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Research Article
Fixed Points of Multivalued Nonself Almost Contractions

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We consider multivalued nonself-weak contractions on convex metric spaces and establish the existence of a fixed point of such mappings. Presented theorem generalizes results of M. Berinde and V. Berinde (2007), Assad and Kirk (1972), and many others existing in the literature.

1. Introduction

The study of fixed points of single-valued self-mappings or multivalued self-mappings satisfying certain contraction conditions has a great majority of results in metric fixed point theory. All these results are mainly generalizations of Banach contraction principle.

The Banach contraction principle guarantees the existence and uniqueness of fixed points of certain self-maps in complete metric spaces. This result has various applications to operator theory and variational analysis. So, it has been extended in many ways until now. One of these is related to multivalued mappings. Its starting point is due to Nadler Jr. [1].

The fixed point theory for multivalued nonself-mappings developed rapidly after the publication of Assad and Kirk's paper [2] in which they proved a non-self-multivalued version of Banach's contraction principle. Further results for multivalued non-self-mappings were proved in, for example, [3–7]. For other related results, see also [8–38].

On the other hand, Berinde [11–13] introduced a new class of self-mappings (usually called weak contractions or almost contractions) that satisfy a simple but general contraction condition that includes most of the conditions in Rhoades' classification [39]. He obtained a fixed point theorem for such mappings which generalized the results of Kannan [40], Chatterjea [41], and Zamfirescu [42]. As shown in [43], the weakly contractive metric-type fixed point result in [12] is "almost" covered by the related altering metric one due to Khan et al. [21].

In [9], M. Berinde and V. Berinde extended Theorem 8 to the case of multivalued weak contractions.

Definition 1. Let (X, d) be a metric space and K a nonempty subset of X. A map $T : K \to CB(X)$ is called a multivalued almost contraction if there exist a constant $\delta \in (0, 1)$ and some $L \geq 0$ such that

$$H(Tx, Ty) \leq \delta \cdot d(x, y) + LD(y, Tx), \quad \forall x, y \in K. \tag{1}$$

Theorem 2 (see [9]). *Let X be a complete metric space and $T : X \to CB(X)$ a multivalued almost contraction. Then T has a fixed point.*

The aim of this paper is to prove a fixed point theorem for multivalued nonself almost contractions on convex metric spaces. This theorem extends several important results (including the above) in the fixed point theory of self-mappings to the case on nonself-mappings and generalizes several fixed point theorems for nonself-mappings.

2. Preliminaries

We recall some basic definitions and preliminaries that will be needed in this paper.

Let (X, d) be a metric space and $CB(X)$ the set of all nonempty bounded and closed subsets of X. For $A, B \in CB(X)$, define

$$D(x, A) = \inf\{d(x, y) : y \in A\},$$
$$D(A, B) = \inf\{d(x, y) : x \in A, y \in B\}, \tag{2}$$
$$H(A, B) = \max\left\{\sup_{x\in A} D(x, B), \sup_{y\in B} D(y, A)\right\}.$$

It is known that H is a metric on $CB(X)$ and H is called the Hausdorff metric or Pompeiu-Hausdorff metric induced by d. It is also known that $(CB(X), H)$ is a complete metric space whenever (X, d) is a complete metric space.

Definition 3. Let $T : X \to CB(X)$ be a multivalued map. An element $x \in X$ is said to be a fixed point of T if $x \in Tx$.

In this paper we assume that (X, d) is a convex metric space which is defined as follows.

Definition 4. A metric space (X, d) is convex if for each $x, y \in X$ with $x \neq y$ there exists $z \in X$, $x \neq z \neq y$, such that

$$d(x, y) = d(x, z) + d(z, y). \tag{3}$$

This notion is similar to the definition of metric space of hyperbolic type. The class of metric spaces of hyperbolic type includes all normed linear spaces and all spaces with hyperbolic metric.

It is known that in a convex metric space each two points are the endpoints of at least one metric segment (see [2]).

Proposition 5 (see [2]). *Let K be a closed subset of a complete and convex metric space X. If $x \in K$ and $y \notin K$, then there exists a point $z \in \partial K$ (the boundary of K) such that*

$$d(x, y) = d(x, z) + d(z, y). \tag{4}$$

The following lemma will be required in the sequel.

Lemma 6 (see [1, 2]). *Let (X, d) be a metric space and $A, B \in CB(X)$. If $x \in A$, then, for each positive number α, there exists $y \in B$ such that*

$$d(x, y) \leq H(A, B) + \alpha. \tag{5}$$

The definition of an almost contraction given by Berinde [12] is as follows.

Definition 7. Let (X, d) be a metric space. A map $T : X \to X$ is called almost contraction if there exist a constant $\delta \in (0, 1)$ and some $L \geq 0$ such that

$$d(Tx, Ty) \leq \delta \cdot d(x, y) + Ld(y, Tx), \quad \forall x, y \in X. \tag{6}$$

Theorem 8 (see [12]). *Let (X, d) be a complete metric space and $T : X \to X$ an almost contraction. Then*

(1) $\mathrm{Fix}(T) = \{x \in X : Tx = x\} \neq \emptyset$;

(2) *for any $x_0 \in X$, the Picard iteration $\{x_n\}_{n=0}^{\infty}$ converges to some $x^* \in \mathrm{Fix}(T)$;*

(3) *the following estimate holds*

$$d(x_{n+i-1}, x^*) \leq \frac{\delta^i}{1-\delta} d(x_n, x_{n-1}), \quad n = 1, 2, \ldots;\ i = 1, 2, \ldots. \tag{7}$$

Let us recall (see [30]) that a mapping T possessing properties (1) and (2) is called a *weakly Picard operator*.

In fact, Theorem 8 generalizes some important fixed point theorems in the literature such as Banach contraction principle, Kannan fixed point theorem [40], Chatterjea fixed point theorem [41], and Zamfirescu fixed point theorem [42].

3. Main Results

Theorem 9. *Let (X, d) be a complete convex metric space and K a nonempty closed subset of X. Suppose that $T : K \to CB(X)$ is a multivalued almost contraction, that is,*

$$H(Tx, Ty) \leq \delta \cdot d(x, y) + LD(y, Tx), \quad \forall x, y \in K, \tag{8}$$

with $\delta \in (0, 1)$ and some $L \geq 0$ such that $\delta(1 + L) < 1$. If T satisfies Rothe's type condition, that is, $x \in \partial K \Rightarrow Tx \subset K$, then there exists $z \in K$ such that $z \in Tz$; that is, T has a fixed point in K.

Proof. We construct two sequences $\{x_n\}$ and $\{y_n\}$ in the following way. Let $x_0 \in K$ and $y_1 \in Tx_0$. If $y_1 \in K$, let $x_1 = y_1$. If $y_1 \notin K$, then there exists $x_1 \in \partial K$ such that

$$d(x_0, x_1) + d(x_1, y_1) = d(x_0, y_1). \tag{9}$$

Thus $x_1 \in K$, and, by Lemma 6 and $\alpha = \delta$, we can choose $y_2 \in Tx_1$ such that

$$d(y_1, y_2) \leq H(Tx_0, Tx_1) + \delta. \tag{10}$$

If $y_2 \in K$, let $x_2 = y_2$. If $y_2 \notin K$, then there exists $x_2 \in \partial K$ such that

$$d(x_1, x_2) + d(x_2, y_2) = d(x_1, y_2). \tag{11}$$

Thus $x_2 \in K$, and, by Lemma 6 and $\alpha = \delta^2$, we can choose $y_3 \in Tx_2$ such that

$$d(y_2, y_3) \leq H(Tx_1, Tx_2) + \delta^2. \tag{12}$$

Continuing the arguments we construct two sequences $\{x_n\}$ and $\{y_n\}$ such that

(i) $y_{n+1} \in Tx_n$;

(ii) $d(y_n, y_{n+1}) \leq H(Tx_{n-1}, Tx_n) + \delta^n$,

where

(iii) $y_n \in K \Rightarrow y_n = x_n$;

(iv) $y_n \neq x_n$ whenever $y_n \notin K$, and then $x_n \in \partial K$ is such that

$$d(x_{n-1}, x_n) + d(x_n, y_n) = d(x_{n-1}, y_n). \tag{13}$$

Now we claim that $\{x_n\}$ is a Cauchy sequence. Suppose that

$$P = \{x_i \in \{x_n\} : x_i = y_i\}, \quad Q = \{x_i \in \{x_n\} : x_i \neq y_i\}. \tag{14}$$

Obviously, if $x_n \in Q$, then x_{n-1} and x_{n+1} belong to P. Now, we conclude that there are three possibilities.

Case 1. If $x_n, x_{n+1} \in P$, then $y_n = x_n$, $y_{n+1} = x_{n+1}$. Thus

$$\begin{aligned} d(x_n, x_{n+1}) &= d(y_n, y_{n+1}) \\ &\leq H(Tx_{n-1}, Tx_n) + \delta^n \\ &\leq \delta \cdot d(x_{n-1}, x_n) + LD(x_n, Tx_{n-1}) + \delta^n \\ &= \delta \cdot d(x_{n-1}, x_n) + \delta^n \end{aligned} \tag{15}$$

since $y_n \in Tx_{n-1}$.

Case 2. If $x_n \in P, x_{n+1} \in Q$, then $y_n = x_n$, $y_{n+1} \neq x_{n+1}$. We have

$$\begin{aligned} d(x_n, x_{n+1}) &\leq d(x_n, x_{n+1}) + d(x_{n+1}, y_{n+1}) \\ &= d(x_n, y_{n+1}) \\ &= d(y_n, y_{n+1}) \\ &\leq H(Tx_{n-1}, Tx_n) + \delta^n \\ &\leq \delta \cdot d(x_{n-1}, x_n) + LD(x_n, Tx_{n-1}) + \delta^n \\ &= \delta \cdot d(x_{n-1}, x_n) + \delta^n. \end{aligned} \tag{16}$$

Case 3. If $x_n \in Q, x_{n+1} \in P$, then $x_{n-1} \in P$, $y_n \neq x_n$, $y_{n+1} = x_{n+1}$, $y_{n-1} = x_{n-1}$, and $y_n \in Tx_{n-1}$. We have

$$\begin{aligned} d(x_n, x_{n+1}) &= d(x_n, y_{n+1}) \\ &\leq d(x_n, y_n) + d(y_n, y_{n+1}) \\ &\leq d(x_n, y_n) + H(Tx_{n-1}, Tx_n) + \delta^n \\ &\leq d(x_n, y_n) + \delta \cdot d(x_{n-1}, x_n) \\ &\quad + LD(x_n, Tx_{n-1}) + \delta^n. \end{aligned} \tag{17}$$

Since $\delta < 1$, then

$$\begin{aligned} d(x_n, x_{n+1}) &\leq d(x_n, y_n) + d(x_{n-1}, x_n) \\ &\quad + LD(x_n, Tx_{n-1}) + \delta^n \\ &= d(x_{n-1}, y_n) + LD(x_n, Tx_{n-1}) + \delta^n \\ &\leq d(x_{n-1}, y_n) + Ld(x_n, y_n) + \delta^n \\ &= d(x_{n-1}, y_n) + Ld(x_{n-1}, y_n) \\ &\quad - Ld(x_{n-1}, x_n) + \delta^n \\ &\leq (1+L)\, d(y_{n-1}, y_n) + \delta^n \\ &\leq (1+L)\, H(Tx_{n-2}, Tx_{n-1}) \\ &\quad + (1+L)\,\delta^{n-1} + \delta^n \\ &\leq (1+L)\,\delta \cdot d(x_{n-2}, x_{n-1}) \\ &\quad + (1+L)\, LD(x_{n-1}, Tx_{n-2}) \\ &\quad + (1+L)\,\delta^{n-1} + \delta^n \\ &= (1+L)\,\delta \cdot d(x_{n-2}, x_{n-1}) + (1+L)\,\delta^{n-1} + \delta^n. \end{aligned} \tag{18}$$

Since

$$h = (1+L)\,\delta < 1, \tag{19}$$

then

$$d(x_n, x_{n+1}) < hd(x_{n-2}, x_{n-1}) + h\delta^{n-2} + \delta^n. \tag{20}$$

Thus, combining Cases 1, 2, and 3, it follows that

$$d(x_n, x_{n+1}) \leq \begin{cases} \alpha \cdot d(x_{n-1}, x_n) + \alpha^n \\ \alpha d(x_{n-2}, x_{n-1}) + \alpha^{n-1} + \alpha^n, \end{cases} \tag{21}$$

where

$$\alpha = \max\{\delta, h\} = h. \tag{22}$$

Following [2], by induction it follows that for $n > 1$

$$d(x_n, x_{n+1}) \leq h^{(n-1)/2}\omega + h^{n/2} n, \tag{23}$$

where

$$\omega = \max\{d(x_0, x_1), d(x_1, x_2)\}. \tag{24}$$

Now, for $n > m$, we have

$$\begin{aligned} d(x_n, x_m) &\leq d(x_n, x_{n-1}) + d(x_{n-1}, x_{n-2}) \\ &\quad + \cdots + d(x_{m-1}, x_m) \\ &\leq \left(h^{(n-1)/2} + h^{(n-2)/2} + \cdots + h^{(m-1)/2}\right)\omega \\ &\quad + \alpha^{n/2} n + \alpha^{(n-1)/2}(n-1) + \cdots + \alpha^{m/2} m. \end{aligned} \tag{25}$$

This implies that the sequence $\{x_n\}$ is a Cauchy sequence. Since X is complete and K is closed, it follows that there exists $z \in K$ such that

$$z = \lim_{n \to \infty} x_n. \tag{26}$$

By construction of $\{x_n\}$, there is a subsequence $\{x_q\}$ such that

$$y_q = x_q \in Tx_{q-1}. \tag{27}$$

We will prove that $z \in Tz$. In fact, by (i), $x_q \in Tx_{q-1}$. Since $x_q \to z$ as $q \to \infty$, we have

$$D(z, Tx_{q-1}) \longrightarrow 0, \tag{28}$$

as $q \to \infty$. Note that

$$D(z, Tz) \le d(z, x_q) + d(x_q, Tz) \tag{29}$$

$$\le d(z, x_q) + H(Tx_{q-1}, Tz) \tag{30}$$

$$\le d(z, x_q) + \delta d(x_{q-1}, z) + LD(z, Tx_{q-1}), \tag{31}$$

which on letting $q \to \infty$ implies that $D(z, Tz) = 0$; it, then, follows that $z \in Tz$. □

By Theorem 9 we obtain as a particular case, a fixed point theorem for multivalued nonself-contractions due to Assad and Kirk [2] that appears to be the first fixed point result for nonself-mappings in the literature.

Corollary 10 (see [2]). *Let (X, d) be a complete convex metric space and K a nonempty closed subset of X. Suppose that $T : K \to CB(X)$ is a multivalued contraction; that is,*

$$H(Tx, Ty) \le \delta d(x, y), \quad \forall x, y \in K, \tag{32}$$

with $\delta \in (0, 1)$. If T satisfies Rothe's type condition, that is, $x \in \partial K \Rightarrow Tx \subset K$, then there exists $z \in K$ such that $z \in Tz$; that is, T has a fixed point in K.

Example 11. Let X be the set of real numbers with the usual norm, $K = [0, 1]$ the unit interval, and $T : K \to CB(X)$ be given by $Tx = \{(1/9)x\}$, for $x \in [0, 1/2)$, $T(1/2) = \{-1\}$, and $Tx = [17/18, (1/9)x + 8/9]$, for $x \in (1/2, 1]$.

In order to show that T is a multivalued almost contraction, we have to discuss 8 possible cases.

Case 1. Consider $(x, y) \in \Omega_1 = [0, 1/2) \times (1/2, 1]$. Then condition (8) reduces to

$$\left|\frac{1}{9}x - \frac{1}{9}y - \frac{8}{9}\right| \le \delta |x - y| + L\left|y - \frac{1}{9}x\right|, \quad (x, y) \in \Omega_1. \tag{33}$$

Since, for $(x, y) \in \Omega_1$, one has $|(1/9)x - (1/9)y - 8/9| \le 1$ and $|y - (1/9)x| > 4/9$, in order to have the previous inequality satisfied, it suffices to take $L \ge 9/4$ and $0 < \delta < 4/13$ arbitrarily.

Case 2. Consider $(x, y) \in \Omega_2 = (1/2, 1] \times [0, 1/2)$. Then condition (8) reduces to

$$\left|\frac{1}{9}x + \frac{8}{9} - \frac{1}{9}y\right| \le \delta |x - y| + L\left|y - \frac{1}{9}x - \frac{8}{9}\right|, \quad (x, y) \in \Omega_2. \tag{34}$$

Since, for $(x, y) \in \Omega_2$, one has $|(1/9)x + 8/9 - (1/9)y| \le 1$ and $|y - (1/9)x - 8/9| > 4/9$, in order to have the previous inequality satisfied, it suffices to take $L \ge 9/4$ and $0 < \delta < 4/13$ arbitrarily.

Case 3. Take $(x, y) \in \Omega_3 = [0, 1/2)^2$. In this case we have

$$H(Tx, Ty) = d\left(\frac{1}{9}x, \frac{1}{9}y\right) = \left|\frac{1}{9}x - \frac{1}{9}y\right|, \tag{35}$$

and so condition (8) is satisfied with $\delta = 1/9$ and $L \ge 0$ arbitrarily.

Case 4. Consider $(x, y) \in \Omega_4 = (1/2, 1]^2$. In this case we have

$$\begin{aligned} H(Tx, Ty) &= H\left(\left[\frac{1}{9}x + \frac{8}{9}, 1\right], \left[\frac{1}{9}y + \frac{8}{9}, 1\right]\right) \\ &= \left|\frac{1}{9}x - \frac{1}{9}y\right|, \end{aligned} \tag{36}$$

and so condition (8) is satisfied with $\delta = 1/9$ and $L \ge 0$ arbitrarily.

Case 5. Take $(x, y) \in \Omega_5 = \{1/2\} \times [0, 1/2)$. Then condition (8) reduces to

$$\left|1 + \frac{1}{9}y\right| \le \delta\left|\frac{1}{2} - y\right| + L|y + 1|, (x, y) \in \Omega_5. \tag{37}$$

Since for $(x, y) \in \Omega_5$, one has $|1+(1/9)y| < 19/18$ and $|1+y| \ge 1$, in order to have the previous inequality satisfied, it suffices to take $L \ge 19/18$ and $0 < \delta < 18/37$ arbitrarily.

Case 6. Consider $(x, y) \in \Omega_6 = [0, 1/2) \times \{1/2\}$. Then condition (8) reduces to

$$\left|1 + \frac{1}{9}x\right| \le \delta\left|x - \frac{1}{2}\right| + L\left|\frac{1}{2} - \frac{1}{9}x\right|, \quad (x, y) \in \Omega_6. \tag{38}$$

Since, for $(x, y) \in \Omega_6$, one has $|1 + (1/9)x| \le 19/18$ and $|1/2 - (1/9)x| \ge 4/9$, in order to have the previous inequality satisfied, it suffices to take $L \ge 19/8$ and $0 < \delta < 8/27$ arbitrarily.

Case 7. Take $(x, y) \in \Omega_7 = \{1/2\} \times (1/2, 1]$. Then condition (8) reduces to

$$\left|1 + \frac{1}{9}y + \frac{8}{9}\right| \le \delta\left|\frac{1}{2} - y\right| + L|y + 1|, \quad (x, y) \in \Omega_7. \tag{39}$$

Since, for $(x, y) \in \Omega_7$, one has $|1 + (1/9)y + 8/9| \le 2$ and $|y+1| \ge 3/2$, in order to have the previous inequality satisfied, it suffices to take $L \ge 4/3$ and $0 < \delta < 3/7$.

Case 8. Consider $(x, y) \in \Omega_8 = (1/2, 1] \times \{1/2\}$. Then condition (8) reduces to

$$\left|1 + \frac{1}{9}x + \frac{8}{9}\right| \le \delta\left|x - \frac{1}{2}\right| + L\left|\frac{1}{2} - \frac{1}{9}x - \frac{8}{9}\right|, \quad (x, y) \in \Omega_8. \tag{40}$$

Since, for $(x, y) \in \Omega_8$, one has $|1 + (1/9)x + 8/9| \le 2$ and $|1/2 - (1/9)x - 8/9| > 4/9$, in order to have the previous inequality satisfied, it suffices to take $L \ge 9/2$ and $0 < \delta < 2/11$ arbitrarily.

Now, by summarizing all cases, we conclude that condition (8) is satisfied with $\delta = 1/9$ and $L = 9/2$. Note that the additional condition $\delta(1 + L) < 1$ is also satisfied.

Hence, T is a multivalued almost contraction that satisfies all assumptions in Theorem 9, and T has two fixed points; that is, $\mathrm{Fix}(T) = \{0, 1\}$.

Note that Corollary 10 cannot be applied to T in Example 11. Indeed, if we take $x = 1$ and $y = 1/2$ in (32), then one obtains

$$H\left(T1, T\frac{1}{2}\right) \leq \delta \left|1 - \frac{1}{2}\right|. \quad (41)$$

That is, $|1 + 1| \leq \delta |1/2|$, which leads to the contradiction $4 \leq \delta < 1$.

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