Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 621623, 11 pages
http://dx.doi.org/10.1155/2013/621623

Research Article

Delay-Distribution-Dependent Consensus for Second-Order Leader-Follower Nonlinear Multiagent Systems via Pinning Control

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Received 25 June 2013; Revised 27 August 2013; Accepted 27 August 2013

Academic Editor: Hamid Reza Karimi

This paper investigates the consensus problem for second-order leader-follower nonlinear multiagent systems with general network topologies. A pinning control algorithm is proposed, where it includes time-varying delay information. By using the information of delay-partition and delay-distribution and constructing an appropriate Lyapunov-Krasovskii functional, the consensus criteria are derived to achieve leader-follower consensus for multiagent systems, which are in the form of linear inequalities that can be solved by employing the semidefinite programme method. Moreover, this paper addresses what kind of agents and how many agents should be pinned. Two numerical examples are presented to further demonstrate the effectiveness of the proposed approach.

1. Introduction

The coordinated and cooperative control of multiple autonomous vehicles is a problem which has been much researched in recent years due to its applications in the formation control of unmanned air vehicles, the cooperative control of mobile robots, the design of distributed sensor networks, and so on. Particularly, consensus in multiagent systems, which means all the agents can reach an agreement on certain concern or interest, has been extensively studied in the past few years [1–8].

A particularly interesting topic is the consensus of a group of agents with a leader, where the leader is a special agent whose motion is independent of all the other agents and thus is followed by all the other ones, and such a problem is commonly called leader-following consensus problem [9–13]. Leader-following consensus problem is investigated in [9], and the corresponding conclusion is that if all the agents are jointly connected with their leader, their states would converge to that of the leader. Reference [11] provides a rigorous proof for the consensus using an extension of LaSalle's invariance principle. A neighbor-based observer design approach is proposed in [12] and discusses a group of mobile agents with an active leader moving with an unknown velocity. In [13], the leader-following consensus problem is considered by designing distributed controllers using local information to ensure that all the agents follow the leader. It is well known that time delay often causes undesirable dynamic behaviors such as oscillation, performance degradation, and instability of the network [14–16]; therefore, various approaches to consensus analysis for multiagent systems with time delay have been investigated in the literature [17, 18]. In [17], two different cases of coupling topologies are investigated for coordination problem of multiagent system with time-varying coupling delays. Reference [18] studies the leader-following consensus for a multiagent system with a varying-velocity leader and time-varying delays. However, less consideration has been paid to the delay-dependent consensus conditions for multiagent systems with time-varying delays. It is noted that all physical systems are nonlinear in nature [19] and that even different agents may have different dynamics. It is very challenging task to discuss consensus problem of nonlinear multiagent systems [20, 21]. Reference [21] proposes leader-following consensus algorithms based on pinning control for second-order nonlinear multiagent systems and provides a pinned-agent selection scheme to determine what kind of followers and how many followers should be pinned. Considering that it is practically impossible to apply control actions to all agents in a large-scale multiagent system, and some authors develop consensus algorithms based on pinning control [22–25]. It is well known that pinning control is an effective method to solve the problem of nonlinear system, it would be beneficial to apply these control techniques to study the consensus problem of multiagent system. In many practical systems, such as networked control systems, the probability distribution of the time delay in the interval is an important characteristic for the network conditions, and its probability distribution can be measured by the statistical method [26–28]. Therefore, the information of probability distribution of the delay should be employed in the model. To the best of the authors' knowledge, no result has been reported for the consensus problem for second-order leader-follower nonlinear multiagent systems via pinning control, when both the information of variation range of the time delay and the information of variation probability of the time delay in an interval can be observed, which motivates the present study.

Motivated by the above discussions, in this paper, we investigate the consensus problem for second-order leader-follower nonlinear multiagent systems. Based on the probability distribution of delay, a new type of multiagent system model with stochastic parameters in the coefficient matrices is proposed. A pinning control algorithm is proposed, and the delay-partition-dependent and delay-distribution-dependent consensus criteria are derived by combining delay-partitioning method and Lyapunov-Krasovskii functional method. Moreover, this paper addresses what kind of agents and how many agents should be pinned.

The main contributions of this paper can be summarized as follows. (1) Based on the probability distribution of delay, a novel multiagent system model with stochastic parameters in the coefficient matrices is introduced. (2) The new pinning control algorithm is proposed, and this paper gives what kind of agents and how many agents should be pinned.

The rest of this paper is organized as follows. In Section 2, problem formulation and preliminaries are briefly outlined. In Section 3, the consensus criteria are derived to achieve leader-follower consensus for multiagent systems. In Section 4, two simulation examples are provided to show the advantages of the obtained results, and some conclusions are drawn in Section 5.

Notation. The notation used in the paper is fairly standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is a set of real $n \times m$ matrices. The notation $X > 0$ (resp., $X < 0$), for $X \in \mathbb{R}^{n\times n}$, means that the matrix X is real symmetric positive definite (resp., negative definite). $\text{diag}\{\cdots\}$ stands for a block-diagonal matrix. $\|\cdot\|$ denotes the Euclidean norm in $\mathbb{R}^n$. The superscript "T" stands for matrix transposition. $E\{\cdot\}$ stands for mathematical expectation. The Kronecker product of matrices $Q \in \mathbb{R}^{m\times n}$ and $R \in \mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{mp\times nq}$ and denoted as $Q \otimes R$. In this paper, if not explicitly stated, matrices are assumed to have compatible dimensions.

2. Preliminaries

Let $g = \{\nu, \varepsilon, A\}$ be a weighted diagraph of order N, with the set of nodes $\nu = \{\nu_1, \nu_2, \ldots, \nu_N\}$, an edge set $\varepsilon \subseteq \nu \times \nu$, and a weighted adjacency matrix $A = (a_{ij})_{N\times N}$ with nonnegative elements, directed edge e_{ij} in network g, is denoted by the ordered pair of nodes (ν_i, ν_j), which means that node ν_j can receive information from node ν_i. The elements of the adjacency matrix A are defined as $a_{ij} > 0$ if and only if there is a directed edge (ν_j, ν_i) in g; otherwise $a_{ij} = 0$. We assume that $a_{ii} = 0$ for all $i \in \nu$. The neighbor set of node i is defined by $N_i = \{j \in \nu \mid (\nu_j, \nu_i) \in \varepsilon\}$, and the in-degree and out-degree of node i are defined as

$$\deg_{\text{in}}(i) = \sum_{j=1, j\neq i}^{N} a_{ij}, \qquad \deg_{\text{out}}(i) = \sum_{j=1, j\neq i}^{N} a_{ji}. \tag{1}$$

A diagraph is called balanced if $\deg_{\text{in}}(i) = \deg_{\text{out}}(i)$ for all $i \in \nu$.

The Laplacian matrix $L = (l_{ij})_{N\times N}$ associated with the adjacency matrix A is defined as

$$\begin{aligned} l_{ij} &= -a_{ij} \quad (i \neq j) \\ l_{ii} &= -\sum_{j=1, j\neq i}^{N} l_{ij} \quad (i, j = 1, 2, \ldots, N) \end{aligned} \tag{2}$$

which ensures the diffusion property that $\sum_{j=1}^{N} l_{ij} = 0$.

A directed path from node ν_i to ν_j in g is a sequence of edges $((\nu_i, \nu_{i1}), (\nu_{i1}, \nu_{i2}), \ldots, (\nu_{il}, \nu_j))$ in the directed graph which is strongly connected if for any two distinct nodes ν_j and ν_i, there exists a directed path form node ν_j to ν_i. A directed graph has a directed spanning tree if there exists at least one node called root which has a directed path to all other nodes.

The dynamics of the virtual leader is described by the following second-order nonlinear system:

$$\begin{aligned} \dot{x}_r(t) &= v_r(t), \\ \dot{v}_r(t) &= f(t, x_r(t), v_r(t)), \end{aligned} \tag{3}$$

where $x_r(t) = (x_{r1}(t), x_{r2}(t), \ldots, x_{rn}(t))^T \in \mathbb{R}^n$ and $v_r(t) = (v_{r1}(t), v_{r2}(t), \ldots, v_{rn}(t))^T \in \mathbb{R}^n$ are the position and velocity states of the virtual leader, respectively. $f(t, x_r(t), v_r(t)) = (f_1(t, x_r(t), v_r(t)), f_2(t, x_r(t), v_r(t)), \ldots, f_n(t, x_r(t), v_r(t)))^T$ is a nonlinear vector-valued continuous bounded function.

The second-order dynamics of multiagent systems consisting of N-coupled autonomous agents is given as follows:

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) + u_{1i}(t), \\ \dot{v}_i(t) &= f(t, x_i(t), v_i(t)) + u_{2i}(t) \quad (i = 1, 2, \ldots, N), \end{aligned} \tag{4}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$ and $v_i(t) = (v_{i1}(t), v_{i2}(t), \ldots, v_{in}(t))^T \in \mathbb{R}^n$ are the position and velocity states of agent i, respectively. $u_{1i}(t)$ and $u_{2i}(t)$ are the control inputs for agent i to be designed.

Definition 1. The multiagent systems (4) are said to achieve second-order leader-following consensus if, for any initial conditions, the state of each following agent i $(i = 1, 2, \ldots, N)$ satisfies

$$\lim_{t\to\infty} \|x_i(t) - x_r(t)\| = 0 \qquad \lim_{t\to\infty} \|v_i(t) - v_r(t)\| = 0. \tag{5}$$

Motivated by the coupling rule used in [17], the consensus algorithm is designed based on pinning control such that all agents in nonlinear multiagent systems (4) can asymptotically follow the virtual leader (3). Without loss of generality, rearrange the order of all agents, and the sort method will be given in Section 3. Let the first l $(1 \leq l < N)$ agents in multiagent systems (4) be controlled, and the controllers have the following form:

$$\begin{aligned} u_{1i}(t) = &-\sum_{j\in N_i} a_{ij}\left\{\left(x_i(t)-x_j(t)\right)\right.\\ &\left.+\left(x_i(t-\tau(t))-x_j(t-\tau(t))\right)\right\}\\ &-\delta_i\left\{\left(x_i(t)-x_r(t)\right)\right.\\ &\left.+\left(x_i(t-\tau(t))-x_r(t-\tau(t))\right)\right\}\\ u_{2i}(t) = &-\sum_{j\in N_i} a_{ij}\left\{\left(v_i(t)-v_j(t)\right)\right.\\ &\left.+\left(v_i(t-\tau(t))-v_j(t-\tau(t))\right)\right\}\\ &-\delta_i\left\{\left(v_i(t)-v_r(t)\right)\right.\\ &\left.+\left(v_i(t-\tau(t))-v_r(t-\tau(t))\right)\right\}, \end{aligned} \tag{6}$$

where the local feedback gains satisfy $\delta_i > 0$ $(i = 1, 2, \ldots, l)$ and $\delta_i = 0$ $(i = l+1, l+2, \ldots, N)$. Also $\tau(t) \in [0, \tau]$ is the bounded time-varying delay, and τ is a positive scalar.

For notational convenience, we define

$$\begin{aligned} x(t) &= \left(x_1^T(t), x_2^T(t), \ldots, x_N^T(t)\right)^T \in \mathbb{R}^{Nn},\\ v(t) &= \left(v_1^T(t), v_2^T(t), \ldots, v_N^T(t)\right)^T \in \mathbb{R}^{Nn},\\ 1_N &= (1, 1, \ldots, 1)^T \in \mathbb{R}^N,\\ F(t, x(t), v(t)) &\\ &= \left(f^T(t, x_1(t), v_1(t)), f^T(t, x_2(t), v_2(t)), \ldots,\right.\\ &\quad\left. f^T(t, x_N(t), v_N(t))\right)^T \in \mathbb{R}^{Nn}\\ \delta &= \operatorname{diag}\{\delta_1, \delta_2, \ldots, \delta_l, 0, \ldots, 0\} \in \mathbb{R}^{N\times N}. \end{aligned} \tag{7}$$

Let $\xi(t) = x(t) - 1_N \otimes x_r(t)$, $\eta(t) = v(t) - 1_N \otimes v_r(t)$. Combining (3) and (4), one has

$$\begin{aligned} \dot{\xi}(t) &= \dot{x}(t) - 1_N \otimes \dot{x}_r(t)\\ &= v(t) - \left[(L+\delta)\otimes I_n\right](\xi(t) + \xi(t-\tau(t)))\\ &\quad - 1_N \otimes v_r(t)\\ &= \eta(t) - \left[(L+\delta)\otimes I_n\right](\xi(t) + \xi(t-\tau(t))),\\ \dot{\eta}(t) &= \dot{v}(t) - 1_N \otimes \dot{v}_r(t)\\ &= F(t, x(t), v(t)) - 1_N \otimes f(t, x_r(t), v_r(t))\\ &\quad - \left[(L+\delta)\otimes I_n\right](\eta(t) + \eta(t-\tau(t))). \end{aligned} \tag{8}$$

Based on (8), the augment system can be obtained as follows:

$$\begin{aligned} \begin{bmatrix}\dot{\xi}(t)\\ \dot{\eta}(t)\end{bmatrix}^T &= \begin{bmatrix}-(L+\delta)\otimes I_n & I_{Nn}\\ 0 & -(L+\delta)\otimes I_n\end{bmatrix}\begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix}\\ &+ \begin{bmatrix}-(L+\delta)\otimes I_n & 0\\ 0 & -(L+\delta)\otimes I_n\end{bmatrix}\begin{bmatrix}\xi(t-\tau(t))\\ \eta(t-\tau(t))\end{bmatrix}\\ &+ \begin{bmatrix}0\\ F(t, x(t), v(t)) - 1_N \otimes f(t, x_r(t), v_r(t))\end{bmatrix}. \end{aligned} \tag{9}$$

Setting

$$\begin{gathered} e(t) = \begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix} \qquad A = \begin{bmatrix}-(L+\delta) & I_N\\ 0 & -(L+\delta)\end{bmatrix},\\ \widetilde{F} = \begin{bmatrix}0\\ \overline{F}\end{bmatrix} \qquad A_\tau = \begin{bmatrix}-(L+\delta) & 0\\ 0 & -(L+\delta)\end{bmatrix},\\ \overline{F} = F(t, x(t), v(t)) - 1_N \otimes f(t, x_r(t), v_r(t)), \end{gathered} \tag{10}$$

then the system (9) can be rewritten in the following compact form:

$$\dot{e}(t) = \left(A \otimes I_n\right)e(t) + \left(A_\tau \otimes I_n\right)e(t-\tau(t)) + \widetilde{F}. \tag{11}$$

Remark 2. In many practical systems, such as networked control systems, the probability distribution of the delay in the interval is an important characteristic. Inspired by [26–28], in this paper, it is assumed that the probability of the delay appearing in some intervals can be observed. Then new multiagent systems with stochastic time-varying delays can be proposed.

For a given positive integer M and a partition $0 = \tau_0 < \tau_1 < \tau_2 \cdots < \tau_M = \tau$, we assume that the probabilities of $\tau(t) \in [\tau_{i-1}, \tau_i)$ $(i = 1, 2, \ldots, N)$ are known. Define the set

$$\Omega_i = \left\{t : \tau(t) \in \left[\tau_{i-1}, \tau_i\right)\right\} \quad (i = 1, 2, \ldots, M). \tag{12}$$

It can easily be seen that $\bigcup \Omega_i = R^+$, $\Omega_i \bigcap \Omega_j = \phi$ $(i \neq j)$. Define

$$\tau_i(t) = \begin{cases}\tau(t) & t \in \Omega_i\\ 0 & t \overline{\in} \Omega_i\end{cases} \qquad \beta_i(t) = \begin{cases}1 & t \in \Omega_i\\ 0 & t \overline{\in} \Omega_i.\end{cases} \tag{13}$$

Assumption 3. Stochastic variable $\beta(t) := [\beta_1(t), \beta_2(t), \ldots, \beta_N(t)]^T$ is a distributed sequence with

$$\operatorname{Prob}\left\{\beta_i(t) = 1\right\} = E\left\{\beta_i(t)\right\} = \beta_i \sum_{i=1}^{M} \beta_i = 1. \tag{14}$$

Remark 4. By some computations, we derived from Assumption 3 that

$$\begin{gathered} E\left(\beta_i(t) - \beta_i\right)^2 = \beta_i\left(1 - \beta_i\right)\\ E\left\{\beta_i(t) - \beta_i\right\} = 0 \qquad (i = 1, 2, \ldots, M)\\ E\left\{\left(\beta_i(t) - \beta_i\right)\left(\beta_j(t) - \beta_j\right)\right\} = -\beta_i\beta_j\\ (i, j = 1, 2, \ldots, M;\ i \neq j). \end{gathered} \tag{15}$$

By using the new functions $\beta_i(t)$ $(i = 1, 2, \ldots, M)$, system (11) can be rewritten as

$$\dot{e}(t) = \left[\mathscr{A} + \sum_{i=1}^{M-1} (\beta_i(t) - \beta_i) \mathscr{A}_{\tau i}\right] \xi(t) + \widetilde{F}, \quad (16)$$

where

$$\begin{aligned} \mathscr{A} &= [A, \beta_1 A_\tau, 0, \beta_2 A_\tau, 0, \ldots, 0, \beta_{M-1} A_\tau, 0, \beta_M A_\tau] \otimes I_n \\ \mathscr{A}_{\tau i} &= [0_{N,(2i-1)N}, A_\tau, 0_{N,(2M-2i-1)N}, -A_\tau], \\ \xi^T(t) &= [e^T(t), e^T(t-\tau_1(t)), e^T(t-\tau_1), \\ &\quad e^T(t-\tau_2(t)), \ldots, e^T(t-\tau_{M-1}), e^T(t-\tau_M(t))]. \end{aligned} \quad (17)$$

Let $C([-\tau, 0]; \mathbb{R}^n)$ denote the family of continuous functions φ from $[-\tau, 0]$ to $\mathbb{R}^n$. It is not difficult to show that, given any $\varphi \in C([-\tau, 0]; \mathbb{R}^n)$, the system (16) has a unique continuous solution $e(t)$ on $t \geq -\tau$. Clearly, $e(t)$ is a continuous $\mathbb{R}^n$-valued stochastic process on $t \in [-\tau, 0]$. Let $e_t = \{e(t+\theta) : -\tau \leq \theta \leq 0\}$ for $t \geq 0$. Then $\{e_t\}$ is a $C([-\tau, 0]; \mathbb{R}^n)$-valued Markov process. Its infinitesimal operator L, acting on functional $V : C([-\tau, 0]; \mathbb{R}^n) \times [0, +\infty) \to \mathbb{R}$, is defined by

$$LV(e_t, t) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} [E(V(e_{t+\Delta, t+\Delta}) \mid e_t) - V(e_t, t)]. \quad (18)$$

The following lemmas and assumption are needed to derive our main results.

Lemma 5. *For matrices A, B, C, and D with appropriate dimensions and a positive scalar α, by the definition of Kronecker product, the following properties can be proved:*

$$\begin{aligned} (\alpha A) \otimes B &= A \otimes (\alpha B), \\ (A + B) \otimes C &= A \otimes C + B \otimes C, \\ (A \otimes B)(C \otimes D) &= (AC) \otimes (BD), \\ (A \otimes B)^T &= A^T \otimes B^T. \end{aligned} \quad (19)$$

Lemma 6. *Let $R \in \mathbb{R}^{n \times n}$ satisfy $R > 0$. If a $\mathbb{R}^n$-valued function $x(t)$ is integral on the interval $[a, b]$, then*

$$-\int_a^b x^T(s) R x(s)\, ds \leq -\frac{1}{b-a} \int_a^b x^T(s)\, ds\, R \int_a^b x(s)\, ds. \quad (20)$$

Assumption 7. For the nonlinear function $f(t, \cdot, \cdot)$ and for all $x, v, y, z \in \mathbb{R}^n$, there exist nonnegative constants ρ_1 and ρ_2, such that the following inequality holds:

$$\|f(t, x, v) - f(t, y, z)\|^2 \leq \rho_1 \|x - y\|^2 + \rho_2 \|v - z\|^2. \quad (21)$$

3. Main Results

Theorem 8. *Suppose that Assumption 7 holds. Second-order leader-following consensus is achieved under the pinning controllers (6), if for some given positive scalars τ_i and $\beta_i \in [0, 1]$ $(i = 1, 2, \ldots, M)$ there exist some positive matrices $\mathbf{P} = \mathrm{diag}\{P, P\}$, $\mathbf{Q}_i = \mathrm{diag}\{Q_i, Q_i\}$ $(i = 1, 2, \ldots, M-1)$, $\mathbf{R}_j = \mathrm{diag}\{R_j, R_j\}$ $(j = 1, 2, \ldots, M+1)$ and a positive $\alpha > 0$ such that the following linear matrix inequality holds:*

$$\Phi = \begin{bmatrix} \Phi_{11} & * & * & * & \cdots & * & * \\ \Phi_{21} & \Phi_{22} & * & * & \cdots & * & * \\ 0 & 0 & \Phi_{33} & * & \cdots & * & * \\ \Phi_{41} & 0 & \Phi_{43} & \Phi_{44} & \cdots & * & * \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \Phi_{2M,1} & 0 & 0 & 0 & \cdots & \Phi_{2M,2M} & * \\ 0 & \Phi_{2M+1,2} & 0 & \Phi_{2M+1,4} & \cdots & \Phi_{2M+1,2M} & \Phi_{2M+1,2M+1} \end{bmatrix} < 0, \quad (22)$$

where

$$\Phi_{11} = \begin{bmatrix} \Omega_1 + \alpha\rho_1 I_N & P - (L+\delta)^T \Delta \\ P - \Delta(L+\delta) & \Omega_1 + \alpha\rho_1 I_N \end{bmatrix},$$

$$\Phi_{21} = \begin{bmatrix} \Omega_2 & -\beta_1 (L+\delta)^T \Delta \\ 0 & \Omega_2 \end{bmatrix},$$

$$\Phi_{22} = \begin{bmatrix} \beta_1 (L+\delta)^T \Delta (L+\delta) - \dfrac{R_1}{\tau_1} & 0 \\ 0 & \beta_1 (L+\delta)^T \Delta (L+\delta) - \dfrac{R_1}{\tau_1} \end{bmatrix},$$

$$\Phi_{33} = \begin{bmatrix} -Q_1 - \dfrac{R_2}{\tau_2 - \tau_1} & 0 \\ 0 & -Q_1 - \dfrac{R_2}{\tau_2 - \tau_1} \end{bmatrix},$$

$$\Phi_{41} = \begin{bmatrix} \Omega_3 & -\beta_2 (L+\delta)^T \Delta \\ 0 & \Omega_3 \end{bmatrix},$$

$$\Phi_{43} = \begin{bmatrix} \dfrac{R_2}{\tau_2 - \tau_1} & 0 \\ 0 & \dfrac{R_2}{\tau_2 - \tau_1} \end{bmatrix},$$

$$\Phi_{44} = \begin{bmatrix} \dfrac{R_2}{\tau_2 - \tau_1} + \beta_2 (L+\delta)^T \Delta (L+\delta) & 0 \\ 0 & \dfrac{R_2}{\tau_2 - \tau_1} + \beta_2 (L+\delta)^T \Delta (L+\delta) \end{bmatrix},$$

$$\Phi_{2M,1} = \begin{bmatrix} \Omega_4 & -\beta_M (L+\delta)^T \Delta \\ 0 & \Omega_4 \end{bmatrix},$$

$$\Phi_{2M,2M} = \begin{bmatrix} \Omega_5 & 0 \\ 0 & \Omega_5 \end{bmatrix},$$
$$\Phi_{2M+1,2} = \begin{bmatrix} P-\Delta(L+\delta) & 0 \\ 0 & P-\Delta(L+\delta) \end{bmatrix},$$
$$\Phi_{2M+1,4} = \begin{bmatrix} -\beta_1\Delta(L+\delta) & 0 \\ 0 & -\beta_1\Delta(L+\delta) \end{bmatrix},$$
$$\Phi_{2M+1,2M} = \begin{bmatrix} -\beta_M\Delta(L+\delta) & 0 \\ 0 & -\beta_M\Delta(L+\delta) \end{bmatrix},$$
$$\Phi_{2M+1,2M+1} = \begin{bmatrix} -\alpha I_N+\Delta & 0 \\ 0 & -\alpha I_N+\Delta \end{bmatrix},$$
$$\Delta = \sum_{i=1}^{M}(\tau_i-\tau_{i-1})R_i + \tau_M R_{M+1},$$
$$\Omega_1 = -P(L+\delta)-(L+\delta)^T P + \sum_{i=1}^{M-1} Q_i - \frac{R_1}{\tau_1} - \frac{R_{M+1}}{\tau_M} + (L+\delta)^T\Delta(L+\delta),$$
$$\Omega_2 = -\beta_1(L+\delta)^T P + \frac{R_1}{\tau_1} + \beta_1(L+\delta)^T\Delta(L+\delta),$$
$$\Omega_3 = -\beta_2(L+\delta)^T P + \beta_2(L+\delta)^T\Delta(L+\delta),$$
$$\Omega_4 = -\beta_M(L+\delta)^T P + \frac{R_{M+1}}{\tau_M} + \beta_M(L+\delta)^T\Delta(L+\delta),$$
$$\Omega_5 = -\frac{R_{M+1}}{\tau_M} - \frac{R_M}{\tau_M-\tau_{M-1}} + \left(\beta_M - 2\sum_{i,j=1;i\neq j}^{M-1}\beta_i\beta_j\right)(L+\delta)^T\Delta(L+\delta). \tag{23}$$

Proof. Consider the following Lyapunov function candidate:

$$V(t,e_t) = V_1(t,e_t) + V_2(t,e_t) + V_3(t,e_t), \tag{24}$$

where

$$V_1(t,e_t) = e^T(t)(\mathbf{P}\otimes I_n)e(t),$$
$$V_2(t,e_t) = \sum_{i=1}^{M-1}\int_{t-\tau_i}^{t} e^T(s)(\mathbf{Q_i}\otimes I_n)e(s)\,ds,$$
$$V_3(t,e_t) = \sum_{i=1}^{M}\int_{t-\tau_i}^{t-\tau_{i-1}}\int_s^t \dot{e}^T(v)(\mathbf{R}_i\otimes I_n)\dot{e}(v)\,dv\,ds + \int_{t-\tau_M}^{t}\dot{e}^T(v)(\mathbf{R}_{M+1}\otimes I_n)\dot{e}(v)\,dv\,ds. \tag{25}$$

Taking time derivative of $V_i(t,e_t)$ $(i=1,2,3)$ along the trajectory (16) yields

$$LV_1(t,e_t) = 2e^T(t)(\mathbf{P}\otimes I_n)\mathscr{A}\xi(t) + 2e^T(t)(\mathbf{P}\otimes I_n)\widetilde{F} = 2e^T(t)(\mathbf{P}\otimes I_n)\mathscr{A}\xi(t) + 2e^T(t) \times\left(\mathbf{P}\begin{bmatrix}0\\ I_N\end{bmatrix}\otimes I_n\right)\overline{F}, \tag{26}$$

$$LV_2(t,e_t) = \sum_{i=1}^{M-1}\left[e^T(t)(\mathbf{Q_i}\otimes I_n)e(t) - e^T(t-\tau_i)(\mathbf{Q_i}\otimes I_n)e(t-\tau_i)\right], \tag{27}$$

$$LV_3(t,e_t) = \dot{e}^T(t)(\Delta\otimes I_n)\dot{e}(t) - \sum_{i=1}^{M}\int_{t-\tau_i}^{t-\tau_{i-1}}\dot{e}^T(s)(\mathbf{R}_i\otimes I_n)\dot{e}(s)\,ds - \int_{t-\tau_M}^{t}\dot{e}^T(s)(\mathbf{R}_{M+1}\otimes I_n)\dot{e}(s)\,ds, \tag{28}$$

where $\Delta = \sum_{i=1}^{M}(\tau_i-\tau_{i-1})\mathbf{R}_i + \tau_M\mathbf{R}_{M+1}$

By Lemma 6, the following inequalities hold:

$$\begin{aligned}
&-\int_{t-\tau_i}^{t-\tau_{i-1}}\dot{e}^T(s)(\mathbf{R}_i\otimes I_n)\dot{e}(s)\,ds \\
&\leq -\frac{1}{\tau_i-\tau_{i-1}}\int_{t-\tau_i}^{t-\tau_{i-1}}\dot{e}^T(s)\,ds(\mathbf{R}_i\otimes I_n)\int_{t-\tau_i}^{t-\tau_{i-1}}\dot{e}(s)\,ds \\
&\leq -\frac{1}{\tau_i-\tau_{i-1}}\int_{t-\tau_i(t)}^{t-\tau_{i-1}}\dot{e}^T(s)\,ds(\mathbf{R}_i\otimes I_n)\int_{t-\tau_i(t)}^{t-\tau_{i-1}}\dot{e}(s)\,ds \\
&= -\frac{1}{\tau_i-\tau_{i-1}}\xi^T(t)\begin{bmatrix}0_{4(i-1)Nn,2Nn}\\ I_{2Nn}\\ -I_{2Nn}\\ 0_{4(M-i)Nn,2Nn}\end{bmatrix} \times(\mathbf{R}_i\otimes I_n)\begin{bmatrix}0_{4(i-1)Nn,2Nn}\\ I_{2Nn}\\ -I_{2Nn}\\ 0_{4(M-i)Nn,2Nn}\end{bmatrix}^T\xi(t), \\
&-\int_{t-\tau_M}^{t}\dot{e}^T(s)(\mathbf{R}_{M+1}\otimes I_n)\dot{e}(s)\,ds \\
&\leq -\frac{1}{\tau_M}\int_{t-\tau_M}^{t}\dot{e}^T(s)\,ds(\mathbf{R}_{M+1}\otimes I_n)\int_{t-\tau_M}^{t}\dot{e}(s)\,ds \\
&\leq -\frac{1}{\tau_M}\int_{t-\tau_M(t)}^{t}\dot{e}^T(s)\,ds(\mathbf{R}_{M+1}\otimes I_n)\int_{t-\tau_M(t)}^{t}\dot{e}(s)\,ds \\
&= -\frac{1}{\tau_M}\xi^T(t)\begin{bmatrix}I_{2Nn}\\ 0_{4(M-1)Nn,2Nn}\\ -I_{2Nn}\end{bmatrix} \times(\mathbf{R}_{M+1}\otimes I_n)\begin{bmatrix}I_{2Nn}\\ 0_{4(M-1)Nn,2\mathrm{N}n}\\ -I_{2Nn}\end{bmatrix}^T\xi(t).
\end{aligned} \tag{29}$$

From (16) and (28), we can obtain

$$\dot{e}^T(t)(\Delta\otimes I_n)\dot{e}(t) = \xi^T(t)\left(\mathscr{A}^T + \sum_{i=1}^{M-1}(\beta_i(t)-\beta_i)\mathscr{A}_{\tau i}^T\right)$$

$$\times (\Delta \otimes I_n)\left(\mathscr{A} + \sum_{i=1}^{M-1}(\beta_i(t) - \beta_i)\mathscr{A}_{\tau i}\right)\xi(t)$$

$$+ 2\xi^T(t)\left(\mathscr{A}^T + \sum_{i=1}^{M-1}(\beta_i(t) - \beta_i)\mathscr{A}_{\tau i}^T\right)(\Delta \otimes I_n)\widetilde{F}$$

$$+ \widetilde{F}^T(\Delta \otimes I_n)\widetilde{F}$$

$$= \xi^T(t)\mathscr{A}^T\Delta\mathscr{A}\xi(t) + 2\sum_{i=1}^{M-1}(\beta_i(t) - \beta_i)\xi^T(t)$$

$$\times \mathscr{A}^T(\Delta \otimes I_n)\mathscr{A}_{\tau i}\xi(t)$$

$$+ 2\sum_{i \neq j}(\beta_i(t) - \beta_i)(\beta_j(t) - \beta_j)\xi^T(t)\mathscr{A}_{\tau i}^T(\Delta \otimes I_n)\mathscr{A}_{\tau i}\xi(t)$$

$$+ 2\sum_{i=1}^{M-1}(\beta_i(t) - \beta_i)^2\xi^T(t)\mathscr{A}_{\tau i}^T(\Delta \otimes I_n)\mathscr{A}_{\tau i}\xi(t)$$

$$+ 2\xi^T(t)\left(\mathscr{A}^T + \sum_{i=1}^{M-1}(\beta_i(t) - \beta_i)\mathscr{A}_{\tau i}^T\right)(\Delta \otimes I_n)\widetilde{F} + \widetilde{F}^T\widetilde{F}. \tag{30}$$

Taking the mathematical expectation on both sides of (30) yields

$$E\left\{\dot{e}^T(t)(\Delta \otimes I_n)\dot{e}(t)\right\}$$

$$= \xi^T(t)\left(\mathscr{A}^T(\Delta \otimes I_n)\mathscr{A} - 2\sum_{i,j=1;i\neq j}^{M-1}\beta_i\beta_j\mathscr{A}_{\tau i}^T(\Delta \otimes I_n)\mathscr{A}_{\tau i}\right.$$

$$\left. + \sum_{i=1}^{M-1}\beta_i(1 - \beta_i)\mathscr{A}_{\tau i}^T(\Delta \otimes I_n)\mathscr{A}_{\tau i}\right)\xi(t)$$

$$+ 2\xi^T(t)\mathscr{A}^T(\Delta \otimes I_n)\begin{bmatrix}0\\ I_N\end{bmatrix}\overline{F} + \overline{F}^T\overline{F}. \tag{31}$$

By Assumption 3, we can obtain the following inequality:

$$\overline{F}^T\overline{F} = \left\|F(t, x(t), v(t)) - 1_N \otimes f(t, x_r(t), v_r(t))\right\|^2$$

$$= \sum_{i=1}^{N}\left\|f(t, x_i(t), v_i(t)) - f(t, x_r(t), v_r(t))\right\|^2$$

$$\leq \rho_1\sum_{i=1}^{N}\left\|x_i(t) - x_r(t)\right\|^2 + \rho_2\sum_{i=1}^{N}\left\|v_i(t) - v_r(t)\right\|^2$$

$$= \rho_1\left\|x(t) - 1_N \otimes x_r(t)\right\|^2 + \rho_2\left\|v(t) - 1_N \otimes v_r(t)\right\|^2$$

$$= \rho_1\xi^T(t)\xi(t) + \rho_2\eta^T(t)\eta(t)$$

$$= e^T(t)\begin{bmatrix}\rho_1 I_{Nn} & 0\\ 0 & \rho_2 I_{Nn}\end{bmatrix}e(t). \tag{32}$$

For a given $\alpha > 0$, the following inequality holds based on (32):

$$\alpha e^T(t)\begin{bmatrix}\rho_1 I_{Nn} & 0\\ 0 & \rho_2 I_{Nn}\end{bmatrix}e(t) - \alpha\overline{F}^T\overline{F} \geq 0. \tag{33}$$

It follows from (26)–(33) that

$$LV(t, e_t)$$

$$\leq 2e^T(t)(\mathbf{P} \otimes I_n)\mathscr{A}\xi(t) + 2e^T(t)\left(\mathbf{P}\begin{bmatrix}0\\ I_N\end{bmatrix} \otimes I_n\right)\overline{F}$$

$$+ \sum_{i=1}^{M-1}\left\{e^T(t)(\mathbf{Q_i} \otimes I_n)e(t) - e^T(t - \tau_i)(\mathbf{Q_i} \otimes I_n)e(t - \tau_i)\right\}$$

$$+ \dot{e}^T(t)(\Delta \otimes I_n)\dot{e}(t)$$

$$- \sum_{i=1}^{M}\frac{1}{\tau_i - \tau_{i-1}}\xi^T(t)\begin{bmatrix}0_{4(i-1)Nn,2Nn}\\ I_{2Nn}\\ -I_{2Nn}\\ 0_{4(M-i)Nn,2Nn}\end{bmatrix}$$

$$\times (\mathbf{R}_i \otimes I_n)\begin{bmatrix}0_{4(i-1)Nn,2Nn}\\ I_{2Nn}\\ -I_{2Nn}\\ 0_{4(M-i)Nn,2Nn}\end{bmatrix}^T\xi(t)$$

$$- \frac{1}{\tau_M}\xi^T(t)\begin{bmatrix}I_{2Nn}\\ 0_{4(M-1)Nn,2Nn}\\ -I_{2Nn}\end{bmatrix}$$

$$\times (\mathbf{R}_{M+1} \otimes I_n)\begin{bmatrix}I_{2Nn}\\ 0_{4(M-1)Nn,2Nn}\\ -I_{2Nn}\end{bmatrix}^T\xi(t)$$

$$+ \alpha e^T(t)\begin{bmatrix}\rho_1 I_{Nn} & 0\\ 0 & \rho_2 I_{Nn}\end{bmatrix}e(t) - \alpha\overline{F}^T\overline{F}$$

$$= \xi^T(t)(\Psi \otimes I_n)\xi(t) + 2e^T(t)\left(\mathbf{P}\begin{bmatrix}0\\ I_N\end{bmatrix} \otimes I_n\right)\overline{F}$$

$$+ 2\xi^T(t)\mathscr{A}^T(\Delta \otimes I_n)\begin{bmatrix}0\\ I_N\end{bmatrix}\overline{F}$$

$$+ \alpha e^T(t)\begin{bmatrix}\rho_1 I_{Nn} & 0\\ 0 & \rho_2 I_{Nn}\end{bmatrix}e(t) + (1 - \alpha)\overline{F}^T\overline{F}, \tag{34}$$

where

$$\Psi = \begin{bmatrix}\overline{\Phi}_{11} & * & * & * & \cdots & *\\ \Phi_{21} & \Phi_{22} & * & * & \cdots & *\\ 0 & 0 & \Phi_{33} & * & \cdots & *\\ \Phi_{41} & 0 & \Phi_{43} & \Phi_{44} & \cdots & *\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ \Phi_{2M,1} & 0 & 0 & 0 & \cdots & \Phi_{2M,2M}\end{bmatrix} \tag{35}$$

$$\overline{\Phi}_{11} = \Phi_{11} - \alpha\begin{bmatrix}\rho_1 I_{Nn} & 0\\ 0 & \rho_2 I_{Nn}\end{bmatrix}.$$

Let $\zeta(t) = [\xi^T(t), \overline{F}^T]^T$, and the inequality (34) can further be written as

$$\begin{aligned} LV(t, e_t) &\leq \zeta^T(t) \left\{ \begin{bmatrix} \Psi & * \\ \Gamma & \Phi_{2M+1,2M+1} \end{bmatrix} \otimes I_n \right\} \zeta(t) \\ &= \zeta^T(t)(\Phi \otimes I_n)\zeta(t) < 0, \end{aligned} \quad (36)$$

where

$$\Gamma = [0, \Phi_{2M+1,2}, \Phi_{2M+1,4}, \ldots, 0, \Phi_{2M+1,2M}]. \quad (37)$$

From (22), we have $LV(t, e_t) < 0$. That is, $\|\xi(t)\| \to 0$, $\|\eta(t)\| \to 0$ as $t \to \infty$. Therefore, the states of pinning-controlled multiagent systems (4) globally asymptotically approach state of virtual leader (3). That is, $\|x_i(t) - x_r(t)\| \to 0$, $\|v_i(t) - v_r(t)\| \to 0$ as $t \to \infty$. So the second-order leader-following consensus in pinning-controlled multiagent systems (4) is achieved. The proof is completed. □

Remark 9. It can be seen from Theorem 8 that the feasibility of (22) dependents on not only τ_i and δ but also the probability distribution of the delay β_i, and more information of time delays is involved (22).

Let us consider one special case. When $f(t, \cdot, \cdot) \equiv 0$, then the virtual leader (3) reduces to

$$\dot{x}_r(t) = v_r(t) \qquad \dot{v}_r(t) = 0, \quad (38)$$

which indicates that the reference velocity is constant.

Each agent in multiagent systems is described by

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) + u_{1i}(t), \\ \dot{v}_i(t) &= u_{2i}(t) \quad (i = 1, 2, \ldots, N). \end{aligned} \quad (39)$$

The pinning controllers are given as follows:

$$\begin{aligned} &u_{1i}(t) \\ &= -\sum_{j \in N_i} a_{ij} \left\{ (x_i(t) - x_j(t)) + (x_i(t - \tau(t)) - x_j(t - \tau(t))) \right\} \\ &\quad - \delta_i \left\{ (x_i(t) - x_r(t)) - (x_i(t - \tau(t)) - x_r(t - \tau(t))) \right\} \\ &u_{2i}(t) \\ &= -\sum_{j \in N_i} a_{ij} \left\{ (v_i(t) - v_j(t)) - (v_i(t - \tau(t)) - v_j(t - \tau(t))) \right\} \\ &\quad - \delta_i \left\{ (v_i(t) - v_r(t)) - (v_i(t - \tau(t)) - v_r(t - \tau(t))) \right\}, \end{aligned} \quad (40)$$

where the parameters δ_i are defined in Section 2.

Let $\xi(t) = x(t) - 1_N \otimes x_r(t)$, $\eta(t) = v(t) - 1_N \otimes v_r(t)$. Then we can obtain from (38)–(40) that

$$\dot{e}(t) = (A \otimes I_n)e(t) + (A_\tau \otimes I_n)e(t - \tau(t)). \quad (41)$$

From Theorem 8 and its proof, we have the following corollary.

Corollary 10. *For some given positive scalars $\beta_i \in [0, 1]$ ($i = 1, 2, \ldots, M$), there exist some positive matrices $\mathbf{P} = \mathrm{diag}\{P, P\}$, $\mathbf{Q}_i = \mathrm{diag}\{Q_i, Q_i\}$ ($i = 1, 2, \ldots, M-1$), $\mathbf{R}_j = \mathrm{diag}\{R_j, R_j\}$ ($j = 1, 2, \ldots, M+1$) such that the following linear matrix inequality holds:*

$$\begin{bmatrix} \Phi_{11} & * & * & * & \cdots & * \\ \Phi_{21} & \Phi_{22} & * & * & \cdots & * \\ 0 & 0 & \Phi_{33} & * & \cdots & * \\ \Phi_{41} & 0 & \Phi_{43} & \Phi_{44} & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \Phi_{2M,1} & 0 & 0 & 0 & \cdots & \Phi_{2M,2M} \end{bmatrix} < 0. \quad (42)$$

Then the second-order leader-following consensus is achieved under the adaptive pinning controllers (40). That is, $x_i(t) \to x_r(0) + v_r(0)t$ and $v_i(t) \to v_r(0)$, as $t \to \infty$.

Up to this point, a question arises naturally: how to choose a set of pinned agents such that pinning conditions (22) and (42) are satisfied?

Proposition 11 (see [21]). *For a diagraph g, let v and D denote the node set of g and the pinned-node set, respectively. All nodes in v/D should have access to the pinned-node set D. That is, for any node $i \in v/D$, one can always find node $j \in D$ such that there exists a directed path from node j to node i.*

According to Proposition 11, we give the following procedure to select a set of pinning-agents and to design the parameters δ_i ($i = 1, 2, \ldots, l$).

(1) Define a degree-difference vector $\mathrm{deg}_{\mathrm{dif}}(i) = \mathrm{deg}_{\mathrm{out}} - \mathrm{deg}_{\mathrm{in}}(i)$ ($i = 1, 2, \ldots, N$).

(2) Pick all agents with zero in-degrees as pinned agents, and rearrange the remaining agents in descending order according to their degree-differences.

(3) Find the minimum number of agents l_0 which satisfies (22) or (42), and let $l = l_0$.

(4) From Theorem 8 and Corollary 10, by using LMI Toolbox in MATLAB, we can obtain δ_i ($i = 1, 2, \ldots, l_0$).

(5) If δ_i ($i = 1, 2, \ldots, l_0$) is not good enough for practical use, add more agents to the pinned-agent set, and repeat STEP (4) until we find proper δ_i.

4. Numerical Results

In this section, two numerical examples are given to verify the effectiveness of the proposed pinning control techniques.

Example 12. We consider multiagent systems consisting of six agents described by

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) + u_{1i}(t), \\ \dot{v}_i(t) &= f(t, x_i(t), v_i(t)) + u_{2i}(t) \quad (i = 1, 2, \ldots, 6), \end{aligned} \quad (43)$$

Table 1: The allowable upper bound of time delay for different M.

System	Pinning nodes	$M = 1$	$M = 2$	$M = 3$
Nonlinear system	6, 2, 4	0.3207	0.3363	0.3626
Linear system	6, 2, 4	0.3287	0.3463	0.3787

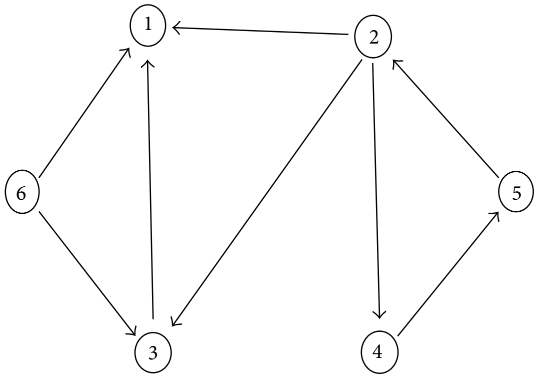

Figure 1: The topology structure of the six agents.

where

$$f(t, x_i(t), v_i(t)) = 2\cos(2t) - \frac{\sqrt{5}}{5}\sin(x_i(t)) - \frac{\sqrt{10}}{5}v_i(t)$$

$$\begin{aligned} u_{1i}(t) &= -\sum_{j\in N_i} a_{ij}\left[\left(x_i(t) - x_j(t)\right) - \left(x_i(t-\tau(t)) - x_j(t-\tau(t))\right)\right] \\ &\quad - \delta_i\left[\left(x_i(t) - x_r(t)\right) - \left(x_i(t-\tau(t)) - x_r(t-\tau(t))\right)\right] \\ u_{2i}(t) &= -\sum_{j\in N_i} a_{ij}\left[\left(v_i(t) - v_j(t)\right) - \left(v_i(t-\tau(t)) - v_j(t-\tau(t))\right)\right] \\ &\quad - \delta_i\left[\left(v_i(t) - v_r(t)\right) - \left(v_i(t-\tau(t)) - v_r(t-\tau(t))\right)\right], \end{aligned} \tag{44}$$

where $x_i(t) \in \mathbb{R}$ and $v_i(t) \in \mathbb{R}$ are the position and velocity states of agent i, respectively.

The interaction diagraph of multiagent systems (43) is shown in Figure 1. We examine what agents should be pinned, and note that agent 6 has zero in-degree and that the out-degree of agents 2 is bigger than their in-degree. According to Proposition 11, agent 6 should be pinned first. Because its state is not affected by others, agent 2 can be chosen as pinned candidates, furthermore. Based on the pinned-agent selection scheme, we rearrange six agents and the new order is 6, 2, 4, 5, 3, 1. By some calculation, we can obtain $\rho_1 = 0.1$, $\rho_2 = 0.2$, for given M, τ_i $(i = 1, 2, \ldots, M-1)$ and β_j $(j = 1, 2, \ldots, M)$ (when $M = 1$, let $\tau_0 = 0$; when $M = 2$, let $\tau_0 = 0$, $\tau_1 = 0.1$, $\beta_1 = 0.1$, $\beta_2 = 0.9$; when $M = 3$, let $\tau_0 = 0$, $\tau_1 = 0.1$, $\tau_2 = 0.2$, $\beta_1 = 0.1$, $\beta_2 = 0.1$, $\beta_3 = 0.8$). By employing the LMI Toolbox in Theorem 8, we can obtain Table 1, it can be seen that (1) the minimum number of pinned agents is three, and we chose agents 6, 2, and 4 as pinned agents; (2) the allowable upper bound τ increases with increasing M. If we can obtain more information of time delay, it will lead to a larger allowable upper bound of time delay. When we chose agents 6, 2, and 4 as pinned agents, the evolutions of positions and velocities of six agents are shown in Figures 2, 3, 4, and 5.

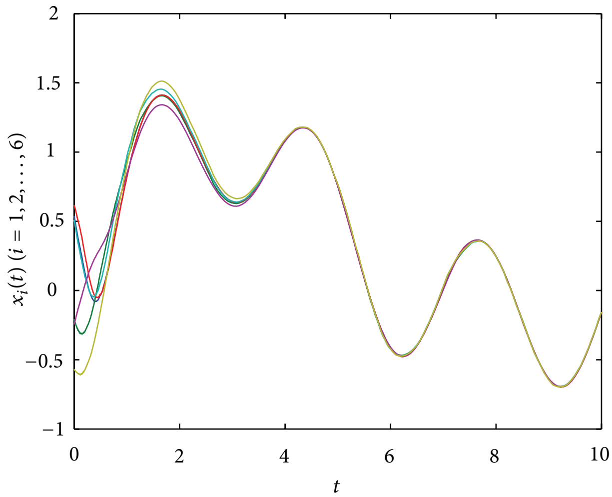

Figure 2: The position curves $x_i(t)$ $(i = 1, 2, \ldots, 6)$ for nonlinear multiagent systems (43) with $\tau_0 = 0$, $\tau_1 = 0.3$.

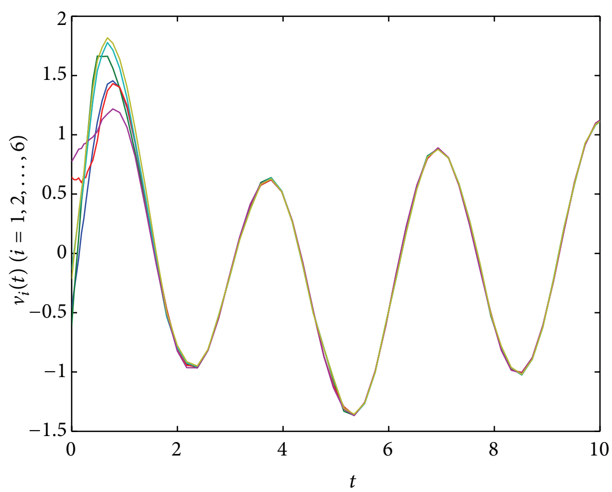

Figure 3: The velocity curves $v_i(t)$ $(i = 1, 2, \ldots, 6)$ for nonlinear multiagent systems (43) with $\tau_0 = 0$, $\tau_1 = 0.3$.

Example 13. We consider the following multiagent systems consisting of six agents described by

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) + u_{1i}(t) \\ \dot{v}_i(t) &= u_{2i}(t) \quad (i = 1, 2, \ldots, 6), \end{aligned} \tag{45}$$

where

$$\begin{aligned} u_{1i}(t) &= -\sum_{j\in N_i} a_{ij}\left[\left(x_i(t) - x_j(t)\right) - \left(x_i(t-\tau(t)) - x_j(t-\tau(t))\right)\right] \\ &\quad - \delta_i\left[\left(x_i(t) - x_r(t)\right) - \left(x_i(t-\tau(t)) - x_r(t-\tau(t))\right)\right] \end{aligned}$$

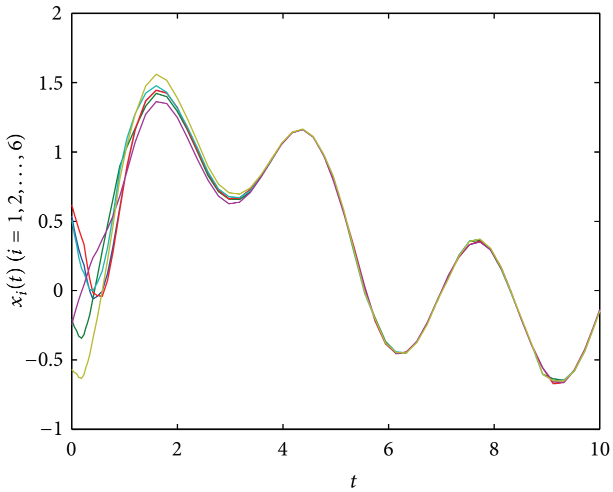

FIGURE 4: The position curves $x_i(t)$ $(i = 1, 2, \ldots, 6)$ for nonlinear multiagent systems (43) with $\tau_0 = 0$, $\tau_1 = 0.2$, $\tau_2 = 0.3$, $\beta_1 = 0.1$, and $\beta_2 = 0.9$.

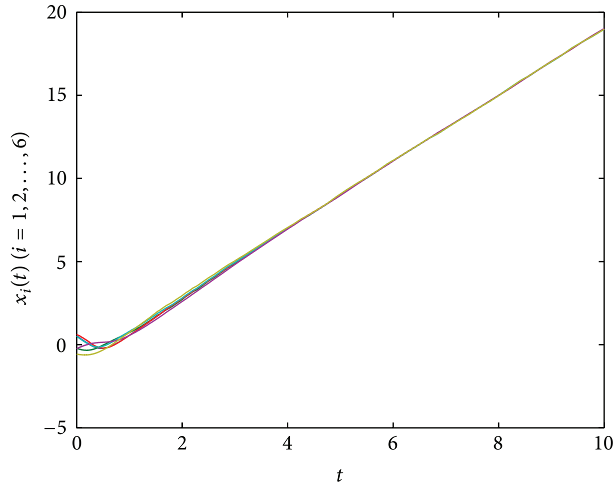

FIGURE 6: The velocity curves $v_i(t)$ $(i = 1, 2, \ldots, 6)$ for nonlinear multiagent systems (45) with $\tau_0 = 0$, $\tau_1 = 0.3$.

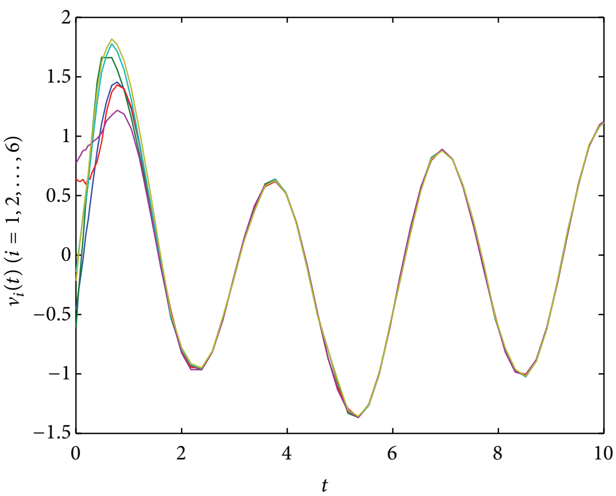

FIGURE 5: The velocity curves $v_i(t)$ $(i = 1, 2, \ldots, 6)$ for nonlinear multiagent systems (43) with $\tau_0 = 0$, $\tau_1 = 0.2$, $\tau_2 = 0.3$, $\beta_1 = 0.1$, and $\beta_2 = 0.9$.

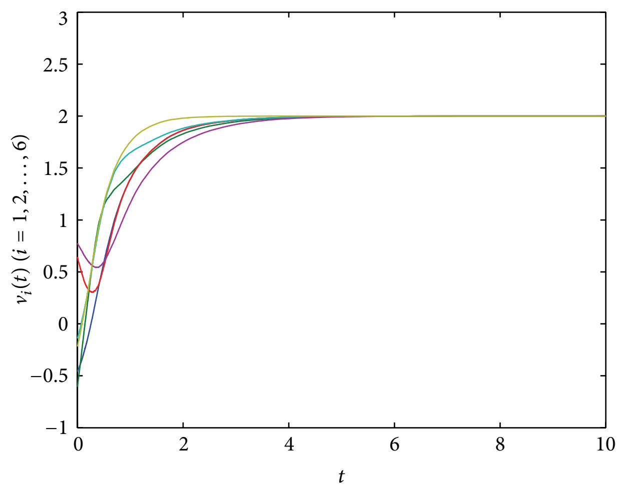

FIGURE 7: The velocity curves $v_i(t)$ $(i = 1, 2, \ldots, 6)$ for nonlinear multiagent systems (45) with $\tau_0 = 0$, $\tau_1 = 0.3$.

$$\begin{aligned} u_{2i}(t) &= -\sum_{j \in N_i} a_{ij} \left[\left(v_i(t) - v_j(t) \right) - \left(v_i(t - \tau(t)) - v_j(t - \tau(t)) \right) \right] \\ &\quad - \delta_i \left[\left(v_i(t) - v_r(t) \right) - \left(v_i(t - \tau(t)) - v_r(t - \tau(t)) \right) \right], \end{aligned} \tag{46}$$

where $x_i(t) \in \mathbb{R}$ and $v_i(t) \in \mathbb{R}$ are the position and velocity states of agent i, respectively.

Similar to Example 12, by using the LMI Toolbox in Corollary 10, we can obtain Table 1. When we chose agents 6, 2, and 4 as pinned agents, the evolutions of positions and velocities of six agents are shown in Figures 6, 7, 8, and 9.

5. Conclusions

By employing the information of the probability distribution of the time delay, this paper investigates the consensus problem for second-order leader-follower nonlinear multiagent systems with general network topologies. Different from the common assumptions on the time delay in the existing literatures, it is assumed in this paper that the delay is random and its probability distribution is known a prior. Based on graph theory, a pinning control algorithm is proposed, and the consensus criteria are derived to achieve leader-follower consensus for multiagent systems with nonlinear second-order dynamics. Moreover, this paper addresses what kind of agents and how many agents should be pinned.

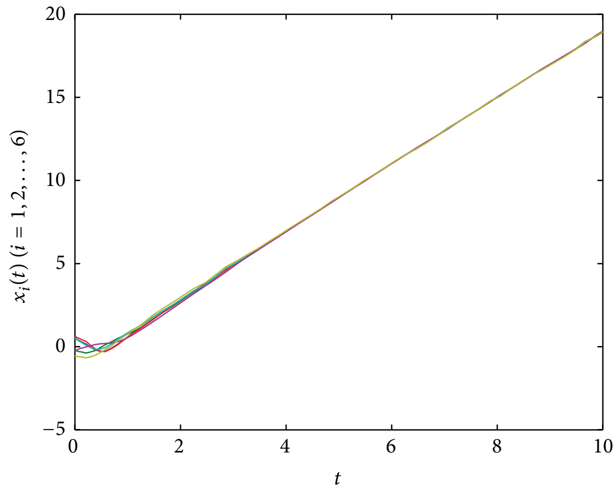

Figure 8: The position curves $x_i(t)$ $(i = 1, 2, \ldots, 6)$ for nonlinear multiagent systems (45) with $\tau_0 = 0$, $\tau_1 = 0.2$, $\tau_2 = 0.3$, $\beta_1 = 0.1$, and $\beta_2 = 0.9$.

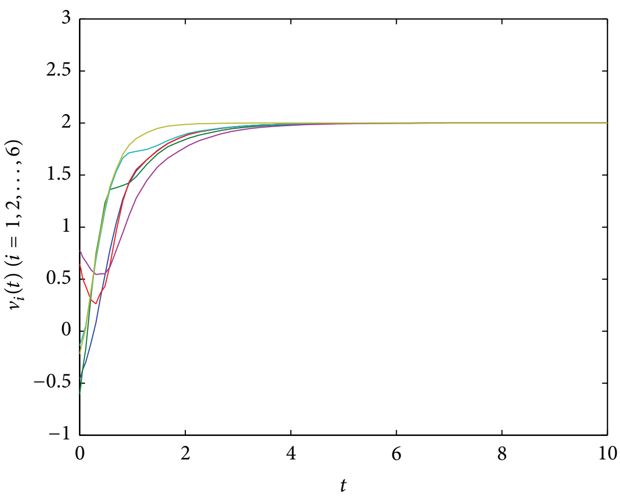

Figure 9: The velocity curves $v_i(t)$ $(i = 1, 2, \ldots, 6)$ for nonlinear multiagent systems (45) with $\tau_0 = 0$, $\tau_1 = 0.2$, $\tau_2 = 0.3$, $\beta_1 = 0.1$, and $\beta_2 = 0.9$.

Acknowledgments

This work was jointly supported by National Natural Science Foundation of China under Grant nos. 61272034 and 11301226 and Zhejiang Provincial Natural Science Foundation of China under Grant no. LY13F030012 and LQ13A010017.

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