under consideration and some preliminaries are provided in Section 2. Section 3 presents the results on stability analysis. Section 4 gives an illustrative example. At last we conclude the paper in Section 5.

Notations used in this paper are fairly standard. Let R^n be the n-dimensional Euclidean space, $R^{n \times m}$ represents the set of $n \times m$ real matrices, the symbol $*$ denotes the elements below the main diagonal of a symmetric block matrix, $A > 0$ means that A is a real symmetric positive definitive matrix, and I denotes the identity matrix with appropriate dimensions. diag$\{\cdots\}$ denotes the diagonal matrix. $E\{\cdot\}$ refers to the expectation operator with respect to some probability measure P. $\|\cdot\|$ refers to the Euclidean vector norm or the induced matrix 2-norm. The superscript T stands for matrix transposition. $L_{n,h} = L([-h, 0], R^n)$ denotes the Banach space of continuous functions mapping the interval $[-h, 0]$ into R^n with the topology of uniform convergence.

2. Model Description and Preliminaries

First, consider the following delay switched system with nonlinear perturbation:

$$\dot{x}(t) - C_{\sigma(t)}\dot{x}(t - d) = A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t - \tau(t))$$

$$+ f_{\sigma(t)}(t, x(t)) + H_{\sigma(t)}u(t),$$

$$u(t) = L_{\sigma(t)}x(t) + r(t), \qquad (1)$$

$$Y(t) = Jx(t),$$

$$x(t_0 + \theta) = \varphi(\theta), \quad \theta \in [-h \ \ 0],$$

where $x(t) \in R^n$ is the state vector, d is the neutral delay, $0 \le \tau(t) \le h$ is the time-varying discrete delay, $\varphi(\theta) \in L_{n,h}$ is the initial condition, $\sigma(t) : [0, +\infty) \to M = \{1, 2, \ldots, m\}$ is the switching signal, $u(t) \in R^l$ is the control input, $Y(t) \in R^m$ is the system output, $r(t) \in R^l$ is the reference input, and $f(t) \in R^n$ is the nonlinear time-varying perturbation, which satisfies $\|f(t, x(t))\| \le \beta\|x(t)\|$, where β is a positive scalar.

Model (1) can be represented as follows:

$$\dot{x}(t) = y(t),$$

$$y(t) - C_{\sigma(t)}y(t - d) = A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t - \tau(t)) \quad (2)$$

$$+ f_{\sigma(t)}(t) + H_{\sigma(t)}u(t).$$

In this paper, the following well-known lemmas and definitions are needed.

Lemma 1 (see [30]). *For any constant matrices E, G, and F with appropriate dimensions with $F^T F \le kI$, then*

$$2x^T EFGy \le cxEE^T x + \frac{k}{c}y^T G^T Gy, \qquad (3)$$

where $x \in R^n$ and $y \in R^n$ and c and k are positive scalars.

Lemma 2 (see [31]). *For any positive definite matrix $\Phi \in R^{n \times n}$, a positive scalar γ, and the vector function $w : [0, \gamma] \to R^n$ such that the integrations concerned are well defined, then*

$$\left(\int_0^\gamma w(s) \, ds \right)^T \Phi \left(\int_0^\gamma w(s) \, ds \right) \le \gamma \int_0^\gamma w^T(s) \Phi w(s) \, ds. \quad (4)$$

Definition 3 (see [32]). A real-valued vector $r(t) \in L_\infty^n$, if $\|r\|_\infty = \sup_{t_0 \le t < \infty} \|r(t)\| < +\infty$.

Definition 4 (see [32]). The control system with reference input $r(t)$ is BIBO stable, if there exist some positive constants θ_1, θ_2, satisfying

$$\|Y(t)\| \le \theta_1 \|r(t)\|_\infty + \theta_2 \qquad (5)$$

for any reference input $r(t) \in L_\infty^n$.

Assumption 5. We assume that for system (1) there exist Hurwitz linear convex combinations of A_i; that is,

$$\gamma_{\alpha_1, \alpha_2, \ldots, \alpha_m}(A_1, A_2, \ldots, A_m)$$

$$= \{\alpha_1 A_1 + \alpha_2 A_2 + \cdots + \alpha_m A_m : \alpha_1, \alpha_2, \ldots \alpha_m \in [0, 1],$$

$$\alpha_1 + \alpha_2 + \cdots + \alpha_m = 1\}. \qquad (6)$$

3. Main Results

In this section, we will establish some BIBO stability criteria using Lyapunov-Krasovskii functional theory and linear matrix inequalities.

Theorem 6. *For given positive scalars h and k, switched system* (1) *is BIBO stable, if there exist $A \in \gamma_{\alpha_1, \alpha_2, \ldots, \alpha_m}(A_1, A_2, \ldots, A_m)$, $B \in \gamma_{\alpha_1, \alpha_2, \ldots, \alpha_m}(B_1, B_2, \ldots, B_m)$, $C \in \gamma_{\alpha_1, \alpha_2, \ldots, \alpha_m}(C_1, C_2, \ldots, C_m)$, $f \in \gamma_{\alpha_1, \alpha_2, \ldots, \alpha_m}(f_1, f_2, \ldots, f_m)$, $H \in \gamma_{\alpha_1, \alpha_2, \ldots, \alpha_m}(H_1, H_2, \ldots, H_m)$, $L \in \gamma_{\alpha_1, \alpha_2, \ldots, \alpha_m}(L_1, L_2, \ldots, L_m)$, positive scalars ε, σ, matrices P_2, P_3, U, V, W,*

and symmetric positive definite matrices P, R, M, S, Q, satisfying

$$\Sigma + \Xi + \Xi^T + he^{kh}W < 0,$$

$$\begin{bmatrix} W & U \\ * & S - R_{22} \end{bmatrix} > 0, \tag{7}$$

$$\begin{bmatrix} W & V \\ * & S \end{bmatrix} > 0,$$

where

$$\Sigma = \begin{bmatrix} \Sigma_{1,1} & \Sigma_{1,2} & \Sigma_{1,3} & \Sigma_{1,4} & \Sigma_{1,5} & \Sigma_{1,6} & \Sigma_{1,7} \\ * & \Sigma_{2,2} & \Sigma_{2,3} & \Sigma_{2,4} & \Sigma_{2,5} & \Sigma_{2,6} & \Sigma_{2,7} \\ * & * & \Sigma_{3,3} & 0 & 0 & 0 & 0 \\ * & * & * & \Sigma_{4,4} & 0 & 0 & 0 \\ * & * & * & * & \Sigma_{5,5} & 0 & 0 \\ * & * & * & * & * & \Sigma_{6,6} & 0 \\ * & * & * & * & * & * & \Sigma_{7,7} \end{bmatrix},$$

$$\Sigma_{1,1} = P_2 A + A^T P_2^T + P_2 HL + L^T H^T P_2^T$$
$$\qquad + Q + \varepsilon\beta^2 + kP + h^2 e^{kh} N,$$

$$\Sigma_{1,2} = P - P_2 + A^T P_3^T + L^T H^T P_3^T,$$

$$\Sigma_{1,3} = P_2 B + R_{12}^T,$$

$$\Sigma_{2,2} = he^{kh}S - P_3 - P_3^T + M,$$

$$\Sigma_{2,3} = P_3 B,$$

$$\Sigma_{3,3} = hR_{11} - R_{12} - R_{12}^T,$$

$$\Sigma_{1,4} = P_2 C, \tag{8}$$

$$\Sigma_{2,4} = P_3 C,$$

$$\Sigma_{4,4} = -e^{-kd}M,$$

$$\Sigma_{1,5} = P_2,$$

$$\Sigma_{2,5} = P_3,$$

$$\Sigma_{5,5} = -\varepsilon I,$$

$$\Sigma_{1,6} = 0,$$

$$\Sigma_{2,6} = 0,$$

$$\Sigma_{6,6} = -e^{-kh}Q,$$

$$\Sigma_{1,7} = P_2 H,$$

$$\Sigma_{2,7} = P_3 H,$$

$$\Sigma_{7,7} = -\sigma I,$$

$$\Xi = \begin{bmatrix} U & 0 & -U+V & 0 & 0 & -V & 0 \end{bmatrix}.$$

Proof. Since

$$A \in \gamma_{\alpha_1,\alpha_2,\ldots,\alpha_m}(A_1, A_2, \ldots, A_m),$$

$$B \in \gamma_{\alpha_1,\alpha_2,\ldots,\alpha_m}(B_1, B_2, \ldots, B_m),$$

$$C \in \gamma_{\alpha_1,\alpha_2,\ldots,\alpha_m}(C_1, C_2, \ldots, C_m),$$

$$L \in \gamma_{\alpha_1,\alpha_2,\ldots,\alpha_m}(L_1, L_2, \ldots, L_m), \tag{9}$$

$$H \in \gamma_{\alpha_1,\alpha_2,\ldots,\alpha_m}(H_1, H_2, \ldots, H_m),$$

$$f \in \gamma_{\alpha_1,\alpha_2,\ldots,\alpha_m}(f_1, f_2, \ldots, f_m),$$

there exist $\alpha_i \in [0,1]$, $i = 1, \ldots, m$, satisfing

$$\sum_{i=1}^{m}\alpha_i = 1, \qquad A = \sum_{i=1}^{m}\alpha_i A_i, \qquad B = \sum_{i=1}^{m}\alpha_i B_i,$$

$$C = \sum_{i=1}^{m}\alpha_i C_i, \qquad L = \sum_{i=1}^{m}\alpha_i L_i, \tag{10}$$

$$H = \sum_{i=1}^{m}\alpha_i H_i, \qquad f = \sum_{i=1}^{m}\alpha_i f_i.$$

From (7), we can obtain

$$\sum_{i=1}^{m}\alpha_i\left(\Sigma_i + \Xi + \Xi^T + he^{kh}W\right) < 0, \tag{11}$$

where

$$\Sigma_i = \begin{bmatrix} \Sigma_{i,1,1} & \Sigma_{i,1,2} & \Sigma_{i,1,3} & \Sigma_{i,1,4} & \Sigma_{i,1,5} & \Sigma_{i,1,6} & \Sigma_{i,1,7} \\ * & \Sigma_{i,2,2} & \Sigma_{i,2,3} & \Sigma_{i,2,4} & \Sigma_{i,2,5} & \Sigma_{i,2,6} & \Sigma_{i,2,7} \\ * & * & \Sigma_{i,3,3} & 0 & 0 & 0 & 0 \\ * & * & * & \Sigma_{i,4,4} & 0 & 0 & 0 \\ * & * & * & * & \Sigma_{i,5,5} & 0 & 0 \\ * & * & * & * & * & \Sigma_{i,6,6} & 0 \\ * & * & * & * & * & * & \Sigma_{i,7,7} \end{bmatrix},$$

$$\Sigma_{i,1,1} = P_2 A_i + A_i^T P_2^T + P_2 H_i L_i$$
$$+ L_i^T H_i^T P_2^T + Q + \varepsilon\beta^2 + kP + h^2 e^{kh} N,$$

$$\Sigma_{i,1,2} = P - P_2 + A_i^T P_3^T + L_i^T H_i^T P_3^T,$$

$$\Sigma_{i,1,3} = P_2 B_i + R_{12}^T,$$

$$\Sigma_{i,2,2} = h e^{kh} S - P_3 - P_3^T + M,$$

$$\Sigma_{i,2,3} = P_3 B_i,$$

$$\Sigma_{i,3,3} = h R_{11} - R_{12} - R_{12}^T,$$

$$\Sigma_{i,1,4} = P_2 C_i,$$

$$\Sigma_{i,2,4} = P_3 C_i,$$

$$\Sigma_{i,4,4} = -e^{kd} M,$$

$$\Sigma_{i,1,5} = P_2,$$

$$\Sigma_{i,2,5} = P_3,$$

$$\Sigma_{i,5,5} = -\varepsilon I,$$

$$\Sigma_{i,1,6} = 0,$$

$$\Sigma_{i,2,6} = 0,$$

$$\Sigma_{i,6,6} = -e^{-kh} Q,$$

$$\Sigma_{i,1,7} = P_2 H_i,$$

$$\Sigma_{i,2,7} = P_3 H_i,$$

$$\Sigma_{i,7,7} = -\sigma I.$$

$$(12)$$

Let

$$\Omega_i = \left\{ q^T \mid q^T \left(\Sigma_i + \Xi + \Xi^T + hW \right) q < 0, \right.$$
$$\left. q(t) = \left[q_1^T, \ldots, q_i^T, \ldots, q_7^T \right]^T, q_i \in R^n \right\}. \quad (13)$$

We obtain

$$\bigcup_{i=1}^m \Omega_i = \frac{R^{7n}}{\{0\}}. \quad (14)$$

Construct a set as

$$\widetilde{\Omega}_1 = \Omega_1,$$

$$\widetilde{\Omega}_2 = \Omega_2 - \widetilde{\Omega}_1, \ldots,$$

$$\widetilde{\Omega}_i = \Omega_i - \bigcup_{j=1}^{i-1} \widetilde{\Omega}_j, \ldots, \quad (15)$$

$$\widetilde{\Omega}_m = \Omega_m - \bigcup_{j=1}^{m-1} \widetilde{\Omega}_j.$$

We get

$$\bigcup_{i=1}^m \widetilde{\Omega}_i = \frac{R^{6n}}{\{0\}}, \quad \widetilde{\Omega}_i \cap \widetilde{\Omega}_j = \phi, \quad i \neq j. \quad (16)$$

Construct the switching rule (SR): $\sigma = i$, for all $q \in \widetilde{\Omega}_i$, $i = 1, \ldots, m$. The ith subsystem is activated when $q \in \widetilde{\Omega}_i$, $i = 1, \ldots, m$. Choose the following Lyapunov-Krasovskii functional candidate:

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$
$$+ V_4(t) + V_5(t) + V_6(t), \quad (17)$$

with

$$V_1(t) = \left(x^T(t) \ \ y^T(t) \right) \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P & 0 \\ P_2^T & P_3^T \end{bmatrix} \left(x^T(t) \ \ y^T(t) \right)^T,$$

$$V_2(t) = \int_{-h}^0 \int_{t+\beta}^t y^T(\alpha) e^{k(\alpha-t+h)} S y(\alpha) \, d\alpha \, d\beta,$$

$$V_3(t) = \int_{-h}^t \int_{\beta-\tau(\beta)}^\beta \eta^T e^{k(\beta-t)} R \eta \, d\alpha \, d\beta,$$

$$V_4(t) = \int_{t-d}^t y^T(s) e^{k(s-t)} M y(s) \, ds,$$

$$V_5(t) = \int_{-h}^0 \int_{t+\beta}^t \xi^T(\alpha) e^{k(\alpha-t+h)} W \xi(\alpha) \, d\alpha \, d\beta,$$

$$V_6(t) = \int_{t-h}^t x^T(s) e^{k(s-t)} Q x(s) \, ds,$$

$$(18)$$

where

$$\eta = \left[x(\beta - \tau(\beta)) \ \ y(\alpha) \right]^T,$$

$$\xi = \left[x^T(t) \ \ y^T(t) \ \ x^T(t-\tau(t)) \ \ y^T(t-d) \ \ f^T(t) \ \ x(t-h) \ \ r(t) \right]^T. \quad (19)$$

The derivative of $V(t)$ along the trajectory of the ith subsystem is given by

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t)$$
$$+ \dot{V}_4(t) + \dot{V}_5(t) + \dot{V}_6(t), \quad (20)$$

where

$$\dot{V}_1(t) = 2\left[x^T(t)\ \ y^T(t)\right]\begin{bmatrix} P & P_2 \\ 0 & P_3 \end{bmatrix}\begin{bmatrix} y(t) \\ 0 \end{bmatrix}$$

$$= 2x^T(t)Py(t) + 2\left(x^T(t)P_2 + y^T(t)P_3\right)$$

$$\times \left(-y(t) + (A_i + H_iL_i)x(t)\right.$$

$$+ B_ix(t - \tau(t)) + C_iy(t-d)$$

$$\left. + f_i(t) + H_ir(t)\right), \tag{21}$$

$$\dot{V}_2(t) = hy^T(t)e^{kh}Sy(t)$$

$$- \int_{t-\tau(t)}^{t} y^T(s)e^{k(s-t+h)}Sy(s)\,ds$$

$$- \int_{t-h}^{t-\tau(t)} y^T(s)e^{k(s-t+h)}Sy(s)\,ds - kV_2(t)$$

$$\leq hy^T(t)e^{kh}Sy(t) \tag{22}$$

$$- \int_{t-\tau(t)}^{t} y^T(s)Sy(s)\,ds$$

$$- \int_{t-h}^{t-\tau(t)} y^T(s)Sy(s)\,ds - kV_2(t),$$

$$\dot{V}_3(t) = \tau(t)x^T(t-\tau(t))R_{11}x(t-\tau(t))$$

$$+ 2x^T(t-\tau(t))R_{12}x(t)$$

$$- 2x^T(t-\tau(t))R_{12}x(t-\tau(t))$$

$$+ \int_{t-\tau(t)}^{t} y(s)R_{22}y(s)\,ds - kV_3(t)$$

$$\leq hx^T(t-\tau(t))R_{11}x(t-\tau(t)) \tag{23}$$

$$+ 2x^T(t)R_{12}^T x(t-\tau(t))$$

$$- 2x^T(t-\tau(t))R_{12}x(t-\tau(t))$$

$$+ \int_{t-\tau(t)}^{t} y(s)R_{22}y(s)\,ds - kV_3(t),$$

$$\dot{V}_4(t) = y^T(t)My(t) - y^T(t-d)e^{-kd}My(t-d) - kV_4(t), \tag{24}$$

$$\dot{V}_5(t) = h\xi^T(t)e^{kh}W\xi(t) - \int_{t-\tau(t)}^{t} \xi^T(s)e^{k(s-t+h)}W\xi(s)\,ds$$

$$- \int_{t-h}^{t-\tau(t)} \xi^T(s)e^{k(s-t+h)}W\xi(s)\,ds - kV_5(t)$$

$$\leq h\xi^T(t)e^{kh}W\xi(t) - \int_{t-\tau(t)}^{t} \xi^T(s)W\xi(s)\,ds$$

$$- \int_{t-h}^{t-\tau(t)} \xi^T(s)W\xi(s)\,ds - kV_5(t), \tag{25}$$

$$\dot{V}_6(t) = x^T(t)Qx(t) - x^T(t-h)e^{-kh}Qx(t-h) - kV_6(t). \tag{26}$$

According to Leibniz-Newton formula, we have

$$2\xi^T U\left[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} y^T(s)\,ds\right] = 0,$$

$$2\xi^T V\left[x(t-\tau(t)) - x(t-h) - \int_{t-h}^{t-\tau(t)} y^T(s)\,ds\right] = 0. \tag{27}$$

From the obtained derivative terms in (21)–(26) and adding the left-hand side of (27) into (20), we obtain the following result:

$$\dot{V}(t) \leq \xi^T\left(\Sigma_i + \Omega + \Omega^T + he^{kh}W\right)\xi$$

$$- \int_{t-\tau(t)}^{t} \zeta^T\Phi_1\zeta\,ds - \int_{t-h}^{t-\tau(t)} \zeta^T\Phi_2\zeta\,ds \tag{28}$$

$$- kV(t) + \sigma\|r(t)\|_\infty^2,$$

where

$$\zeta = \left[\xi^T\ \ y^T(s)\right]^T,$$

$$\Phi_1 = \begin{bmatrix} W & U \\ * & S-R_{22} \end{bmatrix}, \qquad \Phi_2 = \begin{bmatrix} W & V \\ * & S \end{bmatrix}. \tag{29}$$

When $\xi \in \bigcup_{i=1}^{m} \widetilde{\Omega}_i$, $i = 1, \dots, m$, we can obtain

$$\dot{V}(t) \leq \sum_{i=1}^{m} a_i\left(\xi^T\left(\Sigma_i + \Omega + \Omega^T + he^{kh}W\right)\xi\right.$$

$$\left. - \int_{t-\tau(t)}^{t} q^T\Phi_1q\,ds - \int_{t-h}^{t-\tau(t)} q^T\Phi_2q\,ds\right) \tag{30}$$

$$= \xi^T\left(\Sigma + \Xi + \Xi^T + he^{kh}W\right)\xi$$

$$- \int_{t-\tau(t)}^{t} q^T\Phi_1q\,ds - \int_{t-h}^{t-\tau(t)} q^T\Phi_2q\,ds.$$

According to (7), we have

$$\dot{V}(t) \leq -kV(t) + \sigma\|r(t)\|_\infty^2. \tag{31}$$

Hence,

$$\left(V(t)e^{kt}\right)' \leq \left(\dot{V}(t) + kV(t)\right)e^{kt} \leq \sigma\|r(t)\|_\infty^2 e^{kt}. \tag{32}$$

Integrating the previous inequality from t_0 to t yields

$$V(t)e^{kt} \leq V(t_0)e^{kt_0} + \sigma\|r(t)\|_\infty^2 \int_{t_0}^{t} e^{ks}\,ds. \tag{33}$$

Then, we have

$$\lambda_{\min}(P)\|x(t)\|^2 \leq V(t) \leq V(t_0)e^{-k(t-t_0)}$$

$$+ \sigma\|r(t)\|_\infty^2 \int_{t_0}^{t} e^{-k(t-s)}\,ds \tag{34}$$

$$\leq V(t_0)e^{-k(t-t_0)} + \frac{\sigma\|r(t)\|_\infty^2}{k}.$$

Define

$$\psi = \max \left\{ \sup_{h \le \theta \le 0} \left\| \varphi \left(t_0 + \theta \right) \right\|, \sup_{h \le \theta \le 0} \left\| \varphi' \left(t_0 + \theta \right) \right\| \right\}. \quad (35)$$

According to (17), we have

$$V \left(t_0 \right) \le \left[\lambda_{\max} \left(P \right) + h^2 e^{kh} \lambda_{\max} \left(S \right) + h^2 \lambda_{\max} \left(R \right) \right.$$
$$\left. + h^2 e^{kh} \lambda_{\max} \left(W \right) + h \lambda_{\max} \left(Q \right) + h \lambda_{\max} \left(M \right) \right] \psi^2. \quad (36)$$

Hence, the following inequality can be concluded:

$$\left\| x \left(t \right) \right\|^2 \le \frac{a}{\lambda_{\min} \left(P \right)} \psi^2 + \frac{\sigma}{k \lambda_{\min} \left(P \right)} \left\| r \left(t \right) \right\|_{\infty}^2$$
$$\le \left(\sqrt{\frac{a}{\lambda_{\min} \left(P \right)}} \psi + \sqrt{\frac{\sigma}{k \lambda_{\min} \left(P \right)}} \left\| r \left(t \right) \right\|_{\infty} \right)^2, \quad (37)$$

where

$$a = \lambda_{\max} \left(P \right) + h^2 e^{kh} \lambda_{\max} \left(S \right) + h^2 \lambda_{\max} \left(R \right)$$
$$+ h^2 e^{kh} \lambda_{\max} \left(W \right) + h \lambda_{\max} \left(Q \right) + h \lambda_{\max} \left(M \right). \quad (38)$$

Then,

$$\left\| Y \right\| \le \left\| J \right\| \left\| x \right\| \le \theta_1 + \theta_2 \left\| r \left(t \right) \right\|_{\infty}, \quad (39)$$

with

$$\theta_1 = \left\| J \right\| \sqrt{\frac{a}{\lambda_{\min} \left(P \right)}} \psi, \qquad \theta_2 = \left\| J \right\| \sqrt{\frac{\sigma}{k \lambda_{\min} \left(P \right)}}. \quad (40)$$

Therefore, switched system (1) is BIBO stable. This completes the proof. □

## 4. Simulation Results

As an example, let us consider system (1) with the following parameters:

$$A_1 = \begin{bmatrix} -1.5 & 0.5 \\ 0 & -1.5 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -0.2 & 0.3 \\ 0 & -0.4 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0 & -0.2 \end{bmatrix}, \qquad H_1 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -2 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -0.4 & 0 \\ 0.3 & -0.4 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} -0.4 & 0.3 \\ 0 & -0.3 \end{bmatrix}, \quad (41)$$

$$H_2 = \begin{bmatrix} -0.3 & 0 \\ 0 & -0.3 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix},$$

$$J = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad d = 1.5.$$
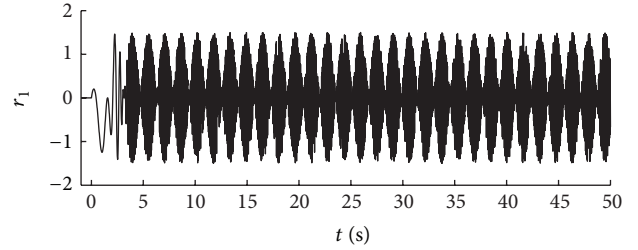
(a)
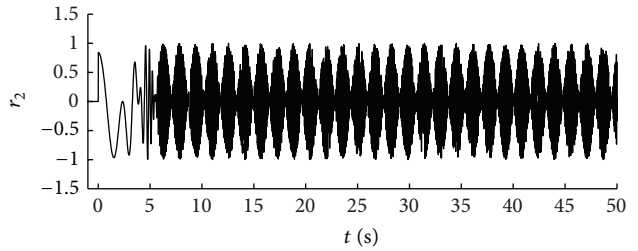
(b)

FIGURE 1: Time response of the reference input variable $r(t)$.

Remark 7. When the BIBO parameter k and the constant parameter β are given, the upper bound of time delay h of system (1) can be determined by solving the following optimization problem:

$$\max h \quad (42)$$
$$\text{when (7) is satisfied}, k \text{ and } \beta \text{ are fixed.}$$

Now we consider the influence of parameters k and β on the maximal allowable delay in Tables 1 and 2.

Remark 8. From Tables 1 and 2, it can be seen that the maximal allowable delay decreases with the rise of the parameter k and increases as the parameter β is reduced.

Then, we carry out some numerical simulation to verify the proposed methodology. The numerical simulation is with initial value $\varphi(\theta) = [-1.5; 1]^T$, $t \in (-1.5, 0)$, and following parameters

$$\tau(t) = 1.1 + 0.4\cos^2(5t),$$

$$f(t, x(t)) = \frac{\beta \left[|x(t) + 1| + |x(t) - 1| \right]}{2},$$

$$r(t) = \left[1.5 \sin(2t) \cos(e^t), \cos(2t) \sin\left(\frac{e^t}{t+1} \right) \right]^T, \quad (43)$$

$$k = 0.1, \qquad \beta = 0.1.$$

The switching signals are produced randomly with switching interval 0.2 seconds.

Remark 9. Figure 1 depicts the time response of system reference input variable $r(t)$, and Figure 2 depicts the time response of switching signals. sw_1 denotes the switching
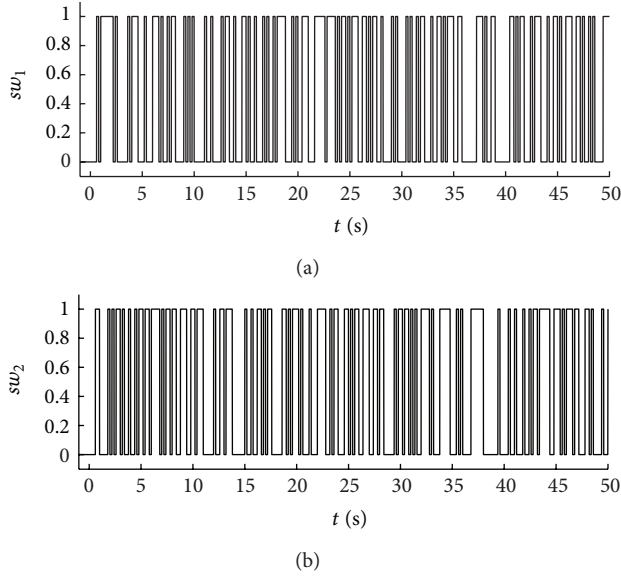
(a)

(b)

FIGURE 2: Time response of the switching variables sw_1 and sw_2.
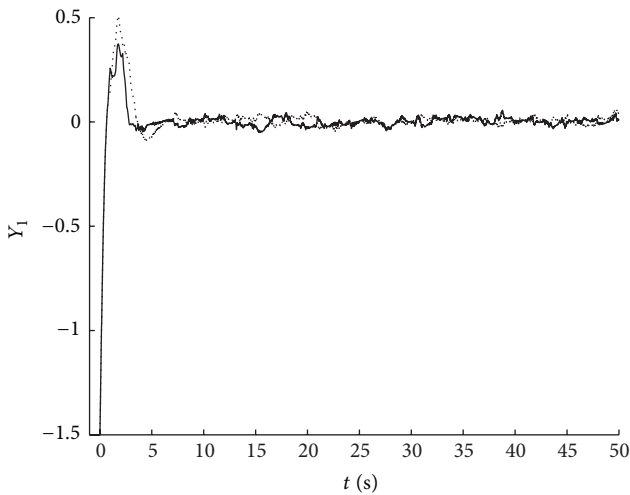

FIGURE 3: Time response of the output variable $Y_1(t)$.

TABLE 1: The maximal allowable delay for different parameters k when β is 0.1.

$k = 0$	$k = 0.1$	$k = 0.2$	$k = 0.5$
$h_{max} = 3.3395$	$h_{max} = 2.5482$	$h_{max} = 2.1313$	$h_{max} = 1.5197$

TABLE 2: The maximal allowable delay for different nonlinear parameters β when k is 0.1.

$\beta = 0$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.5$
$h_{max} = 2.6608$	$h_{max} = 2.5482$	$h_{max} = 2.4370$	$h_{max} = 2.1385$

signal added to the system for the first time, and sw_2 denotes the switching signal added to the system for the second time; Figure 3 depicts the time response of system output variable $Y_1(t)$, and Figure 4 depicts the time response of system output
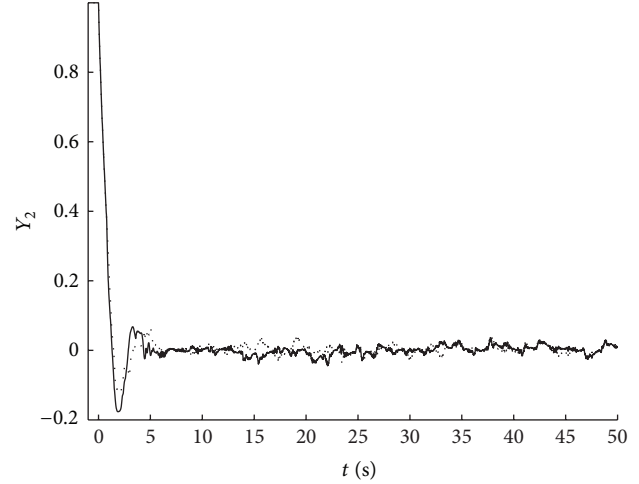

FIGURE 4: Time response of the output variable $Y_2(t)$.

variable $Y_2(t)$. The solid line denotes the output variable of the switched system with the switching signal sw_1, and the dashed line denotes the output variable of the switched system with the switching signal sw_2. From the figures it can be seen that the system output jitters in a range with a given bounded input after a period of time, which means that the system is BIBO stable and demonstrates the effectiveness of our theoretical results.

5. Conclusions

We have studied bounded-input bounded-output stability for a class of delay switched systems with nonlinear perturbation. Based on the Lyapunov-Krasovskii functional theory, new BIBO stabilization criteria were established in terms of delay-dependent linear matrix inequalities. Some numerical simulations have been conducted to demonstrate the effectiveness of the theoretical results obtained in this paper. Future work will investigate fault detection for delay switched systems.

Acknowledgments

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