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Research Article

A Direct Approach Based on C^2-IULOWA Operator for Group Decision Making with Uncertain Additive Linguistic Preference Relations

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With respect to group decision making (GDM) problem with uncertain additive linguistic preference relations (UALPRs), we investigate the efficient aggregation of the uncertain additive linguistic preference information. First, we introduce two measures to assess the consistency level and the consensus level of uncertain additive linguistic preference information, respectively, and study some of their desirable properties. Then, based on both the two measures, we propose a coinduced uncertain linguistic ordered weighted averaging (IULOWA) operator, called the consistency and consensus coinduced uncertain linguistic ordered weighted averaging (C^2-IULOWA) operator, to aggregate individual uncertain additive linguistic preference information, in which the consistency level and the consensus level synergistically serve as inducing variables and then guide the determination of the associated weights. We have proved the collective uncertain linguistic preference information aggregated by the C^2-IULOWA operator that can maintain the fundamental properties of preference relation, such as indifference, reciprocity, and transitivity. By using the C^2-IULOWA operator, we develop a direct GDM approach with UALPRs. Finally, an illustrative example on the selection of chief quality officer is used to demonstrate the effectiveness and rationalitly of the developed approach.

1. Introduction

In group decision making (GDM) problems, preference relations are commonly used by decision makers (DMs) to express their preference information based on pairwise comparisons of alternatives. Three of the most common types of preference relations are multiplicative preference relations [1], fuzzy preference relations [2–7], and linguistic preference relations [8–10]. Due to the complexity and uncertainty involved in real-world decision problems and the inherent subjective nature of human judgments, linguistic preference relations can be more appropriate for capturing the lack of precision in human behavior than others. Hence, decision making based on linguistic preference relation has attracted considerable research interests over the past decades [11–17].

However, sometimes, the DMs are willing or able to provide only uncertain (or interval) linguistic information in some particular situations because of time pressure, lack of knowledge or data, and DMs' limited expertise related to the problem domain [18–24]. Xu [18] proposed the concept of uncertain linguistic variable whose value is interval linguistic information and developed the uncertain linguistic ordered weighted averaging (ULOWA) operator and the uncertain linguistic hybrid aggregation (ULHA) operator to deal with multiple attribute group decision making problems. Xu [19] defined the concept of uncertain additive linguistic preference relation (UALPR) and developed a direct approach to GDM with uncertain linguistic averaging (ULA) and uncertain linguistic weighted averaging (ULWA) operators. Xu [20] introduced the concept of uncertain multiplicative linguistic preference relation and the operational laws of uncertain multiplicative linguistic variables and then proposed the uncertain linguistic weighted geometric mean (ULWGM) operator, uncertain linguistic ordered weighted geometric (ULOWG) operator, and induced uncertain linguistic ordered weighted geometric (IULOWG) operator to deal with GDM problems. Gao and Peng [21] presented a novel quantified SWOT analysis methodology with uncertain linguistic preference relations, interval fuzzy preference relations, and interval multiplicative preference relations. Chen et al. [22] developed a new compatibility for the UALPRs and utilized it to determine the optimal weights of experts in the GDM. Chen and Lee [23] presented an interval linguistic labels ordered weighted average (ILLOWA) operator and a consensus measure for autocratic decision making using group recommendations. Peng et al. [24] presented some multigranular uncertain linguistic prioritized aggregation operators to aggregate directly the uncertain linguistic variables whose values come from the linguistic term sets with different granularities and convey the prioritization phenomenon among the aggregated arguments.

In GDM with all kinds of preference relations, consistency and consensus of preference relations play vital roles. The former refers to the capability of DMs to express their preferences without contradiction; the latter reflects the actual levels of agreement amongst all the individual preferences. Thus, in order to derive a scientific and reasonable decision result, measuring consistency level and measuring consensus level of preference relation are indispensable research topics. To date, although a great deal of research has been conducted on the issues [3, 5–7, 9, 13–17], the contributions of consistency and consensus measures of UALPRs are little.

In the process of practical GDM, on the other hand, it is very difficult for all DMs to construct perfect consistent and consensual preference relations due to the uncertainty and complexity of real world and the subjectivity of DMs' judgments. Also, as pointed out by Saaty, improving consistency does not mean getting an answer closer to real-life situation. It only means that the judgments are closer to being logically related than to being randomly chosen [25]. Furthermore, forcing improvements in consistency may distort the individual's true answer to some extent. The similar situations also exist in consensus. In this paper, we consider the problem that if the DMs are no longer available after constructing preference relations and if the preference relations are not perfect consistent and consensual, then how to determine the scientific and reasonable decision results with a higher level of consistency and consensus. To do so, we shall first give the definitions of the measures of consistency and consensus of UALPRs, present a consistency and consensus-based coinduced uncertain linguistic OWA operator to aggregate UALPRs in such a way that more importance is placed on the most consistent and consensual preference information, and then develop a GDM approach with uncertain linguistic preference information.

The rest of this paper is set out as follows. Section 2 briefly introduces relevant concepts of uncertain linguistic preference information. In Section 3 we define the consistency measure and the consensus measure to assess the consistency level and consensus level of uncertain linguistic preference information, respectively, and study some of their properties. On the basis of the two measures, we present a consistency and consensus co-induced uncertain linguistic OWA operator and discuss some of its desirable properties in Section 4. In Section 5, based on the proposed operator, we develop a direct GDM approach. Section 6 demonstrates the effectiveness and rationally of the developed method with an illustrative example on the selection of chief quality officer. Section 7 concludes this paper.

2. Preliminaries

2.1. Linguistic Variables. Let $S = \{s_\alpha \mid \alpha = 0, 1, 2, \ldots, \tau\}$ be a linguistic term set with odd cardinality, which satisfies the following characteristics: (1) the set is ordered: $s_\alpha > s_\beta$ if $\alpha > \beta$; (2) there is the reciprocal operator $\operatorname{Neg}(s_\alpha) = s_\beta$ such as $\alpha + \beta = \tau$. For example, S can be defined as

$$\begin{aligned} S = \{&s_0 = \text{extremely low}, s_1 = \text{very low}, \\ &s_2 = \text{low}, s_3 = \text{slightly low}, s_4 = \text{fair}, \\ &s_5 = \text{slightly high}, s_6 = \text{high}, \\ &s_7 = \text{very high}, s_8 = \text{extremely high}\}. \end{aligned} \tag{1}$$

To preserve all the given information, Xu [12–14] extended the discrete term set S to a continuous term set $\overline{S} = \{s_\alpha \mid \alpha \in [0, q]\}$, where q $(q > \tau)$ is a sufficiently large positive integer. If $s_\alpha \in S$, then we call s_α the original term otherwise, we call s_α the virtual term. In general, the decision maker uses the original linguistic terms to evaluate alternatives, and the virtual ones can only appear in operation.

Consider any two terms $s_\alpha, s_\beta \in S$ and $\mu, \mu_1, \mu_2 \in [0, 1]$ and some operational laws as follows [13]:

$$\begin{aligned} &(1)\ \mu s_\alpha = s_{\mu\alpha}, \\ &(2)\ s_\alpha \oplus s_\beta = s_{\alpha+\beta}, \\ &(3)\ s_\alpha \ominus s_\beta = s_{\alpha-\beta}, \\ &(4)\ (\mu_1 + \mu_2) s_\alpha = \mu_1 s_\alpha \oplus \mu_2 s_\alpha, \\ &(5)\ \mu (s_\alpha \oplus s_\beta) = \mu s_\alpha \oplus \mu s_\beta, \\ &(6)\ (\mu_1 - \mu_2) s_\alpha = \mu_1 s_\alpha \ominus \mu_2 s_\alpha. \end{aligned} \tag{2}$$

Definition 1 (see [21]). Let $I(s)$ be the lower index of s, and name it as the gradation of s in $\overline{S}$. For example, if $s = s_\alpha$, then $I(s) = \alpha$. Two operational laws of gradation function are given as follows:

$$\begin{aligned} I(s_\alpha) + I(s_\beta) &= I(s_\alpha \oplus s_\beta), \\ I(s_\alpha) \times I(s_\beta) &= I(s_\alpha \otimes s_\beta). \end{aligned} \tag{3}$$

2.2. Additive Uncertain Linguistic Preference Relations. Consider a GDM problem, let $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$ be the set of alternatives, and let $D = \{d_1, d_2, \ldots, d_m\}$ $(m \geq 2)$ be the set of DMs. Every DM compares each pair of alternatives in X and provides his/her preference of the alternatives with UALPRs $\{\widetilde{L}^{(1)}, \ldots, \widetilde{L}^{(k)}, \ldots, \widetilde{L}^{(m)}\}$, where $\widetilde{L}^{(k)} =$

$(\tilde{l}_{ij}^{(k)})_{n\times n} = ([\tilde{l}_{ij}^{(k)L}, \tilde{l}_{ij}^{(k)U}])_{n\times n}, \tilde{l}_{ij}^{(k)L}, \tilde{l}_{ij}^{(k)U} \in [s_0, s_1, \ldots, s_\tau]$ $(k = 1, 2, \ldots, m; i, j = 1, 2, \ldots, n)$. The UALPR can be formally defined as follows.

Definition 2 (see [19, 21, 22]). An UALPR $\tilde{L} = (\tilde{l}_{ij})_{n\times n}$ on a set of alternatives X is characterized by a function $\mu_{\tilde{L}} : X \times X \to \tilde{S}$ and satisfies

$$\tilde{l}_{ij} = \left[\tilde{l}_{ij}^L, \tilde{l}_{ij}^U\right], \qquad s_0 \leq \tilde{l}_{ij}^L \leq \tilde{l}_{ij}^U \leq s_\tau,$$
$$\tilde{l}_{ij}^L \oplus \tilde{l}_{ji}^U = \tilde{l}_{ij}^U \oplus \tilde{l}_{ji}^L = s_\tau, \qquad \tilde{l}_{ii}^L = \tilde{l}_{ii}^U = s_{\tau/2}, \tag{4}$$
$$\forall i, j \in \{1, \ldots, n\},$$

where the preference value $\tilde{l}_{ij} = \mu_{\tilde{L}}(x_i, x_j)$ indicates the preference degree of the alternative x_i over x_j and is interpreted as x_i is $\tilde{l}_{ij}$ as x_j, $\tilde{l}_{ij}^L$ and $\tilde{l}_{ij}^U$ are the lower and upper limits of $\tilde{l}_{ij}$, respectively, $\tilde{l}_{ij}^L, \tilde{l}_{ij}^U \in [s_0, s_1, \ldots, s_\tau]$. In particular, if $\tilde{l}_{ij}^L = \tilde{l}_{ij}^U$, $\forall i, j \in \{1, \ldots, n\}$, then $\tilde{l}_{ij}$ is reduced to a linguistic preference value l_{ij}.

3. Consistency Level and Consensus Level

3.1. Consistency Measure. Consistency is usually characterized by transitivity. Some transitive properties of linguistic preference relations can be described as follows [3, 5, 14]: (1) weak transitivity, (2) max–min transitivity, (3) max–max transitivity, (4) restricted max–min transitivity, (5) restricted max–max transitivity, (6) multiplicative consistency, and (7) additive consistency. Among these concepts, the most commonly used form is additive consistency [5, 6, 13, 15, 26–29]. Recently, Alonso et al. [26] introduced an additive transitivity property of the interval-valued preference relations and constructed a consistency measure; Gong et al. [27] also presented a similar additive transitivity property. Inspired by their ideas, we further redefine an additive transitivity of UALPRs to guarantee the indifference and reciprocity properties simultaneously. Then based on the definition, we propose a consistency measure to assess the consistency level of an UALPR.

Definition 3. An UALPR $\tilde{L} = (\tilde{l}_{ij})_{n\times n}$ is called additive consistent uncertain linguistic preference relation, if it satisfies the following additive transitivity:

$$\tilde{l}_{ij}^L = \min\left(\tilde{l}_{it}^L \oplus \tilde{l}_{tj}^U \ominus s_{\tau/2}, \tilde{l}_{it}^U \oplus \tilde{l}_{tj}^L \ominus s_{\tau/2}\right),$$
$$\tilde{l}_{ij}^U = \max\left(\tilde{l}_{it}^U \oplus \tilde{l}_{tj}^L \ominus s_{\tau/2}, \tilde{l}_{it}^L \oplus \tilde{l}_{tj}^U \ominus s_{\tau/2}\right), \tag{5}$$
$$\forall i, j, t \in \{1, \ldots, n\}.$$

Obviously, the preference value $\tilde{l}_{ij}$ can be estimated by using an intermediate alternative x_t:

$$e\tilde{l}_{ij}^L = \min\left(\tilde{l}_{it}^L \oplus \tilde{l}_{tj}^U \ominus s_{\tau/2}, \tilde{l}_{it}^U \oplus \tilde{l}_{tj}^L \ominus s_{\tau/2}\right),$$
$$e\tilde{l}_{ij}^U = \max\left(\tilde{l}_{it}^U \oplus \tilde{l}_{tj}^L \ominus s_{\tau/2}, \tilde{l}_{it}^L \oplus \tilde{l}_{tj}^U \ominus s_{\tau/2}\right), \tag{6}$$
$$\forall i, j, t \in \{1, \ldots, n\}.$$

Indeed, if the $\tilde{l}_{ij}$ is completely consistent in an UALPR, then $e\tilde{l}_{ij}^t = \tilde{l}_{ij}, \forall t \in \{1, \ldots, n\}$; that is, $e\tilde{l}_{ij}^{Lt} = \tilde{l}_{ij}^L$ and $e\tilde{l}_{ij}^{Ut} = \tilde{l}_{ij}^U$. However, it does not always hold due to the uncertainty and complexity of real-world decision problems and the subjectivity of DMs' judgments. In such case, we can get a deviation.

Definition 4. Let $\tilde{L} = (\tilde{l}_{ij})_{n\times n}$ be an UALPR; then

$$\Delta\tilde{l}_{ij} = \left(\Delta\tilde{l}_{ij}^L, \Delta\tilde{l}_{ij}^U\right) = \left(\frac{\sum_{t=1, t\neq i,j}^{n} \left|I\left(e\tilde{l}_{ij}^{Lt}\right) - I\left(\tilde{l}_{ij}^L\right)\right|}{n-2}, \frac{\sum_{t=1, t\neq i,j}^{n} \left|I\left(e\tilde{l}_{ij}^{Ut}\right) - I\left(\tilde{l}_{ij}^U\right)\right|}{n-2}\right) \tag{7}$$

is called the deviation between $\tilde{l}_{ij}$ and its estimated ones $e\tilde{l}_{ij}^t = \tilde{l}_{ij}$, $1 \leq t \leq n, t \neq i, j$.

Clearly, when $\Delta\tilde{l}_{ij} = (0, 0)$, then there is no inconsistency at all between $\tilde{l}_{ij}$ and the rest of preference values in the UALPR, and the higher is $\Delta\tilde{l}_{ij}$, the more inconsistent is $\tilde{l}_{ij}$ with respect to the rest of information in the UALPR.

Using the deviation $\Delta\tilde{l}_{ij}$, we define a consistency measure to assess the consistent level of $\tilde{l}_{ij}$.

Definition 5. Let $\tilde{L} = (\tilde{l}_{ij})_{n\times n}$ be an UALPR; then

$$\tilde{c}t_{ij} = \frac{\tau - \Delta\tilde{l}_{ij}}{\tau} = \left(\tilde{c}t_{ij}^L, \tilde{c}t_{ij}^U\right) = \left(\frac{\tau - \Delta\tilde{l}_{ij}^L}{\tau}, \frac{\tau - \Delta\tilde{l}_{ij}^U}{\tau}\right) \tag{8}$$

is called the consistency measure of $\tilde{l}_{ij}$.

This consistency measure has a definite physical implication and reflects the consistency level between the preference value $\tilde{l}_{ij}$ and the rest of the preference values in the UALPR. Clearly, when $\tilde{c}t_{ij} = (1, 1)$, then there is no inconsistency at all between $\tilde{l}_{ij}$ and the other preference values, and the lower is the value of $\tilde{c}t_{ij}$, the more inconsistent is $\tilde{l}_{ij}$ with respect to the rest of information.

Theorem 6. *If $\tilde{L} = (\tilde{l}_{ij})_{n\times n}$ is an UALPR, then*

$$\tilde{c}t_{ij}^L = \tilde{c}t_{ji}^U, \quad \tilde{c}t_{ij}^U = \tilde{c}t_{ji}^L, \quad \forall i, j \in \{1, \ldots, n\}. \tag{9}$$

Proof. Since

$$\Delta\tilde{l}^L_{ij} = \left(\sum_{t=1,t\neq i,j}^{n} \left| I\left(e\tilde{l}^{Lt}_{ij}\right) - I\left(\tilde{l}^L_{ij}\right)\right|\right) \times (n-2)^{-1}$$
$$= \left(\sum_{t=1,t\neq i,j}^{n} \left|\min\left(I\left(\tilde{l}^L_{it}\right) + I\left(\tilde{l}^U_{tj}\right) - I\left(s_{\tau/2}\right), \left(\tilde{l}^U_{it}\right) + I\left(\tilde{l}^L_{tj}\right) - I\left(s_{\tau/2}\right)\right) - I\left(\tilde{l}^L_{ij}\right)\right|\right) \times (n-2)^{-1}$$
$$= \left(\sum_{t=1,t\neq i,j}^{n} \left|\min\left(\tau - I\left(\tilde{l}^U_{ti}\right) + \tau - I\left(\tilde{l}^L_{jt}\right) - \frac{\tau}{2}, \tau - I\left(\tilde{l}^L_{ti}\right) + \tau - I\left(\tilde{l}^U_{jt}\right) - s_{\tau/2}\right) - \tau + I\left(\tilde{l}^U_{ji}\right)\right|\right) \times (n-2)^{-1}$$
$$= \left(\sum_{tk=1,t\neq i,j}^{n} \left|\min\left(-I\left(\tilde{l}^U_{ti}\right) - I\left(\tilde{l}^L_{jt}\right) + \frac{\tau}{2}, -I\left(\tilde{l}^L_{ti}\right) - I\left(\tilde{l}^U_{jt}\right) + s_{\tau/2}\right) + I\left(\tilde{l}^U_{ji}\right)\right|\right) \times (n-2)^{-1}$$
$$= \left(\sum_{t=1,t\neq i,j}^{n} \left|(-1)\left(\max\left(I\left(\tilde{l}^U_{ti}\right) + I\left(\tilde{l}^L_{jt}\right) - \frac{\tau}{2}, I\left(\tilde{l}^L_{ti}\right) + I\left(\tilde{l}^U_{jt}\right) - s_{\tau/2}\right) - I\left(\tilde{l}^U_{ji}\right)\right)\right|\right) \times (n-2)^{-1}$$
$$= \left(\sum_{t=1,t\neq i,j}^{n} \left|(-1)\, I\left(\max\left(\tilde{l}^U_{ti} \oplus \tilde{l}^L_{jt} \ominus s_{\tau/2}, \tilde{l}^L_{ti} \oplus \tilde{l}^U_{jt} \ominus s_{\tau/2}\right) \ominus \tilde{l}^U_{ji}\right)\right|\right) \times (n-2)^{-1}$$
$$= \Delta\tilde{l}^U_{ji}, \tag{10}$$

then

$$\tilde{c}t^L_{ij} = \tilde{c}t^U_{ji}. \tag{11}$$

Similarly, we can obtain $\tilde{c}t^U_{ij} = \tilde{c}t^L_{ji}$. This completes the above proof. □

In particular, if $\tilde{l}_{ij}$ reduced to l_{ij}, then the consistency level of l_{ij} has property $ct_{ij} = ct_{ji}$. Furthermore, we can obtain the consistency level of the whole UALPR by applying the arithmetic mean.

Definition 7. Let $\tilde{L} = (\tilde{l}_{ij})_{n\times n}$ be an UALPR; then

$$\widetilde{C}t = \left(\widetilde{C}t^L, \widetilde{C}t^U\right) = \left(\frac{1}{(n^2-n)} \sum_{\substack{i,j=1,\\ i\neq j}}^{n} \tilde{c}t^L_{ij}, \frac{1}{(n^2-n)} \sum_{\substack{i,j=1,\\ i\neq j}}^{n} \tilde{c}t^U_{ij}\right) \tag{12}$$

is called the consistency measure of $\tilde{L}$.

If $\widetilde{C}t = (1,1)$, then the UALPR $\tilde{L}$ is fully consistent; otherwise, the lower $C\tilde{t}$ is the more inconsistent $\tilde{L}$ is.

Theorem 8. *If $\tilde{L} = (\tilde{l}_{ij})_{n\times n}$ is an UALPR, then*

$$\widetilde{C}t^L = \widetilde{C}t^U. \tag{13}$$

Proof. On has

$$\widetilde{C}t^L = \frac{1}{(n^2-n)} \sum_{\substack{i,j=1,\\ i\neq j}}^{n} \tilde{c}t^L_{ij} = \frac{1}{(n^2-n)} \sum_{\substack{i,j=1,\\ i\neq j}}^{n} \tilde{c}t^U_{ji} = \frac{1}{(n^2-n)} \sum_{\substack{i,j=1,\\ i\neq j}}^{n} \tilde{c}t^U_{ij} = \widetilde{C}t^U. \tag{14}$$

By Theorem 8, we find that the consistency level of the whole UALPR is certain. □

3.2. Consensus Measure. Using anonymous ways of the definitions of consistency measure, we define a consensus measure to assess the level of agreement between the individual preference value and the collective ones.

Definition 9. Let $\{\tilde{L}^{(k)} = (\tilde{l}^{(k)}_{ij})_{n\times n}, k = 1, 2, \ldots, m\}$ be a group of UALPRs provided by DMs; then a deviation of the uncertain linguistic preference value $\tilde{l}^{(k)}_{ij}$ provided by d_k

and the ones $\tilde{l}_{ij}^{(p)} \in \tilde{L}^{(p)}$ by d_p $(p = 1, 2, \ldots, m, p \neq k)$ is defined as

$$\Delta' \tilde{l}_{ij}^{(k)} = \left(\Delta' l_{ij}^{(k)L}, \Delta' l_{ij}^{(k)U}\right) = \left(\frac{\sum_{p=1, p\neq k}^{m} \left|I\left(l_{ij}^{(k)L}\right) - I\left(l_{ij}^{(p)L}\right)\right|}{m-1}, \frac{\sum_{p=1, p\neq k}^{m} \left|I\left(l_{ij}^{(k)U}\right) - I\left(l_{ij}^{(p)U}\right)\right|}{m-1}\right). \quad (15)$$

Clearly, $\Delta' \tilde{l}_{ij}^{(k)} = (0, 0)$ indicates that the $\tilde{l}_{ij}$ provided by d_k and the ones provided by d_p $(p = 1, 2, \ldots, m, p \neq k)$ are not contradictory at all, and the higher the $\Delta' \tilde{l}_{ij}^{(k)}$, is the more decentralized the opinions between d_k and d_p $(p = 1, 2, \ldots, m, p \neq k)$ on the preference value $\tilde{l}_{ij}$ are. Based on the deviation $\Delta' \tilde{l}_{ij}^{(k)}$, we develop a consensus measure of $\tilde{l}_{ij}^{(k)}$.

Definition 10. Let $\{\tilde{L}^{(k)} = (\tilde{l}_{ij}^{(k)})_{n\times n}, k = 1, 2, \ldots, m\}$ be a group of UALPRs provided by DMs; then the consensus measure of $\tilde{l}_{ij}^{(k)}$ is defined as follows:

$$\widetilde{ca}_{ij}^{(k)} = \left(\widetilde{ca}_{ij}^{(k)L}, \widetilde{ca}_{ij}^{(k)U}\right) = \left(1 - \frac{\sum_{p=1, p\neq k}^{m} \left|I\left(l_{ij}^{(k)L}\right) - I\left(l_{ij}^{(p)L}\right)\right|}{\tau(m-1)}, 1 - \frac{\sum_{p=1, p\neq k}^{m} \left|I\left(l_{ij}^{(k)U}\right) - I\left(l_{ij}^{(p)U}\right)\right|}{\tau(m-1)}\right). \quad (16)$$

Clearly, when $\widetilde{ca}_{ij}^{(k)} = (1, 1)$, then there are no decentralized opinions at all between d_k and d_p $(p = 1, 2, \ldots, m, p \neq k)$ on the preference value $\tilde{l}_{ij}$, and the lower the $\widetilde{ca}_{ij}^{(k)}$ is the more decentralized the opinions between d_k and d_p $(p = 1, 2, \ldots, m, p \neq k)$ on the preference value $\tilde{l}_{ij}$ are. By applying the arithmetic mean, we have the consensus measures of the whole UALPRs.

Definition 11. Let $\{\tilde{L}^{(k)} = (\tilde{l}_{ij}^{(k)})_{n\times n}, k = 1, 2, \ldots, m\}$ be a group of UALPRs provided by DMs; then the consensus measures of the whole UALPRs are as follows:

$$\widetilde{Ca}_k = \left(\widetilde{Ca}_k^L, \widetilde{Ca}_k^U\right) = \left(\frac{1}{n(n-1)} \sum_{i,j=1, j\neq i}^{n} \widetilde{ca}_{ij}^{(k)L}, \frac{1}{n(n-1)} \sum_{i,j=1, j\neq i}^{n} \widetilde{ca}_{ij}^{(k)U}\right). \quad (17)$$

Clearly, when $\widetilde{Ca}_k = (1, 1)$, then there are no decentralized opinions at all between d_k and d_p $(p = 1, 2, \ldots, m, p \neq k)$, and the lower the $\widetilde{Ca}_k$ is, the more decentralized the opinions of d_k and the others are.

Theorem 12. *If $\{\tilde{L}^{(k)} = (\tilde{l}_{ij}^{(k)})_{n\times n}, k = 1, 2, \ldots, m\}$ is a group of UALPRs, then*

$$\widetilde{ca}_{ij}^{(k)L} = \widetilde{ca}_{ji}^{(k)U}, \quad \widetilde{ca}_{ij}^{(k)U} = \widetilde{ca}_{ji}^{(k)L}, \quad \forall i, j \in \{1, \ldots, n\}. \quad (18)$$

Proof. On has

$$\widetilde{ca}_{ij}^{(k)L} = 1 - \frac{\sum_{p=1, p\neq k}^{m} \left|I\left(l_{ij}^{(k)L}\right) - I\left(l_{ij}^{(p)L}\right)\right|}{\tau(m-1)} = 1 - \frac{\sum_{p=1, p\neq k}^{m} \left|(-1) I\left(l_{ji}^{(k)U}\right) - I\left(l_{ji}^{(p)U}\right)\right|}{\tau(m-1)} = \widetilde{ca}_{ji}^{(k)U}. \quad (19)$$

which completes the proof of Theorem 12. □

In particular, if UALPRs $\tilde{l}_{ij}$ reduced to l_{ij}, then the consensus level of l_{ij} has property $ca_{ij} = ca_{ji}$.

Theorem 13. *If $\{\tilde{L}^{(k)} = (\tilde{l}_{ij}^{(k)})_{n\times n}, k = 1, 2, \ldots, m\}$ is a group of UALPRs, then*

$$\widetilde{Ca}_k^L = \widetilde{Ca}_k^U. \quad (20)$$

Proof. On has

$$\widetilde{Ca}_k^L = \frac{1}{n(n-1)} \sum_{i,j=1, j\neq i}^{n} \widetilde{ca}_{ij}^{(k)L} = \frac{1}{n(n-1)} \sum_{i,j=1, j\neq i}^{n} \widetilde{ca}_{ji}^{(k)U} = \frac{1}{n(n-1)} \sum_{i,j=1, j\neq i}^{n} \widetilde{ca}_{ij}^{(k)U} = \widetilde{Ca}_k^U. \quad (21)$$

By Theorem 13, we find that the consensus level of the whole UALPR is also certain. □

4. Aggregation of Uncertain Linguistic Preference Information Based on Consistency and Consensus

To obtain collective preference information by combining the individual ones, an aggregation operator is needed. The ordered weighted averaging (OWA) and weighted arithmetic averaging (WAA) are the most common aggregation operators. The induced OWA (IOWA) operator provided by Yager and Filev [30] is a more general type of the OWA operator, in which the ordering of the arguments is induced by the order inducing variables, rather than the values of the arguments. The IOWA operator can be defined as follows.

Definition 14. An IOWA operator of dimension n is a mapping IOWA : $R^n \rightarrow R$ can be defined as follows:

$$\text{IOWA}_w\left(\langle \mu_1, a_1\rangle, \langle \mu_2, a_2\rangle, \ldots, \langle \mu_n, a_n\rangle\right) = \sum_{j=1}^{n} w_j a_{\sigma(j)}, \quad (22)$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is an associated weighting vector, such that $0 \le w_j \le 1$, $\sum_{j=1}^{n} w_j = 1$; $a_{\sigma(j)}$ is the a_j value of the OWA pair $\langle \mu_j, a_j \rangle$ having the jth largest μ_j; μ_j in $\langle \mu_j, a_j \rangle$ is referred to as the order inducing variable and a_j as the argument variable.

Note that (22) can be equivalently written as

$$\text{IOWA}_w\left(\langle \mu_1, a_1 \rangle, \langle \mu_2, a_2 \rangle, \ldots, \langle \mu_n, a_n \rangle\right) = \sum_{j=1}^{n} w_{\rho(j)} a_j, \tag{23}$$

where $\rho = \sigma^{-1} : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ is the inverse permutation of σ and a_j is the value of the OWA pair $\langle \mu_j, a_j \rangle$ having the $\rho(j)$th largest μ_j. μ_j in $\langle \mu_j, a_j \rangle$ is referred to as the order inducing variable and a_j as the argument variable.

Based on IOWA operator, Xu [31] presented an induced uncertain linguistic OWA operator to aggregate uncertain linguistic variables.

Definition 15. An induced uncertain linguistic OWA (IULOWA) operator is defined as follows:

$$\begin{aligned} &\text{IULOWA}_w\left(\langle \mu_1, \tilde{s}_1 \rangle, \langle \mu_2, \tilde{s}_2 \rangle, \ldots, \langle \mu_n, \tilde{s}_n \rangle\right) \\ &= \bigoplus_{j=1}^{n} w_j \tilde{s}_{\beta_j} \\ &= \left[\bigoplus_{j=1}^{n} w_j \tilde{s}_{\beta_j}^{L}, \bigoplus_{j=1}^{n} w_j \tilde{s}_{\beta_j}^{U} \right], \end{aligned} \tag{24}$$

where $w = (w_1, w_2, \ldots, w_n)$ is an associated weighting vector, such that $0 \le w_j \le 1$, $\sum_{j=1}^{n} w_j = 1$; $\tilde{s}_{\beta_j}$ is the $\tilde{s}_j$ value of the OWA pair $\langle \mu_j, \tilde{s}_j \rangle$ having the jth largest $\mu_j \cdot \mu_j$ in $\langle \mu_j, \tilde{s}_j \rangle$ is referred to as the order inducing variable and $\tilde{s}_j$ as the argument variable.

The key issues in the IOWA operator theory are to determine the order inducing variables and then derive the associated weights based on them, but quite few studies have been conducted on the issues. Recently, Chiclana et al. [32] introduced a consistency-induced OWA (C-IOWA) operator, in which the ordering of the preference value is based upon its consistency, and it is further studied in [6, 7, 26, 28, 29]. Wu et al. [17] presented a compatibility index ILOWG (CI-ILOWG) operator, which induces the order of argument values by the compatibility index of experts they also verified that the CI-ILOWG operator guarantees the compatibility degree is at least as good as the arithmetic mean of all the individual compatibility degrees. As aforementioned, both consensus and consistency should be considered simultaneously in any rational GDM. To more effectively relieve the influence some individuals may assign lower consistent and lower centralized preference values to their preferred or repugnant objects, it is very reasonable and meaningful that by assigning low weights to those "inconsistent" or "decentralized" opinions in preference aggregation procedure. Based on the discussions, we here propose a consistency and consensus co-induced uncertain linguistic ordered weighted averaging (C^2-IULOWA) operator to aggregate individual preference information in such a way that the greater weight is given the most consistent and consensual preference information.

Definition 16. Let $\{\tilde{L}^{(k)} = (\tilde{l}_{ij}^{(k)})_{n\times n}, k = 1, 2, \ldots, m\}$ be a group of UALPRs to aggregate, where $\tilde{L}^{(k)} = (\tilde{l}_{ij}^{(k)}) = ([\tilde{l}_{ij}^{(k)L}, \tilde{l}_{ij}^{(k)U}])$ and $\{\widetilde{Ct}_1, \widetilde{Ct}_2, \ldots, \widetilde{Ct}_m\}$ and $\{\widetilde{Ca}_1, \widetilde{Ca}_2, \ldots, \widetilde{Ca}_m\}$ are their corresponding consistency level and consensus level, respectively; then a consensus and consistency co-induced uncertain linguistic ordered weighted averaging (C^2-IULOWA) operator is defined as

$$\begin{aligned} &\text{IULOWA}_{C^2 \mapsto w}\left(\left\langle \widetilde{Ca}_1, \widetilde{Ct}_1, \tilde{L}^{(1)} \right\rangle, \left\langle \widetilde{Ca}_2, \widetilde{Ct}_2, \tilde{L}^{(2)} \right\rangle, \ldots, \left\langle \widetilde{Ca}_m, \widetilde{Ct}_m, \tilde{L}^{(m)} \right\rangle\right) \\ &= \left[\frac{\bigoplus_{k=1}^{m} w_{\xi(k)} \tilde{L}^{(k)L} w_{\zeta(k)}}{\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}}, \frac{\bigoplus_{j=1}^{m} w_{\xi(k)} \tilde{L}^{(k)U} w_{\zeta(k)}}{\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}} \right] \\ &= \left[\frac{\bigoplus_{k=1}^{m} w_{\xi(k)} \tilde{l}_{ij}^{(k)L} w_{\zeta(k)}}{\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}}, \frac{\bigoplus_{k=1}^{m} w_{\xi(k)} \tilde{l}_{ij}^{(k)U} w_{\zeta(k)}}{\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}} \right]_{n\times n}, \end{aligned} \tag{25}$$

where $w = (w_1, w_2, \ldots, w_m)^T$ is an associated weighting vector, such that $0 \le w_j \le 1$, $\sum_{j=1}^{m} w_j = 1$ and the consensus level $\widetilde{Ca}_k$ and the consistency level $\widetilde{Ct}_k$ are simultaneously referred to as the order inducing variables and $\tilde{L}^{(k)} = [\tilde{l}_{ij}^{(k)L}, \tilde{l}_{ij}^{(k)U}]$ as the argument variable. $\xi(\cdot)$ and $\zeta(\cdot)$ are permutation functions such that $\tilde{L}^{(k)}$ is the preference relation which has the $\xi(k)$th and $\zeta(k)$th largest $\widetilde{Ca}_k$ and $\widetilde{Ct}_k$. In particular, when $\tilde{l}_{ij}^{L} = \tilde{l}_{ij}^{U}$, the C^2-IULOWA operator reduced to a consensus and consistency co-induced linguistic ordered weighted averaging (C^2-ILOWA) operator:

$$\begin{aligned} &\text{ILOWA}_{C^2 \mapsto w}\left(\left\langle Ca_k, Ct_k, L^{(k)} \right\rangle, k = 1, \ldots, m\right) \\ &= \frac{\bigoplus_{k=1}^{m} w_{\xi(k)} L^{(k)} w_{\zeta(k)}}{\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}} \\ &= \left[\frac{\bigoplus_{k=1}^{m} w_{\xi(k)} l_{ij}^{(k)} w_{\zeta(k)}}{\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}} \right]_{n\times n.} \end{aligned} \tag{26}$$

Theorem 17. *If a group of UALPRs $\{\tilde{L}^{(k)} = (\tilde{l}_{ij}^{(k)})_{n\times n}, k = 1, 2, \ldots, m\}$ to aggregate has identical consistency level, that is, $\tilde{c}t_1 = \tilde{c}t_2 = \cdots = \tilde{c}t_m$, then the C^2-IULOWA operator becomes a consensus-induced uncertain linguistic ordered weighted averaging (Ca-IULOWA) operator:*

$$\begin{aligned} &\mathit{IULOWA}_{\,Ca \mapsto w}\left(\left\langle \widetilde{Ca}_1, \tilde{L}^{(1)} \right\rangle, \left\langle \widetilde{Ca}_2, \tilde{L}^{(2)} \right\rangle, \ldots, \left\langle \widetilde{Ca}_m, \tilde{L}^{(m)} \right\rangle\right) \\ &= \left[\bigoplus_{k=1}^{m} w_{\xi(k)} \tilde{l}_{ij}^{(k)L}, \bigoplus_{k=1}^{m} w_{\xi(k)} \tilde{l}_{ij}^{(k)U} \right]. \end{aligned} \tag{27}$$

Proof. On has

$$\begin{aligned}
&\text{IULOWA}_{C^2\mapsto w}\left(\left\langle \widetilde{C}a_1, \widetilde{C}t_1, \widetilde{L}^{(1)}\right\rangle, \left\langle \widetilde{C}a_2, \widetilde{C}t_2, \widetilde{L}^{(2)}\right\rangle, \dots, \left\langle \widetilde{C}a_m, \widetilde{C}t_m, \widetilde{L}^{(m)}\right\rangle\right) \\
&= \left[\frac{\bigoplus_{k=1}^m w_{\xi(k)} \widetilde{l}_{ij}^{(k)L} w_{\zeta(k)}}{\sum_{k=1}^m w_{\xi(k)} w_{\zeta(k)}}, \frac{\bigoplus_{k=1}^m w_{\xi(k)} \widetilde{l}_{ij}^{(k)U} w_{\zeta(k)}}{\sum_{k=1}^m w_{\xi(k)} w_{\zeta(k)}}\right] \\
&= \left[\frac{\bigoplus_{k=1}^m w_{\xi(k)} \widetilde{l}_{ij}^{(k)L} (1/n)}{\sum_{k=1}^m w_{\xi(k)} (1/n)}, \frac{\bigoplus_{k=1}^m w_{\xi(k)} \widetilde{l}_{ij}^{(k)U} (1/n)}{\sum_{k=1}^m w_{\xi(k)} (1/n)}\right] \\
&= \left[\bigoplus_{k=1}^m w_{\xi(k)} \widetilde{l}_{ij}^{(k)L}, \bigoplus_{k=1}^m w_{\xi(k)} \widetilde{l}_{ij}^{(k)U}\right] \\
&= \text{IULOWA}_{Ca\mapsto w}\left(\left\langle \widetilde{C}a_1, \widetilde{L}^{(1)}\right\rangle, \left\langle \widetilde{C}a_2, \widetilde{L}^{(2)}\right\rangle, \dots, \left\langle \widetilde{C}a_m, \widetilde{L}^{(m)}\right\rangle\right).
\end{aligned} \tag{28}$$

□

Theorem 18. *If a group of UALPRs $\{\widetilde{L}^{(k)} = (\widetilde{l}_{ij}^{(k)})_{n\times n}, k = 1, 2, \dots, m\}$ to aggregate has identical consensus level, that is, $\widetilde{c}a_1 = \widetilde{c}a_2 = \cdots = \widetilde{c}a_m$, then the C^2-IULOWA operator becomes a consistency-induced uncertain linguistic ordered weighted averaging (Ct-IULOWA) operator:*

$$\begin{aligned}
&IULOWA_{Ct\mapsto w}\left(\left\langle \widetilde{C}t_1, \widetilde{L}^{(1)}\right\rangle, \left\langle \widetilde{C}t_2, \widetilde{L}^{(2)}\right\rangle, \dots, \left\langle \widetilde{C}t_m, \widetilde{L}^{(m)}\right\rangle\right) \\
&= \left[\bigoplus_{k=1}^m w_{\zeta(k)} \widetilde{l}_{ij}^{(k)L}, \bigoplus_{k=1}^m w_{\zeta(k)} \widetilde{l}_{ij}^{(k)U}\right].
\end{aligned} \tag{29}$$

Proof. On has

$$\begin{aligned}
&\text{IULOWA}_{C^2\mapsto w}\left(\left\langle \widetilde{C}a_1, \widetilde{C}t_1, \widetilde{L}^{(1)}\right\rangle, \left\langle \widetilde{C}a_2, \widetilde{C}t_2, \widetilde{L}^{(2)}\right\rangle, \dots, \left\langle \widetilde{C}a_m, \widetilde{C}t_m, \widetilde{L}^{(m)}\right\rangle\right) \\
&= \left[\frac{\bigoplus_{k=1}^m w_{\xi(k)} \widetilde{l}_{ij}^{(k)L} w_{\zeta(k)}}{\sum_{k=1}^m w_{\xi(k)} w_{\zeta(k)}}, \frac{\bigoplus_{k=1}^m w_{\xi(k)} \widetilde{l}_{ij}^{(k)U} w_{\zeta(k)}}{\sum_{k=1}^m w_{\xi(k)} w_{\zeta(k)}}\right] \\
&= \left[\frac{\bigoplus_{k=1}^m (1/n) \widetilde{l}_{ij}^{(k)L} w_{\zeta(k)}}{\sum_{k=1}^m (1/n) w_{\zeta(k)}}, \frac{\bigoplus_{k=1}^m (1/n) \widetilde{l}_{ij}^{(k)U} w_{\zeta(k)}}{\sum_{k=1}^m (1/n) w_{\zeta(k)}}\right] \\
&= \left[\bigoplus_{k=1}^m w_{\zeta(k)} \widetilde{l}_{ij}^{(k)L}, \bigoplus_{k=1}^m w_{\zeta(k)} \widetilde{l}_{ij}^{(k)U}\right] \\
&= \text{IULOWA}_{Ct\mapsto w}\left(\left\langle \widetilde{C}t_1, \widetilde{L}^{(1)}\right\rangle, \left\langle \widetilde{C}t_2, \widetilde{L}^{(2)}\right\rangle, \dots, \left\langle \widetilde{C}t_m, \widetilde{L}^{(m)}\right\rangle\right).
\end{aligned} \tag{30}$$

From Definition 16 and the previous theorems, we know the following.

(1) The inducing variables of the C^2-IULOWA operator are characterized by the consistency measure and the consensus measure, which synergistically reflect the consensus level and the consistency level of UALPRs and participate to guide the determination of associated weights.

(2) The C^2-IULOWA operator generalizes the consensus-induced uncertain linguistic OWA operators and consistency-induced uncertain linguistic OWA operators, which comprehensively merge the information associated with the properties of preference relation in the preference aggregation.

Moreover, it is easy to verify that the C^2-IULOWA operator is monotonic, commutative, bounded, and idempotent; the proofs of them are omitted here because they are trivial. In the following, we study the rationality of the C^2-IULOWA operator that means the aggregated uncertain linguistic preference relations maintain the essential properties of preference relations such as the indifference, the reciprocity, and the consistency properties.

Since the C^2-IULOWA operator is idempotent, the indifference property is obvious and its proofs are omitted here. Now we verify the reciprocity property.

Since $\widetilde{L}^{(k)} = ([\widetilde{l}_{ij}^{(k)L}, \widetilde{l}_{ij}^{(k)U}])_{n\times n}, k = 1, \dots, m$, are UALPRs, we have $\widetilde{l}_{ij}^{(k)L} \oplus \widetilde{l}_{ji}^{(k)U} = s_\tau$, $\widetilde{l}_{ij}^{(k)U} \oplus \widetilde{l}_{ji}^{(k)L} = s_\tau$, $\widetilde{l}_{ii}^{(k)U} = \widetilde{l}_{ii}^{(k)L} = s_{\tau/2}$.

Thus

$$\begin{aligned}
\widetilde{l}_{ij}^{(C)L} \oplus \widetilde{l}_{ji}^{(C)U} &= \frac{\bigoplus_{k=1}^m w_{\xi(k)} \widetilde{l}_{ij}^{(k)L} w_{\zeta(k)}}{\sum_{k=1}^m w_{\xi(k)} w_{\zeta(k)}} \oplus \frac{\bigoplus_{j=1}^n w_{\xi(k)} \widetilde{l}_{ji}^{(k)U} w_{\zeta(k)}}{\sum_{k=1}^m w_{\xi(k)} w_{\zeta(k)}} \\
&= \frac{\bigoplus_{k=1}^m w_{\xi(k)} \left(\widetilde{l}_{ij}^{(k)L} \oplus \widetilde{l}_{ji}^{(k)U}\right) w_{\zeta(k)}}{\sum_{k=1}^m w_{\xi(k)} w_{\zeta(k)}} \\
&= \frac{\bigoplus_{k=1}^m w_{\xi(k)} s_\tau w_{\zeta(k)}}{\sum_{k=1}^m w_{\xi(k)} w_{\zeta(k)}} = s_\tau.
\end{aligned} \tag{31}$$

Similarly, we can verify $\widetilde{l}_{ij}^{(C)U} \oplus \widetilde{l}_{ji}^{(C)L} = s_\tau$.

Then we verify the consistency property.

Since $\widetilde{L}^{(k)} = ([\widetilde{l}_{ij}^{(k)L}, \widetilde{l}_{ij}^{(k)U}])_{n\times n}, k = 1, \dots, m$, are consistent, we have

$$\begin{aligned}
\widetilde{l}_{ij}^{(k)L} &= \min\left(\widetilde{l}_{it}^{(k)L} \oplus \widetilde{l}_{tj}^{(k)U} \ominus s_{\tau/2}, \widetilde{l}_{it}^{(k)U} \oplus \widetilde{l}_{tj}^{(k)L} \ominus s_{\tau/2}\right), \\
\widetilde{l}_{ij}^{(k)U} &= \max\left(\widetilde{l}_{it}^{(k)U} \oplus \widetilde{l}_{tj}^{(k)L} \ominus s_{\tau/2}, \widetilde{l}_{it}^{(k)L} \oplus \widetilde{l}_{tj}^{(k)U} \ominus s_{\tau/2}\right), \\
&\forall i, j, t \in \{1, \dots, n\},\ k = 1, \dots, m,
\end{aligned} \tag{32}$$

then we have

$$\begin{aligned}
\tilde{l}_{ij}^{(C)L} &= \frac{\bigoplus_{k=1}^{m} w_{\xi(k)} \tilde{l}_{ij}^{(k)L} w_{\zeta(k)}}{\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}} \\
&= \bigoplus_{k=1}^{m} w_{\xi(k)} \min\left(\tilde{l}_{it}^{(k)L} \oplus \tilde{l}_{tj}^{(k)U} \ominus s_{\tau/2}, \right. \\
&\qquad \left. \tilde{l}_{it}^{(k)U} \oplus \tilde{l}_{tj}^{(k)L} \ominus s_{\tau/2}\right) w_{\zeta(k)} \\
&\quad \times \left(\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}\right)^{-1} \\
&= \min\left(\frac{\bigoplus_{k=1}^{m} w_{\xi(k)} \left(\tilde{l}_{it}^{(k)L} \oplus \tilde{l}_{tj}^{(k)U} \ominus s_{\tau/2}\right) w_{\zeta(k)}}{\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}}, \right. \\
&\qquad \left. \frac{\bigoplus_{k=1}^{m} w_{\xi(k)} \left(\tilde{l}_{it}^{(k)U} \oplus \tilde{l}_{tj}^{(k)L} \ominus s_{\tau/2}\right) w_{\zeta(k)}}{\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}}\right) \\
&= \min\left(\frac{\bigoplus_{k=1}^{m} w_{\xi(k)} \tilde{l}_{it}^{(k)L} w_{\zeta(k)}}{\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}} \right. \\
&\qquad \oplus \frac{\bigoplus_{j=1}^{n} w_{\xi(k)} \tilde{l}_{tj}^{(k)U} w_{\zeta(k)}}{\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}} \ominus s_{\tau/2}, \\
&\qquad \frac{\bigoplus_{k=1}^{m} w_{\xi(k)} \tilde{l}_{it}^{(k)U} w_{\zeta(k)}}{\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}} \\
&\qquad \left. \oplus \frac{\bigoplus_{j=1}^{n} w_{\xi(k)} \tilde{l}_{tj}^{(k)L} w_{\zeta(k)}}{\sum_{k=1}^{m} w_{\xi(k)} w_{\zeta(k)}} \ominus s_{\tau/2}\right) \\
&= \min\left(\tilde{l}_{it}^{(C)L} \oplus \tilde{l}_{tj}^{(C)U} \ominus s_{\tau/2}, \tilde{l}_{it}^{(C)U} \oplus \tilde{l}_{tj}^{(C)L} \ominus s_{\tau/2}\right).
\end{aligned} \tag{33}$$

Similarly, we can verify $\tilde{l}_{ij}^{(C)U} = \max(\tilde{l}_{it}^{(C)L} \oplus \tilde{l}_{tj}^{(C)U} \ominus s_{\tau/2}, \tilde{l}_{it}^{(C)U} \oplus \tilde{l}_{tj}^{(C)L} \ominus s_{\tau/2})$.

Hence, $\tilde{L}^{(C)}$ is also consistent, which verifies the consistency property.

An important step in the process of aggregating preference relations is to determine the associated weight of each preference relation or preference value. Usually, the associated weights are determined by a linguistic quantifier Q as [32]

$$w_j = Q\left(\frac{j}{n}\right) - Q\left(\frac{j-1}{n}\right), \quad j = 1, \ldots, n, \tag{34}$$

with $Q(r) = r^{1/2}$ meaning the quantifier guiding this aggregation to be "most" [4, 6, 32]. It can easily be shown that using this the w_j satisfy the conditions $w_j \in [0, 1]$ and $\sum_j w_j = 1$.

A further important aspect of induced type aggregation operator is the fact that ties in the order inducing variables, that is, when two or more inducing variables have exactly the same value. In order to solve this problem, we recommend adopting Yager's policy [33], is, that using an average of the components of weights over the tied constituents and leaving the arguments alone. □

5. Group Decision Making Based on C^2-IULOWA Operator

In the procedure of GDM, preference relations are usually composed by two phases [4, 6–10]: the aggregation phase and the exploitation phase, and two distinct choice processes are known: one is the direct way, which draws decision conclusion directly from individual preference relations and aggregates those individual choices, and the other is indirect way, which first aggregates individual linguistic preference relations into a collective preference relation and then draws decision conclusion; they can be described as follows:

$$\begin{aligned}
&\left\{L^{(k)}, k = 1, \ldots, m\right\} \xrightarrow{\text{exploitation}} \left\{\text{solution}^{(k)}, k = 1, \ldots, m\right\} \\
&\xrightarrow{\text{aggregation}} \text{solution}^{(C)} \\
&\left\{L^{(k)}, k = 1, \ldots, m\right\} \xrightarrow{\text{aggregation}} \left\{L^{(C)}\right\} \\
&\xrightarrow{\text{exploitation}} \text{solution}^{(C)}.
\end{aligned} \tag{35}$$

Herrera et al. [10] and Delgado et al. [11] developed a direct GDM approach with linguistic preference relation, which ranks or selects the alternative(s) according to linguistic nondominance degrees. The approach can be briefly described as follows.

(1) For each linguistic preference relation of each expert, $L^{(k)}$, find its respective linguistic strict preference relation, ${}^{s}L^{(k)} = ({}^{s}l_{ij}^{(k)})_{n \times n}$, such that

$${}^{s}l_{ij}^{(k)} = \begin{cases} s_0, & \text{if } l_{ij}^{(k)} < l_{ji}^{(k)} \\ s_h \in S, & \text{if } l_{ij}^{(k)} \geq l_{ji}^{(k)}, \text{with } l_{ij}^{(k)} = s_l, \\ & l_{ji}^{(k)} = s_t,\ l = t + h. \end{cases} \tag{36}$$

(2) For each linguistic strict preference relation of each expert, ${}^{s}L^{(k)}$, determine the individual linguistic non-dominance degree of each alternative x_i, called $\text{IND}_i^{(k)}$, according to linguistic OWA operator:

$$\text{IND}_i^{(k)} = \text{LOWA}\left(\text{neg}\left({}^{s}l_{ji}^{(k)}\right), j = 1, \ldots, n, j \neq i\right), \quad i = 1, \ldots, n,\ k = 1, \ldots, m. \tag{37}$$

(3) For each alternative x_i, calculate the social linguistic non-dominance degree, called SND_i, as follows:

$$\text{SND}_i = \text{LOWA}\left(\text{IND}_i^{(k)},\ l = 1,\dots,m\right), \quad i = 1,\dots,n. \tag{38}$$

(4) Rank or select the alternatives with respect to SND_i $(i = 1,2,\dots,n)$.

Here, we extend those definitions to rank or select the alternative(s) for UALPRs. To do so, we give the following definitions.

Definition 19. Let $\widetilde{L} = (\widetilde{l}_{ij})_{n\times n}$ be an UALPR; then the uncertain linguistic dominance relation in terms of $\widetilde{L}$ is defined as

$$\widetilde{L}D_{ij} = \left[\max\left(\widetilde{l}_{ij}^{L} \ominus \widetilde{l}_{ji}^{U}, s_0\right), \max\left(\widetilde{l}_{ij}^{U} \ominus \widetilde{l}_{ji}^{L}, s_0\right)\right]. \tag{39}$$

The linguistic dominance relation is used to measure the "degree of preference" of x_i over x_j that exceeds the "degree of preference" of x_j over x_i.

Definition 20. Let $\widetilde{L} = (\widetilde{l}_{ij})_{n\times n}$ be an UALPR; then the uncertain linguistic non-dominance relations are defined as:

$$\begin{aligned}\widetilde{L}ND_{ij} &= \text{Neg}\left(\widetilde{L}D_{ji}\right)\\ &= \left[\left(\max\left(\widetilde{l}_{ji}^{U} \ominus \widetilde{l}_{ij}^{L}, s_0\right)\right),\right.\\ &\qquad \left.\text{Neg}\left(\max\left(\widetilde{l}_{ji}^{L} \ominus \widetilde{l}_{ij}^{U}, s_0\right)\right)\right].\end{aligned} \tag{40}$$

Since each uncertain linguistic preference value associates with its consistency level and consensus level, at the same time, from Theorem 6, $ct_{ij}^{L} = ct_{ji}^{U}$, $ct_{ji}^{L} = ct_{ij}^{U}$, and Theorem 12, $ca_{ij}^{L} = ca_{ji}^{U}$, $ca_{ji}^{L} = ca_{ij}^{U}$, it is obvious that the uncertain linguistic non-dominance relations have still preserved consistent consistency level and consensus level of its compositions $\widetilde{l}_{ij}$ and $\widetilde{l}_{ji}$. Hence we can utilize the C^2-IULOWA operator to obtain individual uncertain linguistic non-dominance degrees of alternatives.

For simplicity of calculation, the uncertain linguistic non-dominance relations are split into two crisp relations, $\widetilde{L}ND_{ij}^{L} = [\text{Neg}(\max(\widetilde{l}_{ji}^{U} \ominus \widetilde{l}_{ij}^{L}, s_0))]_{n\times n}$ and $\widetilde{L}ND_{ij}^{U} = [\text{Neg}(\max(\widetilde{l}_{ji}^{L} \ominus \widetilde{l}_{ij}^{U}, s_0))]_{n\times n}$; similar ideas can also be found in many existing references [26, 27, 34, 35]. Then motivated by [36, 37], we give alternative uncertain linguistic non-dominance degrees of alternatives by utilizing C^2-IUOWA operator:

$$\begin{aligned}\widetilde{L}ND_i^{(k)} = \text{IULOWA}_{C^2\mapsto w}&\left(\left\langle \widetilde{c}a_{ij}^{(k)}, \widetilde{c}t_{ij}^{(k)}, \text{Neg}\left(\widetilde{L}D_{ji}^{(k)}\right)\right\rangle,\right.\\ &\left. j = 1,\dots,n, j \neq i\right),\\ &k = 1,2,\dots,m,\ i = 1,2,\dots,n.\end{aligned} \tag{41}$$

Finally, we obtain collective uncertain linguistic non-dominance degrees of alternatives by utilizing the C^2-IULOWA operator again:

$$\begin{aligned}\widetilde{L}ND_i^{(C)} = \text{IULOWA}_{C^2\mapsto w}&\left(\left\langle \widetilde{C}a^{(k)}, \widetilde{C}t^{(k)}, \widetilde{L}ND_i^{(k)}\right\rangle,\right.\\ &\left. k = 1,\dots,m\right),\\ &i = 1,2,\dots,n.\end{aligned} \tag{42}$$

To rank the uncertain linguistic non-dominance degrees, we adopt to the degree of possibility of uncertain linguistic variables $\widetilde{a}_i \geq \widetilde{a}_j$ [18–20].

Definition 21. Let $\widetilde{L}ND_i^{(C)} = [\widetilde{L}ND_i^{L(C)}, \widetilde{L}ND_i^{U(C)}] \subseteq \overline{S}$, $i = 1,2,\dots,n$ be n, uncertain linguistic values; then the degree of possibility $Pos(\widetilde{L}ND_i^{(C)})$ of $\widetilde{L}ND_i^{(C)}$ is defined as

$$\begin{aligned}Pos_i &= Pos\left(\widetilde{L}ND_i^{(C)}\right)\\ &= \sum_{j=1}^{n} Pos\left(\widetilde{L}ND_i^{(C)} \geq \widetilde{L}ND_j^{(C)}\right)\\ &= \sum_{j=1}^{n}\left(\left(\max I\left(\widetilde{L}ND_i^{(C)U} \ominus \widetilde{L}ND_j^{(C)L}, s_0\right)\right.\right.\\ &\qquad \left. - \max I\left(\widetilde{L}ND_j^{(C)U} \ominus \widetilde{L}ND_i^{(C)L}, s_0\right)\right)\\ &\qquad \times\left(I\left(\widetilde{L}ND_i^{(C)U} \ominus \widetilde{L}ND_i^{(C)L}\right)\right.\\ &\qquad \left.\left. + I\left(\widetilde{L}ND_j^{(C)U} \ominus \widetilde{L}ND_j^{(C)L}\right)\right)^{-1}\right),\\ &\qquad i = 1,\dots,n.\end{aligned} \tag{43}$$

Now we can rank or select alternatives in descending order in accordance with Pos_i, $i \in \{1,\dots,n\}$.

The main steps of the proposed approach are summarized as follows.

Step 1. Obtain UALPRs over alternatives from DMs, $\widetilde{L}^{(k)} = (\widetilde{l}_{ij}^{(k)})_{n\times n}$, $k = 1,\dots,m$.

Step 2. Calculate the consistency levels of preference values and preference relations $c\widetilde{t}_{ij}^{(k)}, i, j = 1,2,\dots,n$, $\widetilde{C}t_k$, $k = 1,2,\dots,m$, by using (8) and (12).

Step 3. Calculate the consensus levels of preference values and preference relations $c\widetilde{a}_{ij}^{(k)}, i, j = 1,2,\dots,n$, $\widetilde{C}a_k$, $k = 1,2,\dots,m$, by using (16) and (17).

Step 4. Determine the ordered weights by using (34).

Step 5. Calculate the uncertain linguistic non-dominance relations according to the uncertain linguistic preference relations $(\widetilde{L}ND_{ij}^{(k)})_{n\times n}$, $k = 1,2,\dots,m$, by using (40).

Table 1: Consistency levels of preference values provided by e_1.

	x_1	x_2	x_3	x_4
x_1	(1, 1)	(0.8125, 1)	(1, 0.8125)	(0.9375, 0.9375)
x_2	(1, 0.8125)	(1, 1)	(0.8125, 0.9375)	(0.9375, 1)
x_3	(0.8125, 1)	(0.9375, 0.8125)	(1, 1)	(1, 0.9375)
x_4	(0.9375, 0.9375)	(1, 0.9375)	(0.9375, 1)	(1, 1)

Table 2: Consistency levels of preference values provided by e_2.

	x_1	x_2	x_3	x_4
x_1	(1, 1)	(0.5625, 0.625)	(0.375, 0.5)	(0.75, 0.5625)
x_2	(0.625, 0.5625)	(1, 1)	(0.625, 0.5625)	(0.75, 0.875)
x_3	(0.5, 0.375)	(0.5625, 0.625)	(1, 1)	(0.5625, 0.375)
x_4	(0.5625,0.75)	(0.875, 0.75)	(0.375, 0.5625)	(1, 1)

Step 6. Determine the individual uncertain linguistic non-dominance degrees of alternatives $\widetilde{LND}_i^{(k)}$, $k = 1, 2, \dots, m$, $i = 1, 2, \dots, n$, by using (41).

Step 7. Calculate the collective uncertain linguistic non-dominance degree of alternatives $\widetilde{LND}_i^{(C)}$, $i = 1, 2, \dots, n$, by using (42).

Step 8. Rank the alternatives with $\widetilde{LND}_i^{(C)}$, $i = 1, 2, \dots, n$, by using (43).

6. Numerical Examples

In this section, we consider a group decision making problem that concerns the evaluation and selection of chief quality officer to demonstrate the details of the proposed method.

Over the last decades, organizational competition is shifting from price to quality in many industries. The chief quality officer (CQO), similar to the CEO, undertakes overall responsibility of the quality work of enterprise. Hence, the selection of the most appropriate CQO is naturally one of the key factors for an enterprise's survival. A high-tech enterprise wants to introduce a CQO according to a main criterion of the leadership of quality improvements. After preliminary screening, four alternatives x_j $(j = 1, 2, 3, 4)$ have remained in the candidate list. Three experts e_k $(k = 1, 2, 3)$ from a committee act as decision makers; the decision makers compare these four alternatives with respect to the criterion of the leadership of quality improvements by using the linguistic term set

$$\begin{aligned} S = \{ & s_0 = \text{extremely low},\ s_1 = \text{very low}, \\ & s_2 = \text{low},\ s_3 = \text{slightly low},\ s_4 = \text{fair}, \\ & s_5 = \text{slightly high},\ s_6 = \text{high}, \\ & s_7 = \text{very high},\ s_8 = \text{extremely high} \}. \end{aligned} \tag{44}$$

To get the most desirable CQO, the following steps are involved.

Step 1. Three decision makers (e_1, e_2, and e_3) provide their preference on alternatives and form UALPRs as follows, respectively,

$$\begin{aligned} A^1 &= \begin{pmatrix} [s_4, s_4] & [s_0, s_2] & [s_2, s_4] & [s_1, s_3] \\ [s_6, s_8] & [s_4, s_4] & [s_4, s_6] & [s_4, s_5] \\ [s_4, s_6] & [s_2, s_4] & [s_4, s_4] & [s_3, s_4] \\ [s_5, s_7] & [s_3, s_4] & [s_4, s_5] & [s_4, s_4] \end{pmatrix}, \\ A^2 &= \begin{pmatrix} [s_4, s_4] & [s_4, s_6] & [s_0, s_2] & [s_5, s_7] \\ [s_2, s_4] & [s_4, s_4] & [s_4, s_5] & [s_2, s_4] \\ [s_6, s_8] & [s_3, s_4] & [s_4, s_4] & [s_4, s_5] \\ [s_1, s_3] & [s_4, s_6] & [s_3, s_4] & [s_4, s_4] \end{pmatrix}, \\ A^3 &= \begin{pmatrix} [s_4, s_4] & [s_0, s_1] & [s_4, s_5] & [s_5, s_6] \\ [s_7, s_8] & [s_4, s_4] & [s_6, s_7] & [s_2, s_3] \\ [s_3, s_4] & [s_1, s_2] & [s_4, s_4] & [s_7, s_8] \\ [s_2, s_3] & [s_5, s_6] & [s_0, s_1] & [s_4, s_4] \end{pmatrix}. \end{aligned} \tag{45}$$

Step 2. Calculate the consistency levels of preference values by using (8); the consistency levels of preference values provided by three decision makers are listed in Tables 1, 2, and 3, respectively.

According to the results from Tables 1, 2, and 3, calculate the consistency level of preference relations by utilizing (12); the results are as follows:

$$\widetilde{Ct}_1 = 0.927, \qquad \widetilde{Ct}_2 = 0.594, \qquad \widetilde{Ct}_3 = 0.438. \tag{46}$$

Step 3. Calculate the consensus measures of preference values by utilizing (16). The consensus levels of preference values provided by three experts are listed in Tables 4, 5, and 6, respectively.

Table 3: Consistency levels of preference values provided by e_3.

	x_1	x_2	x_3	x_4
x_1	(1, 1)	(0.4375, 0.5625)	(0.4375, 0.4375)	(0.4375, 0.4375)
x_2	(0.5625, 0.4375)	(1, 1)	(0.4375, 0.4375)	(0.3125, 0.5625)
x_3	(0.4375, 0.4375)	(0.4375, 0.4375)	(1, 1)	(0.4375, 0.3125)
x_4	(0.4375, 0.4375)	(0.5625, 0.3125)	(0.3125, 0.4375)	(1, 1)

Table 4: Consensus levels of preference values provided by e_1.

	x_1	x_2	x_3	x_4
x_1	(1, 1)	(0.75, 0.6875)	(0.75, 0.8125)	(0.5, 0.5625)
x_2	(0.6875, 0.75)	(1, 1)	(0.875, 0.875)	(0.75, 0.8125)
x_3	(0.8125, 0.75)	(0.875, 0.875)	(1, 1)	(0.6875, 0.6875)
x_4	(0.5625, 0.5)	(0.8125, 0.75)	(0.6875, 0.6875)	(1, 1)

Then calculate the consistency levels of preference relations by (17); the results are as follows:

$$\widetilde{C}a_1 = 0.729, \qquad \widetilde{C}a_2 = 0.719, \qquad \widetilde{C}a_3 = 0.719. \tag{47}$$

Step 4. Determine associated weights of decision makers and preference values. By using (34) determine the associated weights of three variables:

$$\begin{aligned} w_1 &= Q\left(\frac{1}{3}\right) - Q\left(\frac{0}{3}\right) = 0.577, \\ w_2 &= Q\left(\frac{2}{3}\right) - Q\left(\frac{1}{3}\right) = 0.239, \\ w_3 &= Q\left(\frac{3}{3}\right) - Q\left(\frac{2}{3}\right) = 0.184. \end{aligned} \tag{48}$$

Step 5. Calculate the individual uncertain linguistic non-dominance relations by utilizing (40):

$$\begin{aligned} \widetilde{L}ND_{ij}^{(1)} &= \begin{pmatrix} [-,-] & [s_0,s_4] & [s_4,s_8] & [s_2,s_6] \\ [s_8,s_8] & [-,-] & [s_8,s_8] & [s_8,s_8] \\ [s_8,s_8] & [s_4,s_8] & [-,-] & [s_6,s_8] \\ [s_8,s_8] & [s_6,s_8] & [s_8,s_8] & [-,-] \end{pmatrix}, \\ \widetilde{L}ND_{ij}^{(2)} &= \begin{pmatrix} [-,-] & [s_8,s_8] & [s_0,s_4] & [s_8,s_8] \\ [s_4,s_8] & [-,-] & [s_8,s_8] & [s_4,s_8] \\ [s_8,s_8] & [s_6,s_8] & [-,-] & [s_8,s_8] \\ [s_2,s_6] & [s_8,s_8] & [s_6,s_8] & [-,-] \end{pmatrix}, \\ \widetilde{L}ND_{ij}^{(3)} &= \begin{pmatrix} [-,-] & [s_0,s_2] & [s_8,s_8] & [s_8,s_8] \\ [s_8,s_8] & [-,-] & [s_8,s_8] & [s_4,s_6] \\ [s_6,s_8] & [s_2,s_4] & [-,-] & [s_8,s_8] \\ [s_4,s_6] & [s_8,s_8] & [s_0,s_2] & [-,-] \end{pmatrix}. \end{aligned} \tag{49}$$

Step 6. Determine the individual uncertain linguistic non-dominance degrees of alternatives by (41). For instance, the individual non-dominance degree of x_1 provided by e_1 is calculated as follows:

$$\begin{aligned} \widetilde{L}ND_1^{(1)} = \big[&(0.184 \times 0.408 s_0 \oplus 0.577 \\ &\times 0.408 s_4 \oplus 0.239 \times 0.184 s_2) \\ &\times (0.184 \times 0.408 + 0.577 \\ &\times 0.408 + 0.239 \times 0.184)^{-1}, \\ &(0.577 \times 0.239 s_4 \oplus 0.184 \\ &\times 0.577 s_8 \oplus 0.239 \times 0.184 s_6) \\ &\times (0.577 \times 0.239 + 0.184 \\ &\times 0.577 + 0.239 \times 0.184)^{-1}\big] \\ = \big[&s_{2.90}, s_{5.78}\big]. \end{aligned} \tag{50}$$

Similarly, we can obtain the other individual non-dominance degrees:

$$\begin{aligned} \widetilde{L}ND_2^{(1)} &= [s_8, s_8], \\ \widetilde{L}ND_3^{(1)} &= [s_{5.35}, s_8], \\ \widetilde{L}ND_4^{(1)} &= [s_{6.42}, s_8], \\ \widetilde{L}ND_1^{(2)} &= [s_{6.92}, s_{7.16}], \\ \widetilde{L}ND_2^{(2)} &= [s_{4.96}, s_8], \\ \widetilde{L}ND_3^{(2)} &= [s_{6.72}, s_8], \\ \widetilde{L}ND_4^{(2)} &= [s_{7.16}, s_{7.52}], \\ \widetilde{L}ND_1^{(3)} &= [s_{4.74}, s_{5.71}], \\ \widetilde{L}ND_2^{(3)} &= [s_{6.42}, s_{6.81}], \end{aligned}$$

TABLE 5: Consensus levels of preference values provided by e_2.

	x_1	x_2	x_3	x_4
x_1	(1, 1)	(0.5, 0.4375)	(0.625, 0.6875)	(0.75, 0.6875)
x_2	(0.4375, 0.5)	(1, 1)	(0.875, 0.8125)	(0.875, 0.875)
x_3	(0.6875, 0.625)	(0.8125, 0.875)	(1, 1)	(0.75, 0.75)
x_4	(0.6875, 0.75)	(0.875, 0.875)	(0.75, 0.75)	(1, 1)

TABLE 6: Consensus levels of preference values provided by e_3.

	x_1	x_2	x_3	x_4
x_1	(1, 1)	(0.75, 0.625)	(0.625, 0.75)	(0.75,0.75)
x_2	(0.625, 0.75)	(1, 1)	(0.75, 0.8125)	(0.875, 0.8125)
x_3	(0.75, 0.625)	(0.8125, 0.75)	(1, 1)	(0.5625, 0.5625)
x_4	(0.75, 0.75)	(0.8125, 0.875)	(0.5625, 0.5625)	(1, 1)

TABLE 7: Collective non-dominance degrees of alternatives by utilizing C^2-IULOWA, Ct-IULOWA, and Ca-IULOWA operators.

	C^2-IULOWA	Ct-IULOWA	Ca-IULOWA
x_1	$[s_{3.52}, s_{5.94}]$	$[s_4, s_{5.73}]$	$[s_{3.44}, s_{6.66}]$
x_2	$[s_{7.49}, s_{7.89}]$	$[s_{7.03}, s_{7.86}]$	$[s_{7.01}, s_{7.83}]$
x_3	$[s_{5.39}, s_{7.76}]$	$[s_{6.1}, s_{7.7}]$	$[s_{5.38}, s_{7.51}]$
x_4	$[s_{6.44}, s_{7.83}]$	$[s_{6.3}, s_{7.36}]$	$[s_{6.49}, s_{7.58}]$

$$\widetilde{LND}_3^{(3)} = [s_{4.06}, s_{5.43}],$$
$$\widetilde{LND}_4^{(3)} = [s_{5.68}, s_{6.82}]. \quad (51)$$

Step 7. Aggregate the individual non-dominance degrees of alternatives into collective non-dominance degrees of alternatives by utilizing (42):

$$\widetilde{LND}_1 = [s_{3.52}, s_{5.94}],$$
$$\widetilde{LND}_2 = [s_{7.49}, s_{7.89}],$$
$$\widetilde{LND}_3 = [s_{5.39}, s_{7.76}],$$
$$\widetilde{LND}_4 = [s_{6.44}, s_{7.83}]. \quad (52)$$

Step 8. Obtain the ranking result of the alternatives according to the magnitude of collective non-dominance degrees by utilizing (43). The ranking result of the four alternatives is $x_2 \succ x_4 \succ x_3 \succ x_1$, and then the most appropriate CQO is x_2.

In order to validate the proposed method, in what follows, we use the Ct-IULOWA and the Ca-IULOWA operators for the same decision structure. The final results derived by C^2-IULOWA, Ct-IULOWA, and Ca-IULOWA operators are shown in Table 7, and they are represented graphically in Figure 1.

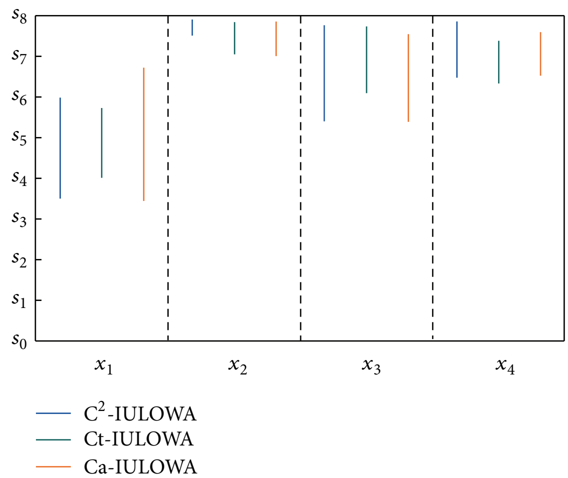

FIGURE 1: Collective non-dominance degrees of alternatives by utilizing C^2-IULOWA, Ct-IULOWA, and Ca- IULOWA operators.

Obviously, the ranking results derived by C^2-IULOWA, Ct-IULOWA, and Ca-IULOWA operators all are $x_2 \succ x_4 \succ x_3 \succ x_1$; the situations verify each other; the results derived from the C^2-IULOWA, Ct-IULOWA, and Ca-IULOWA operators are feasible and effective. The reasons of the differences of the final results are intuitive; that is, as discussed above, the Ct-IULOWA operator focuses solely on the consistency and ignores the consensus of the uncertain linguistic preference information, while the Ca-IULOWA operator focuses only on the consensus and ignores the consistency of the uncertain linguistic preference information. The C^2-IULOWA operator comprehensively considers both the consistency and consensus of the uncertain linguistic preference information. Hence, the results derived by C^2-IULOWA operator are more feasible and effective.

7. Conclusion

In group decision making with uncertain linguistic preference relations, the aggregation of preference information plays important roles in reaching a reasonable decision result. Considering the fact that when carrying out rational decision making, the consensual or consistent information is more appropriate than the inconsistent or decentralized ones, in this paper, we investigated the aggregation of uncertain linguistic preference information by fusing its consensus and consistent information. To do so, we first proposed the consistency and consensus measures, to assess the consistency level and consensus level of uncertain linguistic preference relations and preference values of them, and studied some of their desirable properties. Based on both consistency level and consensus level, we proposed a co-induced uncertain linguistic OWA operator, namely, C^2-IULOWA operator, to aggregate uncertain linguistic preference information, in which consistency level and consensus level synergistically serve as inducing variables and participate in guiding the determination of associated weights in the preference aggregation process. We have verified that the C^2-IULOWA operator is able to maintain the indifference, reciprocity, and consistency properties of uncertain linguistic preference relations after aggregation is carried out. For applying the C^2-IULOWA operator to group decision making with uncertain linguistic preference relations, we developed a direct group decision making approach. The approach is very suitable for the situations with time pressure and decision makers unwilling to revise their initial judgments, in which the inconsistent or decentralized information would be still retained and yet it is considered to be less important for ranking the alternatives. The proposed approach is effective and feasible just as shown in the illustrative example. It is expected that the proposed approach can be applied to the fields of quality management and mobile business.

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