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Research Article

An Analytical Solution for Effect of Magnetic Field and Initial Stress on an Infinite Generalized Thermoelastic Rotating Nonhomogeneous Diffusion Medium

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The problem of generalized magneto-thermoelastic diffusion in an infinite rotating nonhomogeneity medium subjected to certain boundary conditions is studied. The chemical potential is also assumed to be a known function of time at the boundary of the cavity. The analytical expressions for the displacements, stresses, temperature, concentration, and chemical potential are obtained. Comparison was made between the results obtained in the presence and absence of diffusion. The results indicate that the effect of nonhomogeneity, rotation, magnetic field, relaxation time, and diffusion is very pronounced.

1. Introduction

Diffusion can be defined as the spontaneous migration of substances from regions of high concentration to regions of low concentration. There is now a great deal of interest in the study of this phenomenon due to its many applications in geophysics and industrial applications. Thermodiffusion in the solids is one of the transport processes which has great practical importance. Thermodiffusion in an elastic solid is due to the coupling of the fields of temperature, mass diffusion, and that of strain. This mater has attracted the attention of many researchers such as [1–5]. Wave propagation in rotating and nonhomogeneous media was studied by Abd-Alla et al. [6–8]. The extended thermoelasticity theory, introducing one relaxation time in the thermoelastic process, was proposed by Lord and Shulman [9]. In this theory, a modified law of heat conduction including both the heat flux and its time derivative replaces conventional Fourier's law. The heat equation associated with this is a hyperbolic one and hence automatically eliminates the paradox of infinite speeds of propagation inherent in the coupled theory of thermoelasticity. This theory was extended by Dhaliwal and Sherief [10] to include the anisotropic case. Abd-Alla and Mahmoud [11] investigated the magneto-thermoelastic problem in rotating nonhomogeneous orthotropic hollow cylinder under the hyperbolic heat conduction model. Mahmoud [12] investigated wave propagation in cylindrical poroelastic dry bones.

Kumar and Devi [13] studied deformation in porous thermoelastic material with temperature dependent properties. Othman et al. [14] presented the study of the two-dimensional problems of generalized thermoelasticity with one relaxation time with the modulus of elasticity being dependent on the reference temperature for nonrotating and rotating medium, respectively. Kumar and Gupta [15] investigated deformation due to inclined load in an orthotropic micropolar thermoelastic medium with two relaxation times. The temperature-rate dependent theory of thermoelasticity, which takes into account two relaxation times, was developed by Green and Lindsay [16]. Abd-Alla et al. [17, 18] investigated radial vibrations in a nonhomogeneous orthotropic elastic medium subjected to rotation and gravity field. Sherief et al. [19] developed the generalized theory of thermoelastic diffusion with one relaxation time, which allows the finite speed of propagation waves. Sherief and Saleh [20] investigated the problem of a thermoelastic half-space in the context of the theory of generalized thermoelastic diffusion with one relaxation time. The reflection of SV waves from the free surface of an elastic solid in generalized thermoelastic diffusion was discussed by Singh [21]. Kumar and Kansal [22] discussed the propagation of Lamb waves in transversely isotropic thermoelastic diffusive plates. Thermomechanical response of generalized thermoelastic diffusion with one relaxation time due to time harmonic sources was discussed by Ram et al. [23]. Aouadi [24] examined the thermoelastic diffusion problem for an infinite elastic body with spherical cavity. Abd-Alla and Mahmoud [25] investigated analytical solution of wave propagation in nonhomogeneous orthotropic rotating elastic media. Othman et al. [26] discussed the effect of diffusion in a two-dimensional problem of generalized thermoelasticity with Green-Naghdi theory. Xia et al. [27] studied the influence of diffusion on generalized thermoelastic problems of infinite body with a cylindrical cavity. Deswal and Kalkal [28] studied the two-dimensional generalized electromagneto-thermoviscoelastic problem for a half-space with diffusion. Abd-Alla and Abo-Dahab [29] found the time-harmonic sources in a generalized magneto-thermo-viscoelastic continuum with and without energy dissipation. Mahmoud [30] discussed influence of rotation and generalized magnetothermoelastic on Rayleigh waves in a granular medium under effect of initial stress and gravity field. Abd-Alla et al. [31, 32] studied the generalized magneto-thermoelastic Rayleigh waves in a granular medium under the influence of a gravity field and initial stress.

In the present investigation, the temperature, displacements, stresses, diffusion, and concentration as well as chemical potential are obtained in the physical domain using the harmonic vibrations. Also, study of the interaction between the processes of elasticity, nonhomogeneity, rotation, magnetic field, initial stress, heat, and diffusion in an infinite elastic solid with a spherical cavity in the context of the theory of generalized thermoelastic diffusion is presented.

2. Formulation of the Problem

Consider a perfect electric conductor and linearized Maxwell equations governing the electromagnetic field in the absence of the displacement current (SI) in the form as in Kraus [33]. Applying an initial magnetic field vector $\vec{H}(0,0,H_0)$ in spherical coordinates (r,θ,ϕ), $\vec{u} = (u(r,t),0,0)$. One will consider a nonhomogeneous, isotropic medium, occupying the region $a \le r \le b$, where a is the radius of the spherical cavity. The strain tensor has the following components:

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{1a}$$

$$\omega_{ij} = \frac{1}{2}\left(u_{j,i} - u_{i,j}\right), \tag{1b}$$

$$e_{kk} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u\right), \quad i,j,k = 1,2,3, \tag{1c}$$

$$e_{rr} = \frac{\partial u}{\partial r}, \qquad e_{\theta\theta} = e_{\phi\phi} = \frac{u}{r}. \tag{1d}$$

The cubical dilatation is given by $(1/r^2)(\partial/\partial r)(r^2 u)$, where the nonvanishing displacement component is the radial one $u_r = u(r,t)$. The elastic medium is rotating uniformly with an angular velocity $\vec{\Omega} = \Omega\vec{n}$, where $\vec{n}$ is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in the rotating frame has two additional terms: $\vec{\Omega} \times (\vec{\Omega} \times \vec{u})$ which is the centripetal acceleration due to time varying motion only, and $2\vec{\Omega} \times \vec{u}$ is the Coriolis acceleration, where $\vec{\Omega} = (0,\Omega,0)$. Following Sherief's theory of generalized thermoelastic diffusion [19] and Sherief and Saleh [20], one is going to study an isotropic nonhomogeneous elastic medium which suffers thermal shock. Due to spherical symmetry, the stress-displacement-temperature-diffusion relation or constitutive equations are given by

$$\begin{aligned}\sigma_{rr} = {} & \left(2\mu + P_1\right)\frac{\partial u}{\partial r} + \left(\lambda + P_1\right)\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u\right) \\ & - \beta_1\left(\theta - \theta_0\right) - \beta_2 C,\end{aligned} \tag{2a}$$

$$\begin{aligned}\sigma_{\phi\phi} = \sigma_{\theta\theta} = {} & \left(2\mu + P_1\right)\frac{u}{r} + \left(\lambda + P_1\right)\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u\right) \\ & - \beta_1\left(\theta - \theta_0\right) - \beta_2 C.\end{aligned} \tag{2b}$$

The chemical-displacement-temperature-diffusion relation is given by

$$P = -\beta_2\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u\right) + bC - c\left(\theta - \theta_0\right). \tag{3}$$

The governing equation for an isotropic nonhomogeneous elastic solid with generalized magneto-thermoelastic diffusion under effect of rotation is given by

$$\begin{aligned}&\frac{\partial\left(1/r^2\right)\left(\partial/\partial r\right)\left(r^2 u\right)}{\partial r} - \frac{\beta_2}{h_0}\frac{\partial C}{\partial r} - \frac{\beta_1}{h_0}\frac{\partial\left(\theta - \theta_0\right)}{\partial r} + F_r \\ &\quad = \frac{\rho}{h_0}\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega\frac{\partial u}{\partial t}\right],\end{aligned} \tag{4}$$

where λ and μ are Lame's elastic constants, δ_{ij} is Kronecker's delta, P_1 is the initial stress, ρ is the density of the medium, and $\vec{F}$ is defined as Lorentz's force which may be written as

$$\vec{F} = \mu_e\left(\vec{J} \times \vec{H}\right) = \left(\mu_e H_0^2\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right), 0, 0\right), \tag{5}$$

where μ_e is the magnetic permeability, $\vec{H}$ is the magnetic field vector, $\vec{J}$ is the electric current density, $\vec{u}$ is the displacement vector, and t is the time.

Equation of heat conduction is given by

$$K\nabla^2\theta = \left(\frac{\partial}{\partial t} + \tau\frac{\partial^2}{\partial t^2}\right)\left(\rho c_v\theta + \theta_0\beta_1\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u\right) + c\theta_0 C\right), \tag{6}$$

where the Laplacian operator ∇^2 is given by $\nabla^2 = (\partial^2/\partial r^2) + (2/r)(\partial/\partial r)$ and $h_0 = 2\mu + \lambda + (P_1/2) + \mu_e H_0^2$, h_0 is the coefficient of linear diffusion expansion, K is the thermal conductivity, θ is the absolute temperature, θ_0 is the initial uniform temperature, and $|(\theta-\theta_0)/\theta_0| \ll 1$.

Equation of conservation of mass diffusion may be written as

$$D\beta_2\nabla^2\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2u\right)+Dc\nabla^2\theta+\left(\frac{\partial}{\partial t}+\tau\frac{\partial^2}{\partial t^2}\right)C=Db\nabla^2C, \tag{7}$$

where τ is the diffusion relaxation time, τ_0 is the thermal relaxation time, α_t is the coefficient of linear thermal expansion, and θ_0 is constant, where $\beta_1 = (3\lambda+2\mu)\alpha_t$, $\beta_2 = (3\lambda+2\mu)\alpha_c$, σ_{ij} are the components of the stress tensor, τ_{ij} are the components of stress tensor, b and c are the measures of thermodiffusion and diffusive effects, C is the concentration, C_v is the specific heat at constant strain, D is the diffusive coefficient, and e_{ij} are the components of the strain tensor. The thermal relaxation time τ_0 will ensure that the heat conduction equation will predict finite speed of heat propagation. The diffusion relaxation time τ, which will ensure the equation satisfied by the concentration C, will also predict finite speed of propagation of matter from one medium to the other.

3. Dimensionless Quantities

Introduce the following nondimensional parameters:

$$\begin{gathered}
r^*=c_1\eta_0 r,\qquad u^*=c_1\eta_0 u,\qquad T=\frac{\beta_1(\theta-\theta_0)}{h_0},\\
C^*=\frac{\beta_2 C}{h_0},\qquad \Omega^*=\frac{\Omega}{c_1^2\eta_0},\qquad \sigma_{ij}^*=\frac{\sigma_{ij}}{h_0},\\
P^*=\frac{P}{\beta_2},\qquad t^*=c_1^2\eta_0 t,\qquad \tau_0^*=c_1^2\eta_0\tau_0,\\
\tau^*=c_1^2\eta_0\tau,\qquad \eta_0=\frac{\rho c_v}{K},\qquad c_1^2=\frac{h_0}{\rho}.
\end{gathered} \tag{8}$$

The elastic constants λ, μ and the density ρ of nonhomogeneous material in form [32] are as follows:

$$\begin{gathered}
\lambda=r^{2m}\lambda_0,\qquad \mu=r^{2m}\mu_0,\qquad \mu_h=r^{2m}\mu_0,\\
\rho=r^{2m}\rho_0,\qquad p^*=p_0^*r^{2m}.
\end{gathered} \tag{9}$$

Using the above non-dimensional parameters and (9) in (10)–(14), the non-dimensional system becomes

$$\frac{\partial\left(1/r^2\right)(\partial/\partial r)\left(r^2u\right)}{\partial r}-\frac{\partial C}{\partial r}-\frac{\partial T}{\partial r}=\frac{\partial^2u}{\partial t^2}-\Omega^2u-2\Omega\frac{\partial u}{\partial t}, \tag{10}$$

$$\nabla^2T=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(T+\varepsilon_1\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2u\right)+\varepsilon_2C\right), \tag{11}$$

$$\nabla^2\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2u\right)+h_4\nabla^2T+h_6\left(\frac{\partial}{\partial t}+\tau\frac{\partial^2}{\partial t^2}\right)C=h_5\nabla^2C, \tag{12}$$

$$\sigma_{rr}=h_1\frac{\partial u}{\partial r}+h_2\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2u\right)-T-C, \tag{13a}$$

$$\sigma_{\theta\theta}=h_1\frac{u}{r}+h_2\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2u\right)-T-C, \tag{13b}$$

$$\sigma_{\phi\phi}=h_1\frac{u}{r}+h_2\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2u\right)-T-C, \tag{13c}$$

$$P=-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2u\right)+h_3C-h_4T, \tag{14}$$

where

$$\begin{gathered}
\varepsilon_1=\frac{\beta_1^2T_0m}{h_0\rho c_v},\qquad \varepsilon_2=\frac{\beta_1cT_0h_0(m+2)}{\beta_2},\qquad h_1=\frac{2\mu}{h_0},\\
h_2=\frac{\lambda}{h_0},\qquad h_3=\frac{mbh_0}{\beta_2^2},\qquad h_4=\frac{ch_0}{\beta_1\beta_2},\\
h_5=\frac{Dbh_0}{\beta_2},\qquad h_6=\frac{2m+h_0}{\beta_2^2D\eta_0}.
\end{gathered} \tag{15}$$

4. Boundary Conditions

The nonhomogeneous initial conditions are supplemented by the following boundary conditions. The cavity surface is traction free:

$$\sigma_{rr}(r,t)+\overline{\tau}_{rr}(r,t)=0,\qquad r=a. \tag{16a}$$

The cavity surface is subjected to a thermal shock

$$T(a,t)=T_0H(t), \tag{16b}$$

where $H(t)$ is the Heaviside unit step function. The chemical potential is also assumed to be a known function of time at the cavity surface:

$$P(a,t)=P_0H(t),\qquad P_0 \text{ is real constant.} \tag{16c}$$

The displacement function is as follows:

$$u(r,t)=0,\qquad r=a. \tag{16d}$$

5. Solution of the Problem

In this section, one obtains the analytical solution of the problem for a spherical region with boundary conditions by taking the harmonic vibrations. One assumes that the solution of (10)–(12) as follows:

$$C(r,t)=C'(r)e^{i\omega t},\qquad T(r,t)=T'(r)e^{i\omega t}, \tag{17a}$$

$$u(r,t)=u'(r)e^{i\omega t}\cdot\qquad e(r,t)=E'(r)e^{i\omega t}, \tag{17b}$$

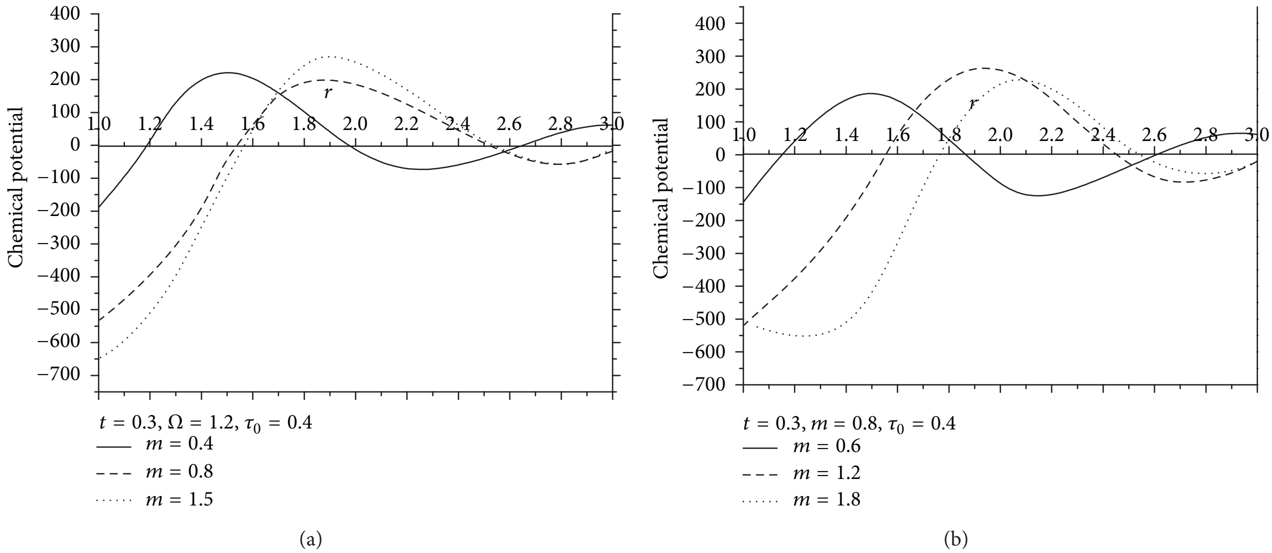


FIGURE 1: Variation of chemical potential P with radius r (thermoelastic diffusion nonhomogeneity medium).

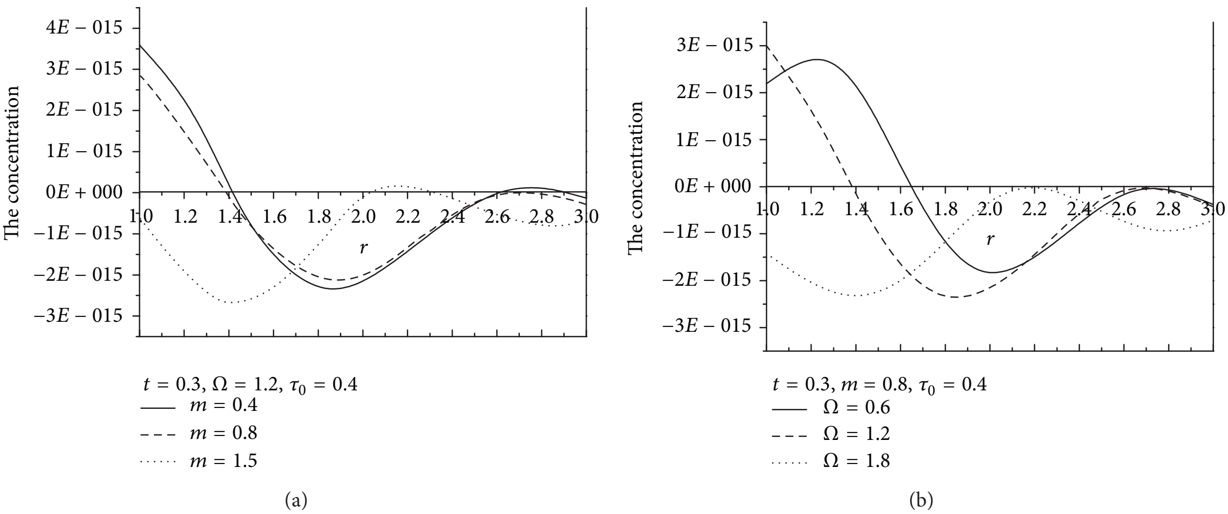


FIGURE 2: Variation of concentration C with radius r (thermoelastic diffusion nonhomogeneity medium).

where $e = (\partial u/\partial r) + (2u/r) = (1/r^2)(\partial/\partial r)(r^2 u)$.

Substituting (17a) and (17b) into (10)–(12) yields

$$\frac{\partial E'}{\partial r} - \frac{\partial C'}{\partial r} - \frac{\partial T'}{\partial r} = \beta u', \tag{18}$$

$$\nabla^2 T' = k_1 \left(T' + \varepsilon_1 E' + \varepsilon_2 C' \right), \tag{19}$$

$$\nabla^2 E' + h_4 \nabla^2 T' + h_6 k_2 G = h_5 \nabla^2 C'. \tag{20}$$

Applying the operator Laplacian operator ∇^2 to (18), we obtain

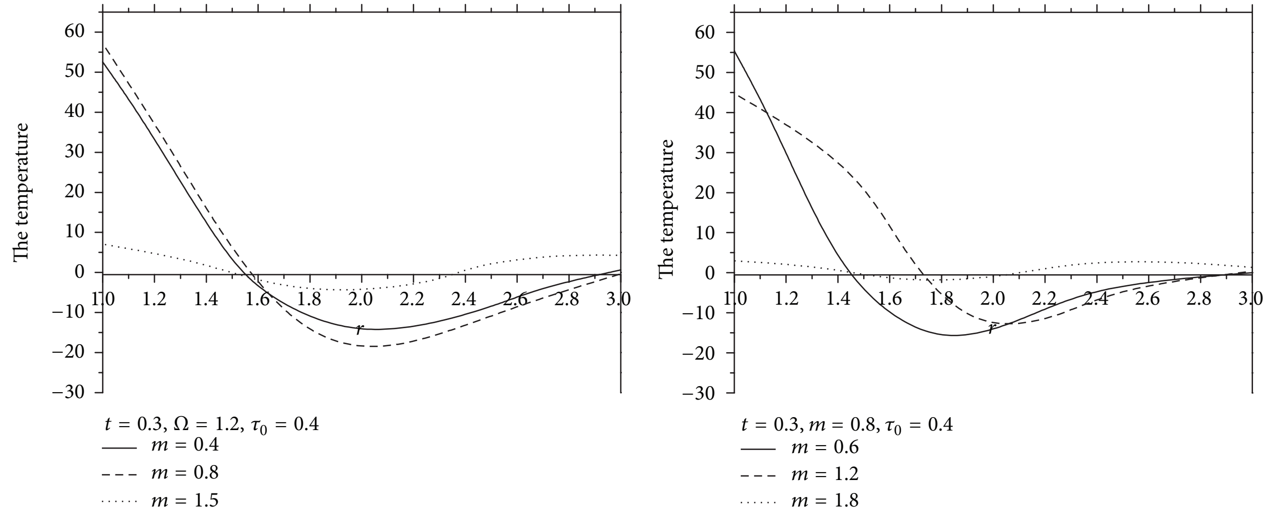
$$\left(\nabla^2 - \beta \right) E' = \nabla^2 C' + \nabla^2 T'. \tag{21}$$

From (19)–(21), we obtain

$$\left(\nabla^6 + b_1 \nabla^4 + b_2 \nabla^2 + b_3 \right) \left(E', T', C' \right) = 0, \tag{22}$$

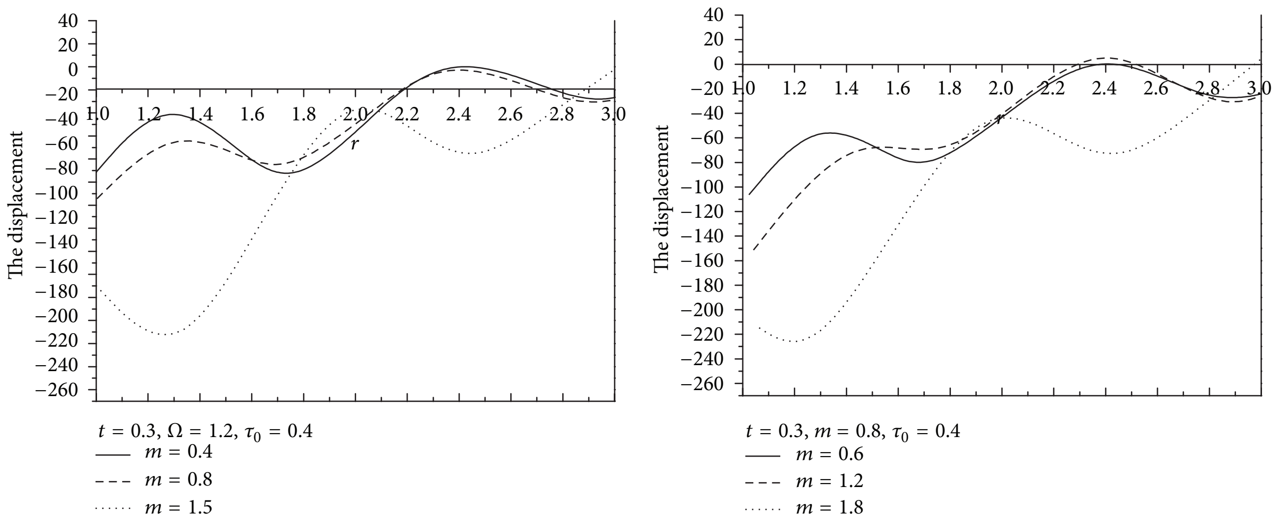
where

$$\begin{gathered} b_1 = \frac{-1}{(h_5 - 1)} \times \left[k_2 h_6 + k_1 \left(h_5 + \varepsilon_2 h_4 \right) + \beta h_5 + k_1 \left(\varepsilon_1 h_5 + \varepsilon_2 \right) \right], \\ b_2 = \frac{1}{(h_5 - 1)} \times \left[k_1 k_2 h_6 + \beta \left(k_2 h_6 + k_1 \left(h_5 + \varepsilon_2 h_4 \right) \right) + k_1 k_2 \varepsilon_1 h_6 \right], \\ b_3 = \frac{-\beta k_1 k_2 h_6}{(h_5 - 1)}, \qquad k_1 = i\omega \left(1 + i\omega\tau_0 \right), \\ k_2 = i\omega \left(1 + i\omega\tau \right), \qquad \beta = -\left(\omega^2 + \Omega^2 + 2i\omega\Omega \right). \end{gathered} \tag{23}$$



(a) (b)

FIGURE 3: Variation of temperature θ with radius r (thermoelastic diffusion nonhomogeneity medium).



(a) (b)

FIGURE 4: Variation of displacement u with radius r (thermoelastic diffusion nonhomogeneity medium).

Equation (22) can be factorized as

$$\left(\nabla^2 + Q_1^2\right)\left(\nabla^2 + Q_2^2\right)\left(\nabla^2 + Q_3^2\right)\left(E', T', C'\right) = 0, \quad (24)$$

where Q_1^2, Q_2^2, and Q_3^2 are the roots of the characteristic equation

$$Q^6 + b_1 Q^4 + b_2 Q^2 + b_3 = 0. \quad (25)$$

The solution of (24) which is bounded at infinity is given by

$$\begin{aligned} T'(r,\omega) &= \frac{1}{\sqrt{r}} \sum_{j=1}^{3} B_j(\omega) K_{1/2}\left(Q_j r\right), \\ E'(r,\omega) &= \frac{1}{\sqrt{r}} \sum_{j=1}^{3} B_j'(\omega) K_{1/2}\left(Q_j r\right), \\ C'(r,\omega) &= \frac{1}{\sqrt{r}} \sum_{j=1}^{3} B_j''(\omega) K_{1/2}\left(Q_j r\right), \end{aligned} \quad (26)$$

where B_j, B_j', and B_j'' are parameters depending only on ω and $K_{1/2}$ is the modified spherical Bessel function of the second

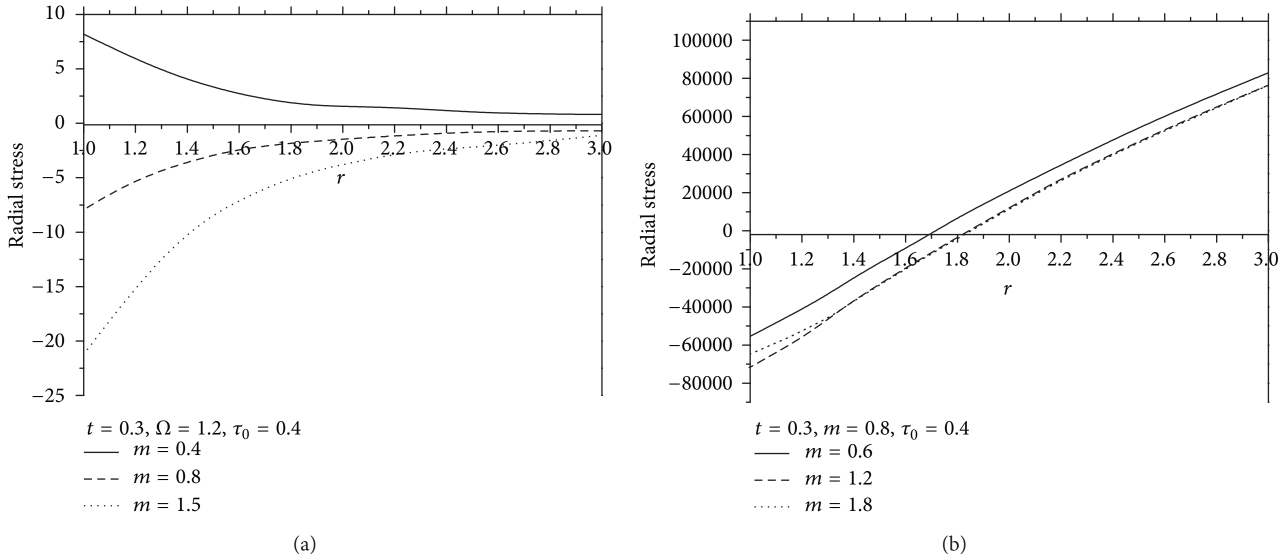


FIGURE 5: Variation of radial stress σ_{rr} with radius r (thermoelastic diffusion nonhomogeneity medium).

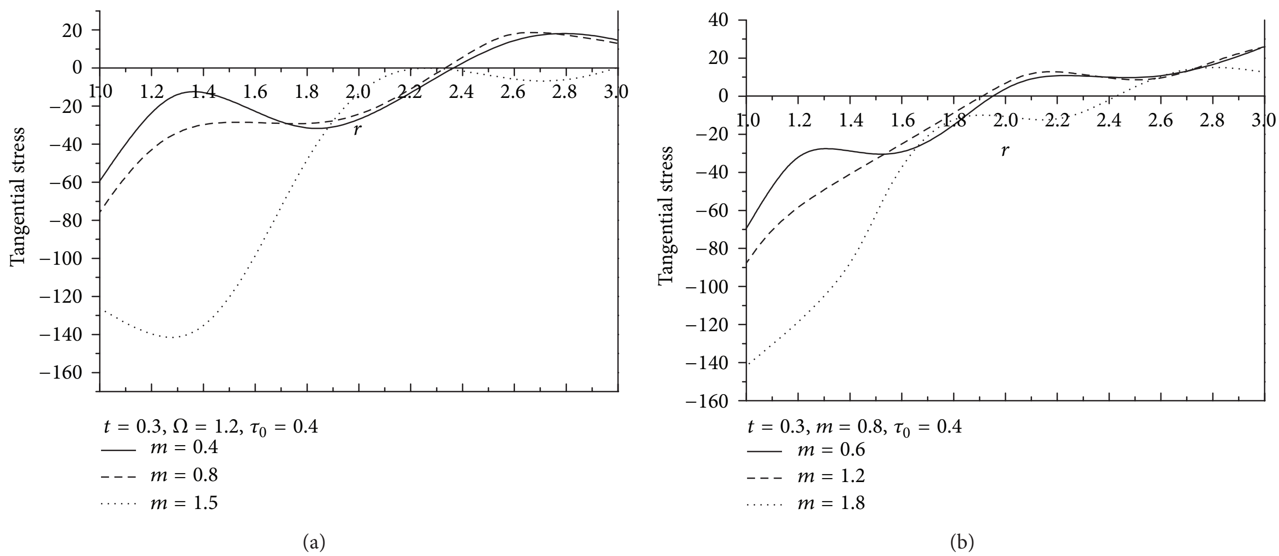


FIGURE 6: Variation of tangential stress $\sigma_{\phi\phi}$ with radius r (thermoelastic diffusion nonhomogeneity medium).

kind of order 1/2. Compatibility between (26) along with (19) and (20) will give rise to

$$B'_j(\omega) = \frac{\left(Q_j^2 - k_1\right)\left(h_5 Q_j^2 - h_6 k_2\right) - \varepsilon_2 k_1 h_4 Q_j^2}{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)} B_j(\omega),$$
$$B''_j(\omega) = \frac{\varepsilon_1 k_1 h_4 Q_j^2 + Q_j^2 \left(Q_j^2 - k_1\right)}{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)} B_j(\omega). \tag{27}$$

Substituting (26) into (17a) and (17b), we obtain

$$T(r,t) = \frac{1}{\sqrt{r}} \sum_{j=1}^{3} B_j(\omega) K_{1/2}\left(Q_j r\right) e^{i\omega t}, \tag{28}$$

$$e(r,t) = \frac{1}{\sqrt{r}} \sum_{j=1}^{3} B'_j(\omega) K_{1/2}\left(Q_j r\right) e^{i\omega t}, \tag{29}$$

$$C(r,t) = \frac{1}{\sqrt{r}} \sum_{j=1}^{3} B''_j(\omega) K_{1/2}\left(Q_j r\right) e^{i\omega t}. \tag{30}$$

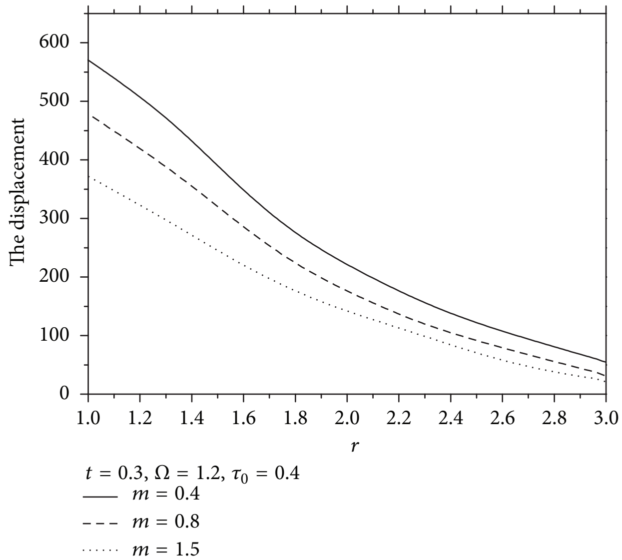


FIGURE 7: Variation of displacement u with radius r (thermoelastic nonhomogeneity medium).

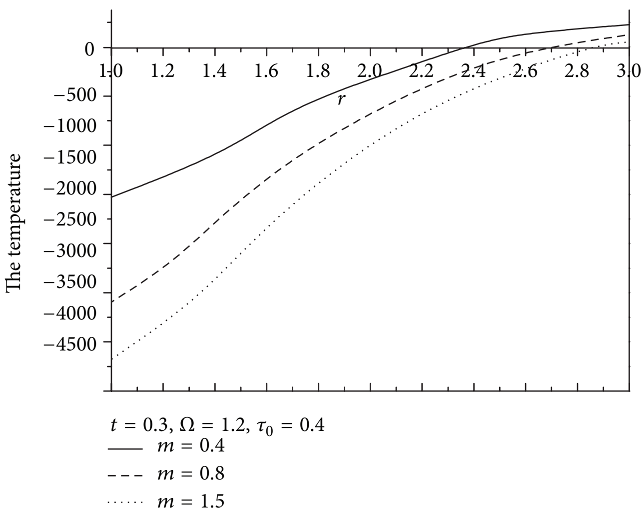


FIGURE 8: Variation of temperature θ with radius r (thermoelastic nonhomogeneity medium).

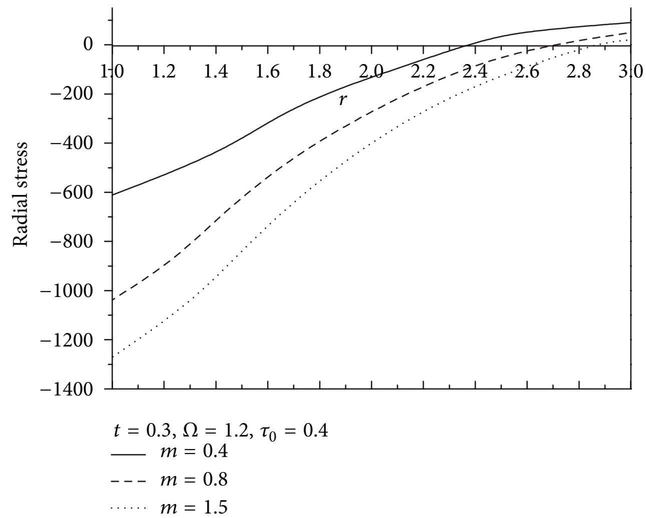


FIGURE 9: Variation of radial stress σ_{rr} with radius r (thermoelastic nonhomogeneity medium).

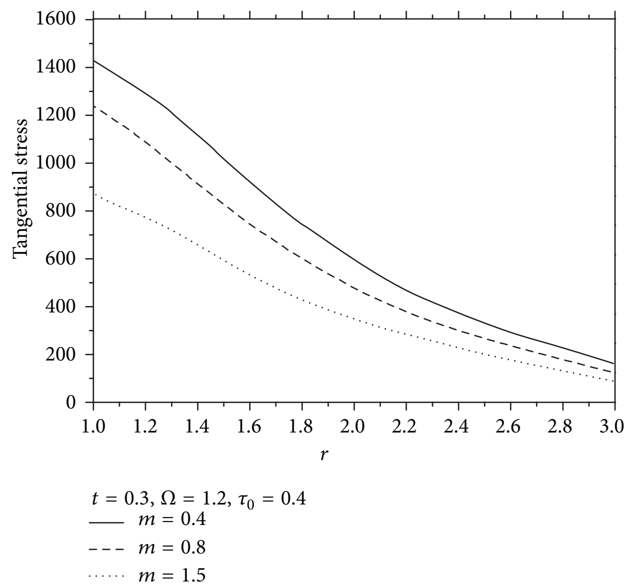


FIGURE 10: Variation of tangential stress $\sigma_{\phi\phi}$ with radius r (thermoelastic nonhomogeneity medium).

Integrating both sides of (29) from r to infinity and assuming that $u(r,t)$ vanishes at infinity, we obtain

$$u(r,t) = \frac{1}{\sqrt{r}} \sum_{j=1}^{3} \frac{\left(Q_j^2 - k_1\right)\left(h_5 Q_j^2 - h_6 k_2\right) - \varepsilon_2 k_1 h_4 Q_j^2}{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)} \times B_j(\omega) K_{3/2}\left(Q_j r\right) e^{i\omega t}. \tag{31}$$

From (13a), (13b), and (13c)-(14), we get

$$\begin{aligned}\sigma_{rr} = {}& \frac{1}{\sqrt{r}} \sum_{j=1}^{3} B_j(\omega) \frac{\left(Q_j^2 - k_1\right)\left(h_5 Q_j^2 - h_6 k_2\right) - \varepsilon_2 k_1 h_4 Q_j^2}{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)} \\ & \times \left\{ \left[\left(h_1 + h_2\right) - \frac{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)}{\left(Q_j^2 - k_1\right)\left(h_5 Q_j^2 - h_6 k_2\right) - \varepsilon_2 k_1 h_4 Q_j^2} \right] \right. \\ & \left. \times K_{1/2}\left(Q_j r\right) - \frac{h_1}{r} K_{3/2}\left(Q_j r\right) \right\} e^{i\omega t},\end{aligned}$$

$$
\begin{aligned}
\sigma_{\phi\phi} &= \sigma_{\theta\theta} \\
&= \frac{1}{\sqrt{r}} \sum_{j=1}^{3} B_j(\omega) \\
&\quad \times \left\{ \frac{h_1}{r} \frac{\left(Q_j^2 - k_1\right)\left(h_5 Q_j^2 - h_6 k_2\right) - \varepsilon_2 k_1 h_4 Q_j^2}{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)} \right. \\
&\quad \times K_{3/2}\left(Q_j r\right) \\
&\quad + \left[h_2 \frac{\left(Q_j^2 - k_1\right)\left(h_5 Q_j^2 - h_6 k_2\right) - \varepsilon_2 k_1 h_4 Q_j^2}{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)} \right. \\
&\quad \left. - 1 - \frac{\varepsilon_1 k_1 h_4 Q_j^2 + Q_j^2 \left(Q_j^2 - k_1\right)}{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)} \right] \\
&\quad \left. \times K_{1/2}\left(Q_j r\right) \right\} e^{i\omega t}
\end{aligned}
$$

$$
\begin{aligned}
P &= \frac{1}{\sqrt{r}} \sum_{j=1}^{3} B_j(\omega) \left\{ - \frac{\left(Q_j^2 - k_1\right)\left(h_5 Q_j^2 - h_6 k_2\right) - \varepsilon_2 k_1 h_4 Q_j^2}{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)} \right. \\
&\quad \left. + h_3 \frac{\varepsilon_1 k_1 h_4 Q_j^2 + Q_j^2 \left(Q_j^2 - k_1\right)}{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)} - h_4 \right\} \\
&\quad \times K_{1/2}\left(Q_j r\right) e^{i\omega t}.
\end{aligned} \tag{32}
$$

Using the boundary conditions, we get

$$
\begin{aligned}
B_1(\omega) &= \left(-N_2 \left[\sqrt{a}\theta_0 W_3 - K_{1/2}\left(aQ_3\right) p_0\right]\right. \\
&\quad \left. + N_3 \left[\sqrt{a}\theta_0 W_2 - K_{1/2}\left(aQ_2\right) p_0\right]\right) \times (M)^{-1}, \\
B_2(\omega) &= \left(\sqrt{a} N_1 \left[\theta_0 W_3 - K_{1/2}\left(aQ_3\right) p_0\right]\right. \\
&\quad \left. + \sqrt{a} N_3 \left[K_{1/2}\left(aQ_1\right) p_0 - \theta_0 W_1\right]\right) \times (M)^{-1}, \\
B_3(\omega) &= \left(\sqrt{a} N_1 \left[K_{1/2}\left(aQ_2\right) p_0 - \theta_0 W_2\right]\right. \\
&\quad \left. - \sqrt{a} N_2 \left[K_{1/2}\left(aQ_1\right) p_0 - \theta_0 W_1\right]\right) \times (M)^{-1}, \\
N_j &= \frac{\left(Q_j^2 - k_1\right)\left(h_5 Q_j^2 - h_6 k_2\right) - \varepsilon_2 k_1 h_4 Q_j^2}{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)} \\
&\quad \times \left[\left(h_1 + h_2\right) \right. \\
&\quad \left. - \frac{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)}{\left(Q_j^2 - k_1\right)\left(h_5 Q_j^2 - h_6 k_2\right) - \varepsilon_2 k_1 h_4 Q_j^2} \right] \\
&\quad \times K_{1/2}\left(Q_j a\right) - \left[\frac{h_1}{a} + \mu_e H_\phi^2 \left(\frac{3}{a^2} - Q_i^2 \right) \right] \\
&\quad \times K_{3/2}\left(Q_j a\right), \\
W_j &= \left\{ - \frac{\left(Q_j^2 - k_1\right)\left(h_5 Q_j^2 - h_6 k_2\right) - \varepsilon_2 k_1 h_4 Q_j^2}{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)} \right. \\
&\quad \left. + h_3 \frac{\varepsilon_1 k_1 h_4 Q_j^2 + Q_j^2 \left(Q_j^2 - k_1\right)}{Q_j^2 \varepsilon_2 k_1 + k_1 \varepsilon_1 \left(h_5 Q_j^2 - h_6 k_2\right)} - h_4 \right\} \\
&\quad \times K_{1/2}\left(Q_j a\right), \\
M &= \left\{ N_1 \left[W_3 K_{1/2}\left(aQ_2\right) - W_2 K_{1/2}\left(aQ_3\right)\right] \right. \\
&\quad - N_2 \left[W_3 K_{1/2}\left(aQ_1\right) - W_1 K_{1/2}\left(aQ_1\right)\right] \\
&\quad \left. + N_3 \left[W_2 K_{1/2}\left(aQ_1\right) - W_1 K_{1/2}\left(aQ_2\right)\right] \right\}, \\
&\qquad j = 1, 2, 3.
\end{aligned} \tag{33}
$$

6. Particular Case

If we neglect the initial stress and diffusion effects by eliminating (3) and (8) and putting $P_1 = \beta_2 = C = 0$ in (4) and (6), we get (T, e), $u(r, t)$, σ_{rr}, $\sigma_{\phi\phi}$, and $\sigma_{\theta\theta}$:

[redacted]

$$
(T, e) = Ae^{-\lambda r + i\omega t} + Be^{-\lambda r + i\omega t}, \tag{34}
$$

where

$$
\begin{aligned}
\eta_1 &= -\left(\ell_1 + \beta\right), \qquad \eta_2 = \ell_1 \left(\beta - \varepsilon_1\right), \\
\left(\lambda_1^2, \lambda_2^2\right) &= \frac{1}{2}\left[\eta_1 \pm \sqrt{\eta_1^2 - 4\eta_2}\right], \\
u(r, t) &= - \left[\frac{r^2 \lambda_1^2 + 2r\lambda_1 + 2}{r^2 \lambda_1^3} A(\omega) e^{-\lambda_1 r} \right. \\
&\quad \left. + \frac{r^2 \lambda_2^2 + 2r\lambda_2 + 2}{r^2 \lambda_2^3} B(\omega) e^{-\lambda_2 r} + C(\omega) \right] e^{i\omega t}, \\
\sigma_{rr} &= \left[\alpha_1 \left(\frac{4 + 4r\lambda_1 + 2r^2 \lambda_1^2 + r^3 \lambda_1^3}{r^3 \lambda_1^3} + \left(\alpha_2 - 1\right) \right) A(\omega) e^{-\lambda_1 r} \right. \\
&\quad + \alpha_1 \left(\frac{4 + 4r\lambda_2 + 2r^2 \lambda_2^2 + r^3 \lambda_2^3}{r^3 \lambda_2^3} + \left(\alpha_2 - 1\right) \right) \\
&\quad \left. \times B(\omega) e^{-\lambda_2 r} \right] e^{i\omega t},
\end{aligned}
$$

$$\sigma_{\phi\phi} = \sigma_{\theta\theta} = - \left\{ \left(\alpha_1 \left(\frac{r^2\lambda_1^2 + 2r\lambda_1 + 2}{r^3\lambda_1^3} \right) - (\alpha_2 - 1) \right) \times A(\omega) e^{-\lambda_1 r} + \left[\alpha_1 \left(\frac{r^2\lambda_2^2 + 2r\lambda_2 + 2}{r^3\lambda_2^3} \right) - (\alpha_2 - 1) \right] \times B(\omega) e^{-\lambda_2 r} \right\} e^{i\omega t}. \quad (35)$$

Using the boundary conditions, we obtain

$$A(\omega) = \frac{-h_2 T_0}{h_1 e^{-\lambda_2 a} - h_2 e^{-\lambda_1 a}}, \qquad B(\omega) = \frac{h_1 T_0}{h_1 e^{-\lambda_2 a} - h_2 e^{-\lambda_1 a}},$$
$$C(\omega) = \frac{(h_2 h_3 - h_1 h_4)\theta_0}{h_1 e^{-\lambda_2 a} - h_2 e^{-\lambda_1 a}},$$
$$h_1 = \alpha_1 \left(\frac{4 + 4a\lambda_1 + 2a^2\lambda_1^2 + a^3\lambda_1^3}{a^3\lambda_1^3} + \left(\alpha_2 - 1 - \mu_e H_\phi^2 \lambda_1 \right) \right) e^{-\lambda_1 a},$$
$$h_2 = \alpha_1 \left(\frac{4 + 4a\lambda_2 + 2a^2\lambda_2^2 + a^3\lambda_2^3}{a^3\lambda_2^3} + \left(\alpha_2 - 1 - \mu_e H_\phi^2 \lambda_2 \right) \right) e^{-\lambda_2 a},$$
$$h_3 = \left(\frac{a^2\lambda_1^2 + 2a\lambda_1 + 2}{a^2\lambda_1^3} \right) e^{-\lambda_1 a},$$
$$h_4 = \left(\frac{a^2\lambda_2^2 + 2a\lambda_2 + 2}{a^2\lambda_2^3} \right) e^{-\lambda_2 a}. \quad (36)$$

7. Numerical Results and Discussion

For the purposes of numerical evaluations. The copper material was chosen. The constants of the problem given by Aouadi [24], Sokolnikoff [34] and Thomas [35] are

$$\mu = 3.86 \times 10^{10}\,\mathrm{kg/ms^3}, \qquad \lambda = 7.76 \times 10^{10}\,\mathrm{kg/ms^3},$$
$$\rho = 8954\,\mathrm{kg/m^3}, \qquad c_v = 383.1\,\mathrm{J/kg \cdot K},$$
$$\alpha_t = 1.78 \times 10^{-5}\,\mathrm{K^{-1}}, \qquad \alpha_c = 1.98 \times 10^{-4}\,\mathrm{m^3/kg},$$
$$k = 386\,\mathrm{W/mK}, \qquad D = 0.85 \times 10^{8}\,\mathrm{kg \cdot s/m^3}, \quad (37)$$
$$T_0 = 293\,\mathrm{K}, \qquad c = 1.2 \times 10^{4}\,\mathrm{m^2/s^2K},$$
$$b = 0.9 \times 10^{6}\,\mathrm{m^5/s^2kg}, \qquad \eta_0 = 8886.73\,\mathrm{s/m^2}.$$

Using the above values, we get $\mu_g = 1$, $\theta_0 = 1$, $P_0 = 1$, $a = 2$, $\omega = 9.5$, $H = 0.7 \times 10^{-5}$, $\tau_0 = 0.1$, and $\tau = 0.2$. The values of radial displacement u, temperature distribution θ, concentration C, stresses σ_{rr}, $\sigma_{\phi\phi}$, and chemical potential distribution P for thermoelastic diffusion and thermoelasticity are studied for force thermal source and chemical potential source. The output is plotted in Figures 1–10. Figure 1 shows that the values of chemical potential distribution P have oscillatory behavior with diffusion in the whole range of radius r. The effects of nonhomogeneity m, rotation Ω, time t and relaxation time τ_0 on chemical potential distribution is shifting from the positive into the negative gradually with the radius r. Figure 2 shows that the value of concentration distribution C has oscillatory behavior for diffusion in the whole range of radius r under the effects of nonhomogeneity, rotation, and relaxation time, while it is decreasing with an increase of nonhomogeneity m. In these figures, it is clear that the distribution has a nonzero value only in the bounded region of space for $t = 0.15$ where the infinite speed of propagation is inherent. The effects of nonhomogeneity, rotation Ω, time t, and relaxation time τ_0 on concentration distribution is shifting from the positive into the negative gradually. This indicates that the equations are satisfied by the concentration C which predict a finite speed of propagation of matter from first medium to another one. Figure 3 shows that the value of temperature distribution θ has an oscillatory behavior for thermoelastic diffusion in the whole range of the radius r, while the solution is notably different inside the sphere. This is due to the fact that, the thermal waves in the coupled theory travel with an infinite speed of propagation as opposed to finite speed in the generalized case. The effects of nonhomogeneity, rotation Ω, time t and relaxation time τ_0 on temperature distribution shift from the positive into the negative gradually. This indicates that the heat propagates as a wave with finite velocity. Figure 4 shows that the value of radial displacement u has oscillatory behavior with diffusion in the whole range of radius r. These figures indicate that the medium along r undergoes expansion deformation due to the thermal shock, while the other one shows the compressive deformation. The effect of nonhomogeneity, rotation Ω, and relaxation time τ_0 on radial displacement becomes large. Increasing the nonhomogeneity, the radial displacement is shifted upward from negative values to positive values. At a given instant, the radial displacement is finite which is due to the effect of nonhomogeneity, rotation, time, and relaxation time. Figures 5 and 6 show the variations of the radial stress σ_{rr} and tangential stress $\sigma_{\phi\phi}$ with respect to the radius r, respectively. The values of radial stress and tangential stress are increased and decreased due to the diffusion in a nonuniform behavior for all values of the radius r. For the values of σ_{rr} and $\sigma_{\phi\phi}$, depicting the effect of nonhomogeneity, diffusion, rotation, and relaxation time, it is shown that the radial stress is compressive in its nature.

Figure 7 shows the values of radial displacement u in thermoelastic medium without diffusion. This figure indicates clearly that the radial displacement at the cavity surface tends to zero which agrees with the boundary conditions prescribed. This coincides with the mechanical boundary condition of the cavity, in case of fixed surface. Figure 8 shows the values of temperature distribution θ without diffusion in the whole range of radius r. It was found that the values of θ under effect of nonhomogeneity and rotation Ω are increase with an increase of nonhomogeneity and rotation Ω but are decreasing with the increase of the values of m. Figures 9 and 10 show the values of radial stress σ_{rr} and the tangential stress $\sigma_{\phi\phi}$ without diffusion in the whole range of radius r, respectively. It was found that the values of σ_{rr} under the effects of nonhomogeneity m and rotation Ω are increasing with an increase of the values of nonhomogeneity m and rotation Ω, while the values of σ_{rr} are decreasing with an increase of nonhomogeneity m, while the tangential stress $\sigma_{\phi\phi}$ is decreasing with the increase of the values of nonhomogeneity m and rotation Ω, but the values of $\sigma_{\phi\phi}$ are increasing with an increase of m. Due to the complicated nature of the governing equations of the generalized magneto-thermoelastic diffusion theory, the done works in this field are unfortunately limited. The method used in this study provides quite a success in dealing with such problems. This method gives exact solutions in the elastic medium without any restrictions on the actual physical quantities that appear in the governing equations of the considered problem.

8. Conclusions

The results presented in this paper will be very helpful for researchers concerned with material science, designers of new materials, and low-temperature physicists, as well as for those working on the development of a theory of hyperbolic propagation of hyperbolic thermodiffusion. Study of the phenomenon of nonhomogeneity, rotation, magnetic field, and diffusion is also used to improve the conditions of oil extractions. It was found that, for values of rotation and nonhomogeneity, the coupled theory and the generalization give close results. The case is quite different when we consider small value of rotation and nonhomogeneity. Comparing Figures 1–6 in case of thermoelastic diffusion medium with the Figures 7–10 in case of thermoelastic medium, it was found that $u, \sigma_{rr}, \sigma_{\phi\phi}, C$, and P have the same behavior in both media. But with the passage of nonhomogeneity and rotation, the numerical values of u, σ_{rr}, $\sigma_{\phi\phi}$, C, and P in thermoelastic diffusion medium are large in comparison with those in thermoelastic medium due to the influences of nonhomogeneity, magnetic field, rotation, and mass diffusion.

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