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Research Article

Analytical Solutions of the One-Dimensional Heat Equations Arising in Fractal Transient Conduction with Local Fractional Derivative

Ai-Ming Yang,[1,2] Carlo Cattani,[3] Hossein Jafari,[4,5] and Xiao-Jun Yang[6]

[1] *College of Science, Hebei United University, Tangshan 063009, China*
[2] *College of Mechanical Engineering, Yanshan University, Qinhuangdao 066004, China*
[3] *Department of Mathematics, University of Salerno, Via Ponte don Melillo, Fisciano, 84084 Salerno, Italy*
[4] *Department of Mathematics, Faculty of Mathematical Sciences, University of Mazandaran, Babolsar, Iran*
[5] *International Institute for Symmetry Analysis and Mathematical Modelling, Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X 2046, Mmabatho 2735, South Africa*
[6] *Department of Mathematics and Mechanics, China University of Mining and Technology, Xuzhou, Jiangsu 221008, China*

Correspondence should be addressed to Ai-Ming Yang; aimin_heut@163.com

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The one-dimensional heat equations with the heat generation arising in fractal transient conduction associated with local fractional derivative operators are investigated. Analytical solutions are obtained by using the local fractional Adomian decomposition method via local fractional calculus theory. The method in general is easy to implement and yields good results. Illustrative examples are included to demonstrate the validity and applicability of the new technique.

1. Introduction

The Adomian decomposition method [1–3] was applied to process linear and nonlinear problems in the fields of science and engineering. Tatari and Dehghan [4] applied Adomian decomposition method to process the multipoint boundary value problem. Wazwaz [5] used Adomian decomposition method to deal with the Bratu-type equations. Daftardar-Gejji and Jafari [6] considered Adomian decomposition method to analyze the Bagley Torvik equation. Larsson [7] presented the solution for Helmholtz equation by using the Adomain decomposition method. Tatari and coworkers [8] investigated solution for the Fokker-Planck equation by Adomain decomposition method.

Fractional calculus [9–12] was applied to model the physical and engineering problems for expressions of stress-strain constitutive relations of different viscoelastic fractional order properties of materials, diffusion processes with fractional order properties, fractional order flows, analytical mechanics of fractional order discrete system vibrations [13–15], and so on. Recently, the application of Adomian decomposition method for solving the linear and nonlinear fractional partial differential equations in the fields of the physics and engineering had been established in [16, 17]. Adomian decomposition method was applied to handle the time-fractional Navier-Stokes equation [18], fractional space diffusion equation [19], fractional KdV-Burgers equation [20], linear and nonlinear fractional diffusion and wave equations [21], KdV-Burgers-Kuramoto equation [22], fractional Burgers' equation [23], and so on. For more details on some methods for solving fractional differential equations, see [24–28].

Recently, local fractional calculus theory was applied to model some nondifferentiable problems for mathematical physics (see [29–36] and the references therein). The Adomian decomposition method, as one of efficient tools for solving the linear and nonlinear differential equations, was extended to find the solutions for local fractional differential equations [37–40] and nondifferentiable solutions were obtained.

The partial differential equationfs describing thermal process of fractal heat conduction were suggested in [30, 38] in the following form:

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} - \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} = 0. \quad (1)$$

The initial and boundary conditions are

$$u(0,t) = f(t), \quad \frac{\partial^{\alpha} u(0,t)}{\partial x^{\alpha}} = g(t), \quad (2)$$

where the operator is the local fractional differential operator [29, 30, 34, 37, 38], which is applied to model the heat conduction problems in fractal media, fractal materials, fractal fracture mechanics, fractal wave behavior, Navier-Stokes equations on Cantor sets, Schrödinger equation with local fractional derivative, and diffusion equations on cantor space-time.

The one-dimensional heat equations with the heat generation arising in fractal transient conduction were considered in [30] as

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} - \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} = \phi(x,t), \quad (3)$$

where $\phi(x,t)$ is the heat generation term.

We use initial and boundary conditions as follows:

$$u(0,t) = f(t), \quad \frac{\partial^{\alpha} u(0,t)}{\partial x^{\alpha}} = g(t). \quad (4)$$

The aim of this paper is to investigate the one-dimensional heat equations with the heat generation arising in fractal transient conduction by using the local fractional Adomian decomposition method.

This paper is structured as follows. In Section 2, we give the basic notations and definitions of local fractional operators. In Section 3, local fractional Adomian decomposition method for heat generation arising in fractal transient conduction is presented. Three examples are shown in Section 4. Finally, Section 5 presents conclusions.

2. Preliminaries

In this section we present some basic definitions and notations of the local fractional operators which are used further through the paper.

Let us denote local fractional continuity of $f(x)$ as

$$f(x) \in C_{\alpha}(a,b). \quad (5)$$

Definition 1. Local fractional derivative operator of $f(x)$ at the point x_0 is given by [29, 30, 34–38]:

$$f^{(\alpha)}(x_0) = \left.\frac{d^{\alpha} f(x)}{dx^{\alpha}}\right|_{x=x_0} = \lim_{x\to x_0} \frac{\Delta^{\alpha}(f(x)-f(x_0))}{(x-x_0)^{\alpha}}, \quad (6)$$

where $\Delta^{\alpha}(f(x)-f(x_0)) \cong \Gamma(1+\alpha)\Delta(f(x)-f(x_0))$ and $f(x) \in C_{\alpha}(a,b)$.

Local fractional derivative of high order and local fractional partial derivative of high order are written in the form [29, 30, 38]

$$f^{(k\alpha)}(x) = \overbrace{D_x^{(\alpha)} \cdots D_x^{(\alpha)}}^{k \text{ times}} f(x), \quad (7)$$

$$\frac{\partial^{k\alpha}}{\partial x^{k\alpha}} f(x,y) = \overbrace{\frac{\partial^{\alpha}}{\partial x^{\alpha}} \cdots \frac{\partial^{\alpha}}{\partial x^{\alpha}}}^{k \text{ times}} f(x,y), \quad (8)$$

respectively.

As inverse of local fractional differential operator, the local fractional integral operator of $f(x)$ in the interval $[a,b]$ is defined as [29, 30, 36–38]

$$\begin{aligned} {}_aI_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)(dt)^{\alpha} \\ &= \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t\to 0} \sum_{j=0}^{j=N-1} f(t_j)(\Delta t_j)^{\alpha}, \end{aligned} \quad (9)$$

where a partition of the interval $[a,b]$ is denoted as $\Delta t_j = t_{j+1} - t_j$, $\Delta t = \max\{\Delta t_0, \Delta t_1, \Delta t_j, \ldots\}$ and $j = 0, \ldots, N-1$, $t_0 = a$, and $t_N = b$.

The properties are only presented as follows [29, 30, 37]:

$$\begin{aligned} & D_x^{(\alpha)}[f(x)g(x)] \\ &\quad = \left(D_x^{(\alpha)} f(x)\right) g(x) + f(x)\left(D_x^{(\alpha)} g(x)\right), \\ & {}_aI_x^{(\alpha)} f(x) g^{(\alpha)}(x) \\ &\quad = \left[f(x)g(x)\right]\Big|_a^x - {}_aI_x^{(\alpha)} f^{(\alpha)}(x) g(x), \\ & D_x^{(\alpha)} \frac{x^{k\alpha}}{\Gamma(1+k\alpha)} = \frac{x^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]}, \\ & {}_0I_b^{(\alpha)} \frac{x^{k\alpha}}{\Gamma(1+k\alpha)} = \frac{x^{(k+1)\alpha}}{\Gamma[1+(k+1)\alpha]}. \end{aligned} \quad (10)$$

3. Analysis of the Method

Let us rewrite the heat equations with the heat generation arising in fractal transient conduction in the form

$$L_t^{(\alpha)} u - L_{xx}^{(2\alpha)} u = \phi, \quad (11)$$

subject to the initial and boundary conditions

$$u(0,t) = f(t), \quad \frac{\partial^{\alpha} u(0,t)}{\partial x^{\alpha}} = g(t), \quad (12)$$

where $\partial^{\alpha}/\partial t^{\alpha}$ and $\partial^{2\alpha}/\partial x^{2\alpha}$ symbolize $L_t^{(\alpha)}$ and $L_{xx}^{(2\alpha)}$, respectively.

By defining the twofold local fractional integral operator as $L_{xx}^{(-2\alpha)}$, we have

$$L_{xx}^{(-2\alpha)}\left[L_t^{(\alpha)}u-\phi\right]=L_{xx}^{(-2\alpha)}L_{xx}^{(2\alpha)}u, \quad (13)$$

so that

$$u=L_{xx}^{(-2\alpha)}L_t^{(\alpha)}u-L_{xx}^{(-2\alpha)}\phi+\frac{x^\alpha}{\Gamma(1+\alpha)}g(t)+f(t). \quad (14)$$

Hence, we get

$$u(x,t)=u_0(x,t)+L_{xx}^{(-2\alpha)}\left[L_t^{(\alpha)}u(x,t)\right], \quad (15)$$

where

$$u_0(x,t)=-L_{xx}^{(-2\alpha)}\phi+\frac{x^\alpha}{\Gamma(1+\alpha)}g(t)+f(t). \quad (16)$$

So, from (15) we have iterative formula as follows:

$$u_{n+1}(x,t)=L_{xx}^{(-2\alpha)}\left[L_t^{(\alpha)}u_n(x,t)\right], \quad n\geq 0, \quad (17)$$

where $u_0(x,t)=-L_{xx}^{(-2\alpha)}\phi+(x^\alpha/\Gamma(1+\alpha))g(t)+f(t)$.

Finally, the exact solution can be constructed as follows:

$$u(x,t)=\lim_{n\to\infty}\sum_{i=1}^{n}u_i(x,t). \quad (18)$$

4. Illustrative Examples

Example 1. In view of (3), we consider $\phi(x,t)=1$, $f(t)=t^\alpha/\Gamma(1+\alpha)$, and $g(t)=t^\alpha/\Gamma(1+\alpha)$.

We have

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}-\frac{\partial^{2\alpha}u(x,t)}{\partial x^{2\alpha}}=1, \quad (19)$$

subject to the initial value condition

$$u_0(x,t)=-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{x^\alpha}{\Gamma(1+\alpha)}\frac{t^\alpha}{\Gamma(1+\alpha)}+\frac{t^\alpha}{\Gamma(1+\alpha)}. \quad (20)$$

From (19) we have the following recursive relations:

$$u_{n+1}(x,t)=L_{xx}^{(-2\alpha)}\left[L_t^{(\alpha)}u_n(x,t)\right]. \quad (21)$$

In view of (21), the first few terms of the decomposition series read

$$u_0(x,t)=-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{x^\alpha}{\Gamma(1+\alpha)}\frac{t^\alpha}{\Gamma(1+\alpha)}+\frac{t^\alpha}{\Gamma(1+\alpha)},$$
$$u_1(x,t)=\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \quad (22)$$

From (25) we get

$$u_2(x,t)=u_3(x,t)=\cdots=u_n(x,t)=0. \quad (23)$$

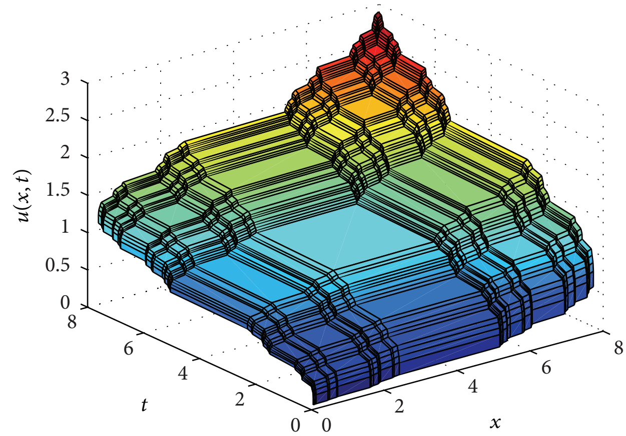

FIGURE 1: Solution for the one-dimensional heat equations with a fixed value $\alpha=\ln 2/\ln 3$.

Therefore, the exact solution of (19) can be written as

$$u(x,t)=\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{x^\alpha}{\Gamma(1+\alpha)}\frac{t^\alpha}{\Gamma(1+\alpha)}+\frac{t^\alpha}{\Gamma(1+\alpha)}. \quad (24)$$

The value of the fractal-dimension order $\alpha=\ln 2/\ln 3$ of the behavior of the solution is shown in Figure 1.

Example 2. When $\phi(x,t)=1$, $f(t)=t^\alpha/\Gamma(1+\alpha)$, and $g(t)=0$, we get

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}-\frac{\partial^{2\alpha}u(x,t)}{\partial x^{2\alpha}}=1. \quad (25)$$

We give the initial value condition as follows:

$$u_0(x,t)=-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{t^\alpha}{\Gamma(1+\alpha)}. \quad (26)$$

From (19) we have the following recursive relations:

$$u_{n+1}(x,t)=L_{xx}^{(-2\alpha)}\left[L_t^{(\alpha)}u_n(x,t)\right]. \quad (27)$$

From (27), we have the first few terms of the decomposition series as follows:

$$u_0(x,t)=-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{t^\alpha}{\Gamma(1+\alpha)},$$
$$u_1(x,t)=\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \quad (28)$$

Hence, we get

$$u_2(x,t)=u_3(x,t)=\cdots=u_n(x,t)=0. \quad (29)$$

So, the exact solution of (19) reads

$$u(x,t)=\frac{t^\alpha}{\Gamma(1+\alpha)}. \quad (30)$$

The solution with fractal-dimension order $\alpha=\ln 2/\ln 3$ is shown in Figure 2.

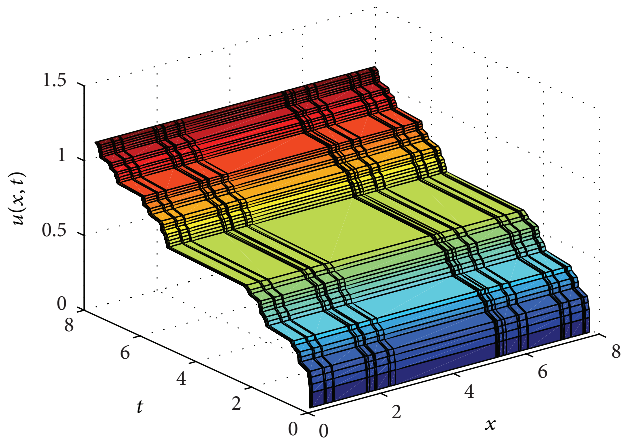

FIGURE 2: Solution for the one-dimensional heat equations with a fixed value $\alpha = \ln 2/\ln 3$.

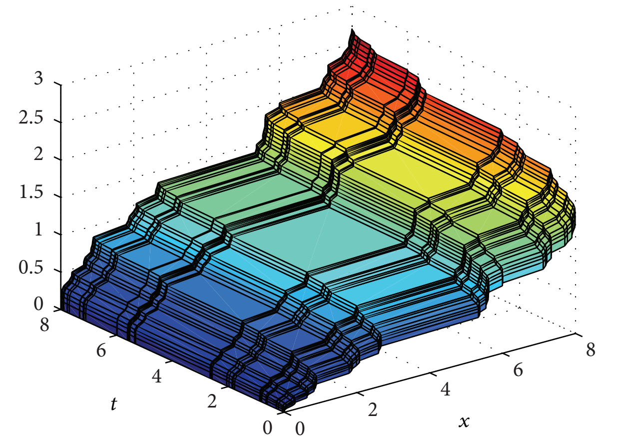

FIGURE 3: The surface shows the exact solution $u(x,t)$ with a fixed value $\alpha = \ln 2/\ln 3$.

Example 3. When $\phi(x,t) = 1$, $f(t) = 0$, and $g(t) = t^{\alpha}/\Gamma(1+\alpha)$, we get

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} - \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} = 1. \tag{31}$$

The initial value condition is presented as follows:

$$u_0(x,t) = -\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{x^{\alpha}}{\Gamma(1+\alpha)}\frac{t^{\alpha}}{\Gamma(1+\alpha)}. \tag{32}$$

From (19) the recursive relations follow

$$u_{n+1}(x,t) = L_{xx}^{(-2\alpha)}\left[L_t^{(\alpha)} u_n(x,t)\right]. \tag{33}$$

In view of (27), we get the few terms of the series; namely,

$$\begin{aligned} u_0(x,t) &= -\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{x^{\alpha}}{\Gamma(1+\alpha)}\frac{t^{\alpha}}{\Gamma(1+\alpha)}, \\ u_1(x,t) &= \frac{x^{3\alpha}}{\Gamma(1+3\alpha)}. \end{aligned} \tag{34}$$

Hence, we get

$$u_2(x,t) = u_3(x,t) = \cdots = u_n(x,t) = 0. \tag{35}$$

So, the exact solution of (19) reads

$$u(x,t) = \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{x^{\alpha}}{\Gamma(1+\alpha)}\frac{t^{\alpha}}{\Gamma(1+\alpha)}. \tag{36}$$

Figure 3 shows the exact solution when $\alpha = \ln 2/\ln 3$.

5. Conclusions

In this work, analytical solutions for the one-dimensional heat equations with the heat generation arising in fractal transient conduction associated with local fractional derivative operators were discussed. The obtained solutions are nondifferentiable functions, which are Cantor functions and they discontinuously depend on the local fractional derivative. It is shown that the local fractional Adomian decomposition method is an efficient and simple tool for solving local fractional differential equations.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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