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Research Article

Li-Yorke Sensitivity of Set-Valued Discrete Systems

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Consider the surjective, continuous map $f : X \to X$ and the continuous map $\overline{f}$ of $\mathscr{K}(X)$ induced by f, where X is a compact metric space and $\mathscr{K}(X)$ is the space of all nonempty compact subsets of X endowed with the Hausdorff metric. In this paper, we give a short proof that if $\overline{f}$ is Li-Yoke sensitive, then f is Li-Yorke sensitive. Furthermore, we give an example showing that Li-Yorke sensitivity of f does not imply Li-Yorke sensitivity of $\overline{f}$.

1. Introduction

Throughout this paper a dynamical system (X, f) is a pair where X is a compact metric space with metric d and $f : X \to X$ is a surjective, continuous map.

The idea of sensitivity from the work [1, 2] by Ruelle and Takens was applied to topological dynamics by Auslander and Yorke in [3] and popularized later by Devaney in [4]. A system (X, f) is called *ε-sensitive* if there exists a positive ε such that any $x \in X$ is a limit of points $y \in X$ satisfying the condition $d(f^n(x), f^n(y)) > \varepsilon$ for some positive integer n. According to Li and Yorke (see [5]), a subset $S \subset X$ is a *scrambled set* (for f), if any different points x and y from S are *proximal* and not *asymptotic*; that is,

$$\liminf_{n\to\infty} d\left(f^n(x), f^n(y)\right) = 0,$$
$$\limsup_{n\to\infty} d\left(f^n(x), f^n(y)\right) > 0. \tag{1}$$

Li-Yoke sensitivity is introduced by Akin and Kolyada in [6]. A system is *Li-Yorke sensitive* if there exists $\varepsilon > 0$ such that every $x \in X$ is a limit of points $y \in X$ such that the pair (x, y) is proximal but $\sup_{n>N}\{d(f^n(x), f^n(y))\} > \varepsilon$ for any $N > 0$, and the positive ε is said to be a Li-Yorke sensitive constant of the system. A pair (x, y) is ε-Li-Yorke sensitive if the pair (x, y) is proximal but whose orbits are frequently at least ε apart.

A dynamical system (X, f) is called *spatiotemporal chaotic* (see [6] or [7]) if every point is a limit point for points which are proximal to but not asymptotic to it. That is, for any $x \in X$ and any open subset U with $x \in U$, there is $y \in U$ such that x and y are proximal and not asymptotic. It is easy to see that Li-Yorke sensitivity implies spatiotemporal chaos and sensitivity.

Román-Flores [8] and Fedeli [9] studied the interplay of chaos for discrete dynamical systems (individual chaos) with the corresponding set-valued versions (collective chaos). Recall that the map $\overline{f} : \mathscr{K}(X) \to \mathscr{K}(X)$ induced by f on $\mathscr{K}(X) = \{K \subset X : K \text{ is a nonempty compact subset}\}$ is defined by $\overline{f}(K) = f(K) = \{f(x) : x \in K\}$, $K \in \mathscr{K}(X)$. Then the pair $(\mathscr{K}(X), \overline{f})$ is a dynamical system with the space $\mathscr{K}(X)$ endowed with the Hausdorff distance:

$$\begin{aligned} H_d\left(K_1, K_2\right) &= \max\left\{\sup\left\{d\left(x_1, K_2\right) : x_1 \in K_1\right\},\right. \\ &\qquad \left.\sup\left\{d\left(x_2, K_2\right) : x_2 \in K_2\right\}\right\}, \end{aligned} \tag{2}$$

and $K_1, K_2 \in \mathscr{K}(X)$. And various concepts of chaos in set-valued discrete systems have been researched recently (see [10–16]).

In this paper, we discuss the relationship between Li-Yorke sensitivity of f and Li-Yorke sensitivity of $\overline{f}$. It will be shown that if $\overline{f}$ is Li-Yoke sensitive, then f is Li-Yorke sensitive. Furthermore, we give an example showing that

Li-Yorke sensitivity of f does not imply Li-Yorke sensitivity of $\overline{f}$. This paper discusses the further work of [17]. And by suing the obtained results, we give positive answers to Sharma and Nagar's question in [18].

2. The Denjoy Homeomorphism and an Interval Map

Let (X, d) be a compact metric space. For any nonempty subsets Y, Y' of X and any $r > 0$, write $d(Y, Y') = \inf\{d(x, y) : x \in Y, y \in Y'\}$, $\operatorname{diam}(Y) = \sup\{d(x, y) : x, y \in Y\}$, and $B(Y, r) = \{x \in X : d(x, Y) < r\}$, where $d(x, Y) = \inf\{d(x, y) : y \in Y\}$. When $Y = \{y\}$ is a singleton, we write $B(y, r)$ (resp., $d(y, Y')$) for $B(Y, r)$ (resp., $d(Y, Y')$). For any nonempty subset $\mathbb{K}$ of $\mathbb{N}$ and any $i \in \mathbb{N}$, write $i + \mathbb{K} = \{i + n : n \in \mathbb{K}\}$.

Write $N(U, V) = \{n \in \mathbb{N} : U \cap f^{-n}(V) \neq \Phi\}$, where U, V are nonempty subsets in X. A subset $\mathbb{K} \subset \mathbb{N}$ is *syndetic* (or *relative dense*) if there is $N \in \mathbb{N}$ such that $\{i, i+1, \ldots, i+N\} \cap \mathbb{K} \neq \Phi$ for every $i \in \mathbb{N}$. A point $x \in X$ is *almost periodic* if for any $\varepsilon > 0$, $N(x, B(x, \varepsilon))$ is syndetic. A subset $\mathbb{K} \subset \mathbb{N}$ is *thick* if it contains arbitrarily long runs of positive integers. A dynamical system is *transitive* if for each pair of nonempty open subsets A, B of X, $N(A, B)$ is nonempty. A point $x \in X$ is *transitive* if the *orbit*$O(x, f) \equiv \{f^n(x) : n = 0, 1, 2, \ldots\}$ is dense in X. A system (X, f) is *minimal* if any $x \in X$ is transitive. We say (X, f) is *mixing* if for each pair of nonempty open subsets U, V, $N(U, V)$ is cofinite, and (X, f) is *weakly mixing* if $(X \times X, f \times f)$ is transitive. The set $\omega(x, f) \equiv \{y$: there exists an increasing sequence $\{n_i\}$ such that $y = \lim_{i\to\infty} f^{n_i}(x)\}$ is said to be the *ω-limit set* of x.

Lemma 1. *If the system (X, f) is minimal, then for any $x \in X$ and any open subset $U \subset X$, $N(x, U)$ is syndetic. For some $x \in X$, if $V \subset \omega(x, f)$ is an invariant closed set with $f(V) = V$, then for any $\delta > 0$, $N(x, B(V, \delta))$ is thick.*

Proof. For any $x \in X$ and any open subset $U \subset X$ there are $\delta > 0$ and $n_0 \in \mathbb{N}$ such that $f^{n_0}(B(x, \delta)) \subset U$. It is well known that if the system (X, f) is minimal, then every $v \in X$ is almost periodic. So $N(x, B(x, \delta))$ is syndetic. Then $N(x, U) \supset \{i + n_0 : i \in N(B(x, \delta))\}$ is syndetic.

Since $f(V) = V$ and f is uniformly continuous, then for any $\delta > 0$ and any $N \in \mathbb{N}$, there is $\delta' \in (0, \delta)$ such that

$$f^{j}\left(B\left(V, \delta'\right)\right) \subset B(V, \delta), \quad \text{for } j = 1, \ldots, 2N. \tag{3}$$

So for some $m \in \mathbb{N}$ with $f^m(x) \in B(V, \delta')$, $\{m, m+1, \ldots, m+2N\} \subset N(x, B(V, \delta))$. □

We will use $\mathbb{R}/\mathbb{Z}$ as a model for the circle S^1. The metric d' is defined by $d'(a, b) = \min\{|a - b|, 1 - |a - b|\}$. Rigid rotation by the real number α is then given by

$$R_\alpha : S^1 \longrightarrow S^1, \quad R_\alpha = t + \alpha \,(\operatorname{mod} 1). \tag{4}$$

Corresponding to the irrational α, the Denjoy homeomorphism $d_\alpha : S^1 \to S^1$ is an orientation preserving homeomorphism of the circle characterized by the following properties: the rotation number of d_α is α; there is a Cantor set $C_\alpha \subset S^1$ on which d_α acts minimally; and if u and v are any two components of $S^1 \setminus C_\alpha$, then $d_\alpha^n(u) = v$ for some integer n (see [19]). There is a Cantor function $h_\alpha : S^1 \to S^1$ that semiconjugates d_α with R_α: h_α being a monotone surjection that collapses the components of $S^1 \setminus C_\alpha$ (and so maps C_α onto S^1) with $R_\alpha \circ h_\alpha = h_\alpha \circ d_\alpha$.

Lemma 2. *Let (C_α, d_α) be the minimal subsystem of a Denjoy homeomorphism, and $c = \max\{\operatorname{diam}(u) : u$ is a connected component of $S^1 \setminus C_\alpha$ with $\operatorname{diam}(u) < 1/4\}$. Then (C_α, d_α) is c-sensitive. Furthermore, for any $x \in C_\alpha$ and any $\delta > 0$, there is $y \in B(x, \delta)$ such that $N_c(x, y) \equiv \{n : d'(d_\alpha^n(x), (d_\alpha^n(y))) > c\}$ is syndetic.*

Proof. For any $x \in C_\alpha$ and any $\delta > 0$, there is $y \in B(x, \delta)$ such that $h_\alpha(x) \neq h_\alpha(y)$. Let $[h_\alpha(x), h_\alpha(y)]$ be the arc in S^1 whose endpoints are x and y and whose length is $d'(h_\alpha(x), h_\alpha(y))$. Then there exist w and $\delta' > 0$ such that $w \in B(w, \delta') \subset [h_\alpha(x), h_\alpha(y)]$. Let u be one of the connected components of $S^1 \setminus C_\alpha$ with $\operatorname{diam}(u) = c$ and $p = d_\alpha(u)$. By Lemma 1, $N(w, B(p, \delta'))$ is syndetic. For any $i \in N(w, B(p, \delta'))$, $p \in [R_\alpha^i(h_\alpha(x)), R_\alpha^i(h_\alpha(y))]$. So $d'(d_\alpha^i(x), (d_\alpha^i(y))) > c$. □

Lemma 3. *Let (C_α, d_α) be the minimal subsystem of a Denjoy homeomorphism R_α, and $c = \max\{\operatorname{diam}(u) : u$ is a connected component of $S^1 \setminus C_\alpha$ with $\operatorname{diam}(u) < 1/4\}$. Then for any $x \in C_\alpha$, there is $\delta > 0$ such that for any $y \in B(x, \delta)$ with $y \neq x$, $\liminf_{n\to\infty} d'(d_\alpha^n(x), d_\alpha^n(y)) > 0$.*

Proof. Let $\{u_i\}_{i\in\mathbb{Z}}$ be an arrangement of the connected components of $S^1 \setminus C_\alpha$ with $d_\alpha(u_i) = u_{i+1}$, $i \in \mathbb{Z}$, and $\operatorname{diam}(u_0) = c$. For any $x \in S^1$, $h_\alpha^{-1}(x)$ has two elements at most. So for any $v \in C_\alpha$, there is $\delta > 0$ such that for any $y \in B(v, \delta)$ with $y \neq v$, $h_\alpha(v) \neq h_\alpha(y)$. For $v' \in B(v, \delta)$ and $v' \neq v$, let $[w, w'] = h_\alpha([v, v'])$ be an arc, and $p = h_\alpha(u_0)$. For the irrational α, there exists $k, l \in \mathbb{N}$ such that $k\alpha \bmod 1 < \operatorname{diam}([w, w'])$ and $k\alpha \times l \bmod 1 < \operatorname{diam}([w, w'])$. So for any $i \in \mathbb{N}$, there is $0 \leq j \leq l$ such that $R_{k\alpha}^j(p) \in R_\alpha^i([w, w'])$. So $u_{k\times j} \subset d_\alpha^i([v, v'])$. Let $\varepsilon_0 = \min\{\operatorname{diam}(u_0), \operatorname{diam}(u_k), \ldots, \operatorname{diam}(u_{k\times l})\}$. Then $\liminf_{n\to\infty} d'(d_\alpha^n(v), d_\alpha^n(v')) \geq \varepsilon_0 > 0$. □

Lemma 4 (see [17]). *$(\mathscr{K}(C_\alpha), \overline{d}_\alpha)$ is not sensitive (C_α is a stable point).*

Lemma 5 (see [6]). *If a nontrivial system (X, f) is weakly mixing then it is Li-Yorke sensitive.*

Lemma 6. *Let $f : I \to I$ be the tent map which is $f(x) = 1 - |1 - 2x|$. Then f is Li-Yorke sensitive.*

Proof. It is well known that the tent map is mixing [12]. Apply Lemma 5. □

Example 7. $f : I \to I$ is given by $f|_{[0,1/3]}$ and $f|_{[2/3,1]}$ which are the tent maps; $f|_{[a,b]}$ is a constant mapping, $f|_{[1/3,a]}$ and $f|_{[b,2/3]}$ are linear where $1/3 < a < b < 2/3$ and $f(a)$ is a transitive point of $f|_{[0,1/3]}$ (see Figure 1).

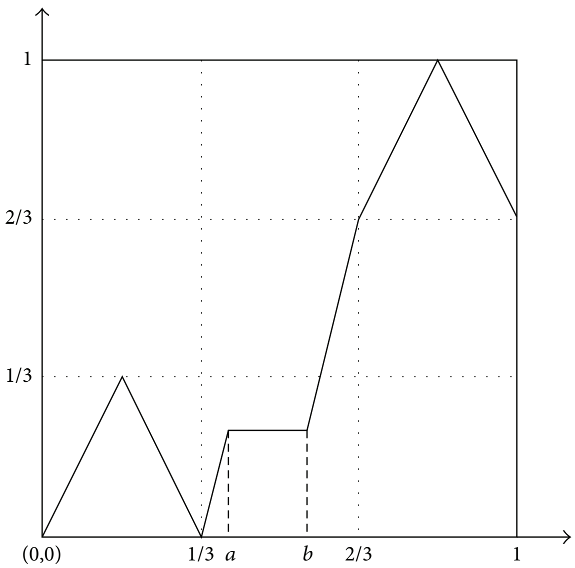

Figure 1

Lemma 8. *There is a positive* $\varepsilon > 0$, *for any* $x \in (I \setminus \bigcup_{i=0}^{\infty} f^{-i}([a,b])) \cup \{2/3\}$ *and any* $\delta > 0$, *there exists* y *with* $d(x,y) < \delta$ *such that the pair* (x,y) *is* ε*-Li-Yorke sensitive.*

Proof. By Lemma 6, $f|_{I\setminus(1/3,2/3)}$ is Li-Yorke sensitive. Let $\varepsilon > 0$ be a Li-Yorke sensitive constant of $f|_{I\setminus(1/3,2/3)}$. Then for any $x \in I \setminus (1/3,2/3)$, the lemma holds. For any $x \in (1/3,2/3) \setminus \bigcup_{i=0}^{\infty} f^{-i}([a,b])$ and any $\delta > 0$, there exist an open interval U with $x \in U \subset B(x,\delta) \cap ((1/3,2/3) \setminus \bigcup_{i=0}^{\infty} f^{-i}([a,b]))$ and $n_0 \in \mathbb{N}$ such that $f^{n_0}(U) \subset [0,1/3]$. It is easy to see that $f^{n_0}(U)$ is connected open neighborhood of $f^{n_0}(x)$. Because $f|_{[0,1/3]}$ is Li-Yorke sensitive, there is $y \in U$ such that (x,y) is ε-Li-Yorke sensitive. □

3. Li-Yorke Sensitivity

Lemma 9 (see [12]). *Let* (X,f) *be a system. Then the following statements are equivalent:*

(i) f *is weakly mixing;*

(ii) $\overline{f}$ *is weakly mixing;*

(iii) $\overline{f}$ *is transitive.*

Theorem 10. *If a nontrivial system* (X,f) *is weakly mixing, then* $\overline{f}$ *is Li-Yorke sensitive.*

Proof. By Lemma 9, $\overline{f}$ is weakly mixing. Apply Lemma 5. □

Theorem 11. *If* $\overline{f}$ *is Li-Yorke sensitive, then* f *is Li-Yorke sensitive.*

Proof. Let $(\mathcal{K}(X),\overline{f})$ be Li-Yorke sensitive. There exists $\varepsilon > 0$, for any $\{y\} = Y \in \mathcal{K}(X)$ and any $\delta > 0$, and there is a contract subset K with $H(Y,K) < \delta$ (so, for any $x \in K$, $d(x,y) < \delta$) such that (Y,K) is an ε-Li-Yorke sensitive pair of $\overline{f}$. So there exists a point $y' \in K$ with $y' \neq y$ such that (y,y') is an ε-Li-Yorke sensitive pair of f. □

4. A Counter Example

Example 12. Let (C_α, d_α) be the minimal subsystem of a Denjoy homeomorphism, and $c = \max\{\mathrm{diam}(u) : u$ is a connected component of $S^1 \setminus C_\alpha$ with $\mathrm{diam}(u) < 1/4\}$, and let (I,f) be the interval map given in Example 7. Let $S = \{(r,2\pi\theta) : r \in I, \theta \in C_\alpha\}$ be a subset in polar coordinate system with metric ρ defined by

$$\rho\left((r,\theta),(r',\theta')\right) = \left(r^2 + r'^2 - 2rr'\cos\left(\theta-\theta'\right)\right)^{1/2}. \quad (5)$$

And let the map $F : S \to S$ be defined by $F(r,2\pi\theta) = (f(r), 2\pi d_\alpha(\theta))$. It is easy to see that (S,F) is a dynamical system.

Proposition 13. (S,F) *is Li-Yorke sensitive.*

Proof. For any $(r,2\pi\theta) \in S$, either $r \in (I \setminus \bigcup_{i=0}^{\infty} f^{-i}([a,b])) \cup \{2/3\}$ or $r \in \bigcup_{i=0}^{\infty} f^{-i}([a,b]) \setminus \{2/3\}$.

If $r \in \bigcup_{i=0}^{\infty} f^{-i}([a,b]) \setminus \{2/3\}$, then there exists $k \in \mathbb{N}$ such that $f^k(r) = f(a)$ is a transitive point of $f|_{[0,1/3]}$ and so $\omega(r,f) = [0,1/3]$. Since $2/9$ is a fixed point of f, by Lemma 1, $N(r, B(2/9,1/9))$ is thick. By Lemma 2, for any $\delta > 0$, there exists $\theta' \in B(x,\delta/2\pi)$ such that $N_c(\theta,\theta')$ is syndetic, so there is $m \in N(r,B(2/9,1/9)) \cap N_c(\theta,\theta')$; that is, $\rho(F^m(r,2\pi\theta), F^m(r,2\pi\theta')) = \sqrt{2}r(1-\cos 2\pi(\theta-\theta'))^{1/2} \geq \sqrt{2}/9(1-\cos 2\pi c)^{1/2}$. On the other hand, there is a sequence $\{n_i\} \subset \mathbb{N}$ such that $\lim_{i\to\infty} f^{n_i}(r) = 0$. So $\lim_{i\to\infty}\rho(F^{n_i}(r,2\pi\theta), F^{n_i}(r,2\pi\theta')) = \lim_{i\to\infty}\sqrt{2}f^{n_i}(r)(1-\cos 2\pi(\theta-\theta'))^{1/2} = 0$. So $((r,2\pi\theta),(r,2\pi\theta'))$ is a $\sqrt{2}/9(1-\cos 2\pi c)^{1/2}$-Li-Yorke sensitive pair.

If $r \in I \setminus \bigcup_{i=0}^{\infty} f^{-i}([a,b]) \cup \{2/3\}$, by Lemma 8, there is a positive ε, for any $\delta > 0$, and there exists a point r' with $d(r,r') < \delta$ such that (r,r') is ε-Li-Yorke sensitive. It is not difficult to verify that $((r,\theta),(r',\theta))$ is an ε-Li-Yorke sensitive.

To sum up, $\varepsilon_0 = \min\{\sqrt{2}/9(1-\cos 2\pi c)^{1/2}, \varepsilon\}$ is a Li-Yorke sensitive constant of F. □

Proposition 14. $(\mathcal{K}(S),\overline{F})$ *is not sensitive.*

Proof. Write $r_0 = (a+b)/2$. By Lemma 4, C_α is a stable point of $(\mathcal{K}(C_\alpha), \overline{d}_\alpha)$. So for any $\varepsilon > 0$, there exists $0 < \delta < (b-a)/2$ such that for every $K \in \mathcal{K}(C_\alpha)$ with $H_{d'}(K,C_\alpha) < \delta$ and all $i \in \mathbb{N}$, $H_{d'}(\overline{d}_\alpha^{\,i}(K),C_\alpha) < \varepsilon$.

We will prove that (r_0,C_α) is a stable point of $\overline{F}$. Let $\pi_1 : S \to I$ be the natural map defined by $\pi_1((r,\theta)) = r$ and $\pi_2 : S \to C_\alpha$ be the natural map defined by $\pi_2((r,\theta)) = \theta$. By the continuities of π_1,π_2, there exists $0 < \delta' < \delta$ such that for any $M \in \mathcal{K}(S)$ with $H_\rho(M,(r_0,C_\alpha)) < \delta'$, $\pi_1(M) \subset [a,b]$ and $H_{d'}(\pi_2(M),C_\alpha) < \delta$. Then for any $i \in \mathbb{N}$,

$$\begin{aligned} &H_\rho\left(\overline{F}^i(M), \overline{F}^i(r_0,C_\alpha)\right) \\ &\quad = H_\rho\left(\overline{F}^{i-1}\left(f(a), \overline{d}_\alpha(\pi_2(M))\right), \overline{F}^{i-1}\left(f(a),C_\alpha\right)\right) \end{aligned}$$

$$\le H_{d'}\left(\overline{d}_{\alpha}^{i}\left(\pi_{2}\left(M\right)\right), C_{\alpha}\right)$$
$$< \varepsilon. \tag{6}$$

□

Proposition 15. *$(\mathcal{K}(S), \overline{F})$ is not spatiotemporal chaotic.*

Proof. For any point $(r_0, 2\pi\theta)$, $\theta \in C_\alpha$. By Lemma 3, there is $\delta > 0$ such that for any $\theta' \in B(\theta, \delta)$ with $\theta' \neq \theta$, $\liminf_{n\to\infty} d'(d_\alpha^n(\theta), d_\alpha^n(\theta')) > 0$. By the continuities of π_1, π_2, there exists $0 < \delta' < \delta$ such that for any $M \in \mathcal{K}(S)$ with $H_\rho(M, (r_0, \theta)) < \delta'$, $\pi_1(M) \subset [a, b]$ and $H_{d'}(\pi_2(M), \theta) < \delta$. Let $\theta_0 \in B(\theta, \delta)$ with $\theta_0 \neq \theta$. Then,

$$\liminf_{i\to\infty} H_\rho\left(\overline{F}^{i}\left(M\right), \overline{F}^{i}\left(r_0, \theta\right)\right)$$
$$= \liminf_{i\to\infty} H_\rho\left(\overline{F}^{i-1}\left(f\left(a\right), \overline{d}_{\alpha}\left(\pi_2\left(M\right)\right)\right), \overline{F}^{i-1}\left(f\left(a\right), d_{\alpha}\left(\theta\right)\right)\right)$$
$$\le H_{d'}\left(\overline{d}_{\alpha}^{i}\left(\pi_{2}\left(M\right)\right), C_{\alpha}\right)$$
$$< \varepsilon. \tag{7}$$

□

From Propositions 13 and 14 or from Propositions 13 and 15, we obtain the following at once.

Theorem 16. *There is a dynamical system (X, f) such that (X, f) is Li-Yorke sensitive, but $(\mathcal{K}(X), \overline{f})$ the set-valued discrete system induced by (X, f) is not sensitive.*

5. Li-Yorke Sensitivity of Interval Maps

Lemma 17 (see [20]). *Let $f : [a, b] \to [a, b]$ be a transitive interval map. Then one of the following conditions holds:*

(i) *f is mixing;*

(ii) *there is $c \in (a, b)$ such that if $f([a, c]) = [c, b]$ and $f([c, b]) = [a, c]$, in addition, c is the unique fixed point of f, and both $f^2|_{[a,c]}$ and $f^2|_{[c,b]}$ are mixing.*

Theorem 18. *Let $f : [a, b] \to [a, b]$ be a transitive interval map. Then f is Li-Yorke sensitive.*

Proof. By Lemma 17, either f is mixing or there is the unique fixed point c such that $f^2|_{[a,c]}$ and $f^2|_{[c,b]}$ are mixing.

If f is mixing, then f is weakly mixing. Apply Lemma 5.

If there is the unique fixed point c such that $f^2|_{[a,c]}$ and $f^2|_{[c,b]}$ are mixing, by Lemma 5, then $f^2|_{[a,c]}$ and $f^2|_{[c,b]}$ are Li-Yorke sensitive. It is easy to see that f is Li-Yorke sensitive. □

Example 19. $f : I \to I$ is given by $f|_{[0,1/3]}$ and $f|_{[2/3,1]}$ which are the tent maps; $f|_{[1/3,2/3]}$ is linear. It is easy to see that f is Li-Yorke sensitive but is not transitive. So the converse version of Theorem 18 does not hold.

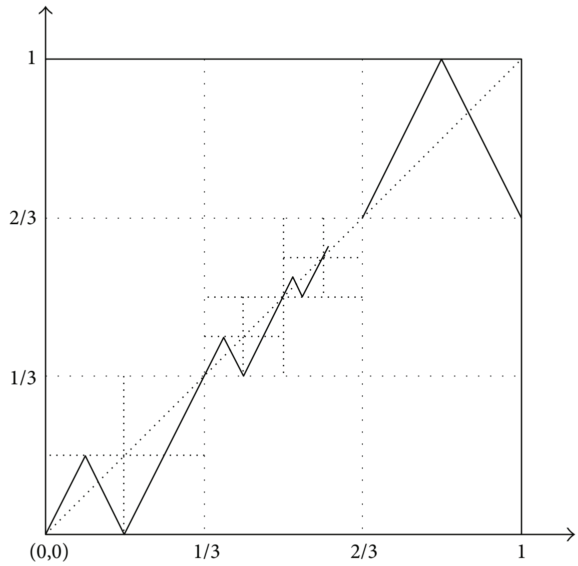

Figure 2

Example 20. $f : I \to I$ is given by $f|_{[0,1/2]}$ which is the tent map and $f|_{[1/2,1]}$ which is linear. It is not difficult to get that f is sensitive but is not Li-Yorke sensitive (1 is a distal point).

The following example is an interval map which is spatiotemporal chaotic but is not Li-Yorke sensitive.

Example 21. $f : I \to I$ is given by $f|_{[0,1/6]}$, $f|_{I_i}$ and $f|_{[2/3,1]}$ which are the tent maps, and $f|_{[1/6,1/3]}$, $f|_{I_i'}$ are linear, where $I_i = [(2/3)(1 - (1/2)^i), (2/3)(1 - (1/2)^i) + (1/2)^{i+1}(1/3)]$, $I_i' = [(2/3)(1 - (1/2)^i) + (1/2)^{i+1}(1/3), (2/3)(1 - (1/2)^{i+1})]$, $i = 1, 2, \ldots$ (see Figure 2).

For any $x \in I$ and any $\delta > 0$, there is $n, n' \in \mathbb{N}$ such that $f^{n'}(x) \in I_n$. Since $f|_{I_n}$ is mixing, there is $y \in B(x, \delta)$ such that x, y is proximal but is not asymptotic. So f is spatiotemporal chaotic.

On the other hand, for any $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $\operatorname{diam} I_i < \varepsilon$ for all $n > n_0$. Since $f(I_i) = I_i$, for all $i \in \mathbb{N}$, then any $x \in \bigcup_{i=n_0+3}^{\infty} I_i$ is not ε-unstable (i.e., there exists $\delta > 0$ such that $\operatorname{diam}(f^i(B(x, \delta))) < \varepsilon$, for all $i \in \mathbb{N}$), so f is not sensitive; especially, f is not Li-Yorke sensitive.

Acknowledgments

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References

[1] D. Ruelle and F. Takens, "On the natureof turbulence," *Communications in Mathematical Physics*, vol. 20, pp. 167–192, 1971.

[2] D. Ruelle and F. Takens, "Note concerning our paper: 'on the nature of turbulence'," *Communications in Mathematical Physics*, vol. 23, pp. 343–344, 1971.

[3] J. Auslander and J. A. Yorke, "Interval maps, factors of maps, and chaos," *The Tôhoku Mathematical Journal*, vol. 32, no. 2, pp. 177–188, 1980.

[4] R. Devaney, *Chaotic Dynamical Systems*, Addison-Wesley, Reading, Mass, USA, 1989.

[5] T. Y. Li and J. A. Yorke, "Period three implies chaos," *The American Mathematical Monthly*, vol. 82, no. 10, pp. 985–992, 1975.

[6] E. Akin and S. Kolyada, "Li-Yorke sensitivity," *Nonlinearity*, vol. 16, no. 4, pp. 1421–1433, 2003.

[7] F. Blanchard, E. Glasner, S. Kolyada, and A. Maass, "On Li-Yorke pairs," *Journal für die Reine und Angewandte Mathematik*, vol. 547, pp. 51–68, 2002.

[8] H. Román-Flores, "A note on transitivity in set-valued discrete systems," *Chaos, Solitons and Fractals*, vol. 17, no. 1, pp. 99–104, 2003.

[9] A. Fedeli, "On chaotic set-valued discrete dynamical systems," *Chaos, Solitons and Fractals*, vol. 23, no. 4, pp. 1381–1384, 2005.

[10] A. Peris, "Set-valued discrete chaos," *Chaos, Solitons and Fractals*, vol. 26, no. 1, pp. 19–23, 2005.

[11] D. Kwietniak and P. Oprocha, "Topological entropy and chaos for maps induced on hyperspaces," *Chaos, Solitons and Fractals*, vol. 33, no. 1, pp. 76–86, 2007.

[12] G. F. Liao, L. D. Wang, and Y. C. Zhang, "Transitivity, mixing and chaos for a class of set-valued mappings," *Science in China A*, vol. 49, no. 1, pp. 1–8, 2006.

[13] H. Román-Flores and Y. Chalco-Cano, "Robinson's chaos in set-valued discrete systems," *Chaos, Solitons and Fractals*, vol. 25, no. 1, pp. 33–42, 2005.

[14] J. Banks, "Chaos for induced hyperspace maps," *Chaos, Solitons and Fractals*, vol. 25, no. 3, pp. 681–685, 2005.

[15] R. B. Gu, "Kato's chaos in set-valued discrete systems," *Chaos, Solitons and Fractals*, vol. 31, no. 3, pp. 765–771, 2007.

[16] W. Bauer and K. Sigmund, "Topological dynamics of transformations induced on the space of probability measures," *Monatshefte für Mathematik*, vol. 79, pp. 81–92, 1975.

[17] H. Liu, E. Shi, and G. Liao, "Sensitivity of set-valued discrete systems," *Nonlinear Analysis. Theory, Methods & Applications*, vol. 71, no. 12, pp. 6122–6125, 2009.

[18] P. Sharma and A. Nagar, "Inducing sensitivity on hyperspaces," *Topology and Its Applications*, vol. 157, no. 13, pp. 2052–2058, 2010.

[19] C. Robinson, *Dynamical Systems*, Studies in Advanced Mathematics, CRC Press, Boca Raton, Fla, USA, 1995.

[20] M. Barge and J. Martin, "Chaos, periodicity, and snakelike continua," *Transactions of the American Mathematical Society*, vol. 289, no. 1, pp. 355–365, 1985.