Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 101764, 6 pages
http://dx.doi.org/10.1155/2013/101764

Research Article

Global Attractivity of a Diffusive Nicholson's Blowflies Equation with Multiple Delays

Xiongwei Liu and Xiao Wang

Department of Mathematics and System Science, College of Science, National University of Defense Technology, Changsha 410073, China

Correspondence should be addressed to Xiongwei Liu; liuxiongwei@yahoo.com.cn and Xiao Wang; wxiao_98@yahoo.com.cn

Received 29 January 2013; Accepted 3 April 2013

Academic Editor: Chuangxia Huang

The present paper considers a diffusive Nicholson's blowflies model with multiple delays under a Neumann boundary condition. Delay independent conditions are derived for the global attractivity of the trivial equilibrium and the positive equilibrium, respectively. Two open problems concerning the stability of positive equilibrium and the occurrence of Hopf bifurcation are proposed.

1. Introduction

Since blowflies are important parasites of the sheep industry in some countries such as Australia, based on the experimental data of Nicholson [1, 2], Gurney et al. [3] first proposed Nicholson's blowflies equation

$$\dot{N}(t) = -\delta N(t) + pN(t-\tau)e^{-aN(t-\tau)}, \quad t > 0, \tag{1}$$

where $N(t)$ is the size of the adult blowflies population at time t; p is the maximum per capita daily egg production rate; $1/a$ is the size at which the blowflies population reproduces at its maximum rate; δ is the per capita daily adult death rate; τ is the generation time. For this equation, global attractivity and oscillation of solutions have been investigated by several authors (see [4–9]).

It is impossible that the size of the adult blowflies population is independent of a spatial variable; therefore, Yang and So [10] investigated both temporal and spatial variations of the diffusive Nicholson's blowflies equation

$$\begin{aligned}\frac{\partial N(t,x)}{\partial t} &= \Delta N(t,x) - \delta N(t,x) \\ &\quad + pN(t-\tau,x)e^{-aN(t-\tau,x)}, \\ &\quad \text{in } D \triangleq (0,\infty)\times\Omega\end{aligned} \tag{2}$$

under Neumann boundary condition and gave the similar sufficient conditions for oscillation of all positive solutions about the positive steady state. Whereafter, many authors studied the various dynamical behaviors for this equation; we refer to Lin and Mei [11], Saker [12], Wang and Li [13], and Yi and Zou [14].

Meanwhile, one can consider a nonlinear equation with several delays because of variability of the generation time; for this purpose, Györi and Ladas [15] and Kulenović and Ladas [6] proposed the following generalized Nicholson's blowflies model:

$$N'(t) = -\delta N(t) + \sum_{i=1}^{n} p_i N(t-\tau_i)e^{-a_i N(t-\tau_i)}, \quad t > 0. \tag{3}$$

Luo and Liu [16] studied the global attractivity of the nonnegative equilibria of (3).

It is of interest to investigate both several temporal and spatial variations of the blowflies population using mathematical models. Hereby, in this paper, we consider the following system:

$$\begin{aligned}&\frac{\partial N(t,x)}{\partial t} \\ &\quad = \Delta N(t,x) - \delta N(t,x) \\ &\qquad + \sum_{i=1}^{n} p_i N(t-\tau_i,x)e^{-a_i N(t-\tau_i,x)}, \quad \text{in } D\end{aligned} \tag{4}$$

with Neumann boundary condition

$$\frac{\partial N(t,x)}{\partial \nu} = 0, \quad \text{on } \Gamma \triangleq (0,\infty)\times\partial\Omega, \tag{5}$$

and initial condition

$$N(\theta,x) = \psi(\theta,x) \geq 0, \quad \text{in } D_\tau \triangleq [-\tau,0]\times\overline{\Omega}, \tag{6}$$

where $\tau_i \geq 0, \tau = \max_{1\leq i\leq n}\{\tau_i\}$, p_i and $a_i = a$, $i = 1,2,\ldots,n$, are all positive constants, $\Omega \subset \mathbb{R}^m$ is a bounded domain with a smooth boundary $\partial\Omega$, $\Delta N(t,x) = \sum_{i=1}^m((\partial_i^2 N(t,x))/(\partial x_i^2))$, $(\partial/\partial\nu)$ denotes the exterior normal derivative on $\partial\Omega$, and $\psi(\theta,x)$ is *Hölder* continuous in D_τ with $\psi(0,x)\in C^1(\overline{\Omega})$.

Though the global attractivity of the nonnegative equilibria of (2) has been studied by Yang and So [10] and Wang and Li [13, 17], they just gave some sufficient conditions. Furthermore, as far as we know, the stability for partial functional differential equations with several delays was investigated by few papers. Motivated by the above excellent works, in this paper, we consider the global attractivity of the nonnegative equilibria of the systems (4)–(6) and present some conditions which depend on coefficients of the systems (4)–(6). When $n = 1$, our results complement those in Yang and So [10] and Wang and Li [13].

It is not difficult to see that if $\sum_{i=1}^n p_i \leq \delta$, then (4) has a unique nonnegative equilibrium $N_0 \equiv 0$ and if $\sum_{i=1}^n p_i > \delta$, then (4) has a unique positive equilibrium $N^* = (1/a)\ln((\sum_{i=1}^n p_i)/\delta)$.

The rest of the paper is organized as follows. We give some lemmas and definitions in Section 2 and state and prove our main results in Section 3. In Section 4, several simulations are obtained to testify our results, and some unsolved problems are discussed.

2. Preliminaries

In this section, we will give some lemmas which can be proved by using the similar methods as those in Yang and So [10].

Lemma 1. *(i) The solution $N(t,x)$ of (4)–(6) satisfies $N(t,x)\geq 0$ for $(t,x)\in(0,\infty)\times\overline{\Omega}$.*

(ii) If $\psi(\theta,x)\not\equiv 0$ on D_τ, then the solution $N(t,x)$ of (4)–(6) satisfies $N(t,x) > 0$ for $(t,x)\in(\tau,\infty)\times\overline{\Omega}$.

Next, we will introduce the concept of lower-upper solution due to Redlinger [18] as adapted to (4)–(6).

Definition 2. A lower-upper solution pair for (4)–(6) is a pair of suitably smooth function v and w such that

(i) $v \leq w$ in $\overline{D}$,

(ii) v and w satisfy

$$\frac{\partial w}{\partial t} \geq \Delta w(t,x) - \delta w + \sum_{i=1}^n p_i\varphi(t-\tau_i,x)e^{-a\varphi(t-\tau_i,x)}, \quad (t,x)\in D,$$

$$\frac{\partial w}{\partial \nu} \geq 0, \quad (t,x)\in\Gamma,$$

$$\frac{\partial v}{\partial t} \leq \Delta v(t,x) - \delta v + \sum_{i=1}^n p_i\varphi(t-\tau_i,x)e^{-a\varphi(t-\tau_i,x)}, \quad (t,x)\in D,$$

$$\frac{\partial v}{\partial \nu} \leq 0, \quad (t,x)\in\Gamma \tag{7}$$

for all $\varphi\in C(D_\tau\cup\overline{D})$ with $v\leq\varphi\leq w$, $(t,x)\in D_\tau\cup\overline{D}$, and

(iii) $v(\theta,x)\leq\varphi(\theta,x)\leq w(\theta,x)$, $(\theta,x)\in D_\tau$.

The following lemma is a special case of Redlinger [19].

Lemma 3. *Let (v,w) be a lower-upper solution pair for the initial boundary value problem (4)–(6). Then, there exists a unique regular solution $N(t,x)$ of (4)–(6) such that $v\leq N\leq w$ on $D_\tau\cup\overline{D}$.*

The following lemma gives us boundedness of the solution $N(t,x)$.

Lemma 4. *(i) The solution $N(t,x)$ of (4)-(6) satisfies*

$$\limsup_{t\to\infty} N(t,x) \leq \sum_{i=1}^n \frac{p_i}{ae\delta}, \quad \textit{uniformly in } x. \tag{8}$$

(ii) There exists a constant $K = K(\psi) \geq 0$ such that $N(t,x)\leq K$ on $D_\tau\cup\overline{D}$.

Proof. Let $w(t)$ be the solution of the following Cauchy problem:

$$\frac{dw}{dt} = -\delta w + \sum_{i=1}^n \frac{p_i}{ae}, \quad t>0,$$
$$w(0) = \max_{(\theta,x)\in D_\tau}\psi(\theta,x). \tag{9}$$

Solving the equation, we have

$$w(t) = \sum_{i=1}^n \frac{p_i}{ae\delta} + e^{-\delta t}\left(w(0) - \sum_{i=1}^n\frac{p_i}{ae\delta}\right), \quad t\geq 0. \tag{10}$$

Taking

$$\overline{w}(t) = \begin{cases} w(0), & t\in[-\tau,0], \\ w(t), & t>0, \end{cases} \tag{11}$$

then $(\overline{w}(t),0)$ is a lower-upper solution pair for (4)–(6). In fact, for any $\varphi\in C(D_\tau\cup\overline{D})$ with $0\leq\varphi\leq\overline{w}(t)$, $(t,x)\in D_\tau\cup\overline{D}$, one can get

$$\begin{aligned}&\frac{\partial\overline{w}(t)}{\partial t} - \Delta\overline{w}(t) + \delta\overline{w}(t) - \sum_{i=1}^n p_i\varphi(t-\tau_i,x)e^{-a\varphi(t-\tau_i,x)} \\ &\quad\geq \frac{\partial\overline{w}(t)}{\partial t} + \delta\overline{w}(t) - \sum_{i=1}^n\frac{p_i}{ae} \\ &\quad= \frac{dw}{dt} + \delta w - \sum_{i=1}^n\frac{p_i}{ae} = 0.\end{aligned} \tag{12}$$

By Lemma 3, there is a unique regular solution $N(t,x)$ such that

$$0 \le N(t,x) \le \overline{w}(t), \quad (t,x) \in D_\tau \cup \overline{D}. \tag{13}$$

Note that

$$\lim_{t\to+\infty} \overline{w}(t) = \lim_{t\to+\infty} w(t) = \sum_{i=1}^{n} \frac{p_i}{ae\delta}. \tag{14}$$

Therefore, the formula (8) is correct, and there exists one $K(\psi) > 0$ such that $\overline{w}(t) \le K(\psi)$ for any $t \in (-\tau,\infty)$ and

$$0 \le N(t,x) \le K(\psi), \quad (t,x) \in D_\tau \cup \overline{D}. \tag{15}$$

So we complete Lemma 4. □

3. Main Results and Proofs

Theorem 5. *Assume that $\sum_{i=1}^n p_i \le \delta$, then every solution $N(t,x)$ of (4)–(6) tends to $N_0 = 0$ (uniformly in x) as $t \to +\infty$.*

Proof. By Lemma 4, without loss of generality, let $0 < N(t,x) \le \sum_{i=1}^n (p_i/ae\delta)$ for $(t,x) \in D_\tau \cup \overline{D}$. Under the condition $\sum_{i=1}^n p_i \le \delta$, we can get

$$0 < N(t,x) \le \frac{1}{ae} < \frac{1}{a} \quad \text{for } (t,x) \in D_\tau \cup \overline{D}. \tag{16}$$

Define $m(t)$ and $y(t)$ to be the solutions of the following two delay equations, respectively:

$$\begin{aligned} m'(t) &= -\delta m(t) + \sum_{i=1}^{n} p_i m(t-\tau_i) e^{-am(t-\tau_i)}, \quad t > 0, \\ m(\theta) &= \min_{x\in\overline{\Omega}} \psi(\theta,x), \quad \theta \in [-\tau,0], \\ y'(t) &= -\delta y(t) + \sum_{i=1}^{n} p_i y(t-\tau_i) e^{-ay(t-\tau_i)}, \quad t > 0, \\ y(\theta) &= \max_{x\in\overline{\Omega}} \psi(\theta,x), \quad \theta \in [-\tau,0]. \end{aligned} \tag{17}$$

By using the similar methods to prove Lemma 4, we can get that

$$\limsup_{t\to\infty} m(t) \le \sum_{i=1}^{n} \frac{p_i}{ae\delta} < \frac{1}{a}, \qquad \limsup_{t\to\infty} y(t) \le \sum_{i=1}^{n} \frac{p_i}{ae\delta} < \frac{1}{a} \tag{18}$$

under the condition $\sum_{i=1}^n p_i \le \delta$, and here $m(t)$ and $y(t)$ are the solutions of (17).

Because of $N(t,x) < 1/a$, for any $\varphi \in C(D_\tau \cup \overline{D})$, $m(t) \le \varphi \le y(t) < 1/a$, one can get

$$\begin{aligned} &\frac{\partial m(t)}{\partial t} - \Delta m(t) + \delta m(t) - \sum_{i=1}^{n} p_i \varphi(t-\tau_i,x) e^{-a\varphi(t-\tau_i,x)} \\ &\quad \le \frac{\partial m(t)}{\partial t} + \delta m(t) - \sum_{i=1}^{n} p_i m(t-\tau_i) e^{-am(t-\tau_i)} \\ &\quad = 0, \\ &\frac{\partial y(t)}{\partial t} - \Delta y(t) + \delta y(t) - \sum_{i=1}^{n} p_i \varphi(t-\tau_i,x) e^{-a\varphi(t-\tau_i,x)} \\ &\quad \ge \frac{\partial y(t)}{\partial t} + \delta y(t) - \sum_{i=1}^{n} p_i y(t-\tau_i) e^{-ay(t-\tau_i)} \\ &\quad = 0. \end{aligned} \tag{19}$$

Therefore, from Definition 2, $(m(t), y(t))$ is a lower-upper pair of (4)-(5) with initial condition $m(\theta) \le \psi(\theta,x) \le y(\theta)$ on D_τ. Consequently, by Lemma 3, we have

$$m(t) \le N(t,x) \le y(t) \quad \text{on } [-\tau,+\infty) \times \overline{\Omega}. \tag{20}$$

By Theorem 1 of Luo and Liu [16], it follows from $\sum_{i=1}^n p_i \le \delta$ that the solutions $m(t)$ and $y(t)$ of (17) both satisfy

$$\lim_{t\to\infty} m(t) = 0, \qquad \lim_{t\to\infty} y(t) = 0. \tag{21}$$

Hence, we complete the proof of Theorem 5. □

Theorem 6. *If $1 < \sum_{i=1}^n (p_i/\delta) \le e$, then every nontrivial solution $N(t,x)$ of (4)–(6) satisfies*

$$\lim_{t\to\infty} N(t,x) = N^*, \quad \textit{uniformly in } x. \tag{22}$$

Proof. Let $f(x) = xe^{-ax}$, then the function $f(x)$ is increasing on $(0,(1/a))$ and decreasing on $((1/a),+\infty)$, $f(1/a) = \max_{x\in[0,\infty)} f(x)$, $N^* = (1/a)\ln(\sum_{i=1}^n (p_i/\delta)) \le 1/a$ for $1 < \sum_{i=1}^n (p_i/\delta) \le e$. Let $g(y) = \sum_{i=1}^n p_i f(y)$, then it is not difficult to verify that the function $g(y)$ satisfies the following conditions:

(g_1) the function $g(y)$ is increasing on $(0,(1/a))$ and decreasing on $((1/a),+\infty)$, $\max_{x\in[0,\infty)} g(x) = g(1/a) = \sum_{i=1}^n (p_i/ae)$,

(g_2) $g(y) > \delta y$ for $y \in (0,N^*)$ and $g(y) < \delta y$ for $y \in (N^*,+\infty)$.

There are now two possible cases to consider.

Case 1 ($N^* < 1/a$). In view of Lemma 4, we may also assume without loss of generality that every solution $N(t,x)$ of (4)–(6) satisfies

$$0 \le N(t,x) \le \frac{g(1/a)}{\delta} = \sum_{i=1}^{n} \frac{p_i}{ae\delta} < \frac{1}{a}, \quad \text{on } D_\tau \cup \overline{D}. \tag{23}$$

Let $\underline{N}(t) = \min_{x\in\overline{\Omega}} N(t,x)$, $\overline{N}(t) = \max_{x\in\overline{\Omega}} N(t,x)$, $\underline{N} = \liminf_{t\to\infty}\underline{N}(t)$ and $\overline{N} = \limsup_{t\to\infty}\overline{N}(t)$. By (23), we have

$$0 \le \underline{N} \le \overline{N} \le \frac{g(1/a)}{\delta} = \sum_{i=1}^{n} \frac{p_i}{ae\delta} < \frac{1}{a}. \tag{24}$$

From Lemma 1(ii), let

$$z_0 = \min\left\{ \min_{(t,x)\in[2\tau,\infty)\times\overline{\Omega}} N(t,x), N^* \right\} > 0,$$
$$y_0 = \frac{1}{a}. \tag{25}$$

Let $I_\infty = \{1, 2, \ldots\}$. Now, we define two sequences $\{z_k\}$ and $\{y_k\}$ to satisfy, respectively,

$$z_k = \frac{g(z_{k-1})}{\delta}, \quad k \in I_\infty,$$
$$y_k = \frac{g(y_{k-1})}{\delta}, \quad k \in I_\infty. \tag{26}$$

We prove that $\{z_k\}$ and $\{y_k\}$ are monotonic and bounded. First of all, we prove that $\{z_k\}$ is monotonically increasing, and N^* is the least upper bounded. Note (g_1) and (g_2), we have

$$z_1 = \frac{g(z_0)}{\delta} > z_0, \qquad z_1 = \frac{g(z_0)}{\delta} < \frac{g(N^*)}{\delta} = N^*. \tag{27}$$

By induction and direct computation, we have

$$0 < z_0 < z_1 < \cdots < \lim_{k\to\infty} z_k = N^*. \tag{28}$$

Similarly, we have

$$0 > y_0 > y_1 > \cdots > \lim_{k\to\infty} y_k = N^*. \tag{29}$$

Define $v_1(t)$ and $w_1(t)$ to be the solutions of the following differential equations, respectively:

$$v_1'(t) = -\delta\left[v_1(t) - z_1\right], \quad t \ge 3\tau,$$
$$v_1(\theta) = z_0 < N^*, \quad \theta \in [2\tau, 3\tau],$$
$$w_1'(t) = -\delta\left[w_1(t) - y_1\right], \quad t \ge 3\tau,$$
$$w_1(\theta) = y_0 > N^*, \quad \theta \in [2\tau, 3\tau]. \tag{30}$$

It follows from (24) and (25) that $z_0 \le N(t,x) \le y_0$ for any $(t,x) \in [2\tau,\infty)\times\overline{\Omega}$. Consider (30), for any $(t,x) \in [2\tau,\infty]\times\overline{\Omega}$, we have

$$\frac{\partial v_1(t)}{\partial t} = \Delta v_1(t) - \delta v_1(t) + g(z_0) \le \Delta v_1(t) - \delta v_1(t) + g(N(t-\tau,x)),$$
$$\frac{\partial w_1(t)}{\partial t} = \Delta w_1(t) - \delta w_1(t) + g(y_0) \ge \Delta w_1(t) - \delta w_1(t) + g(N(t-\tau,x)). \tag{31}$$

Therefore, from Definition 2, $(v_1(t), w_1(t))$ is a lower-upper pair of (4)-(5) with initial condition $z_0 \le N(t,x) \le y_0$ on $[2\tau, 3\tau]\times\overline{\Omega}$. Consequently, by Lemma 3, we have

$$v_1(t) \le N(t,x) \le \omega_1(t) \quad \text{on } [2\tau,\infty]\times\overline{\Omega}. \tag{32}$$

Note that $w_1(t)$ is monotonically decreasing for $t \ge 3\tau$ and $\lim_{t\to\infty} w_1(t) = y_1$, while $v_1(t)$ is monotonically increasing for $t \ge 3\tau$ and $\lim_{t\to\infty} v_1(t) = z_1$. Hence,

$$z_1 = \lim_{t\to\infty} v_1(t) \le \underline{N} \le \overline{N} \le \lim_{t\to\infty} w_1(t) = y_1. \tag{33}$$

Define $v_n(t)$ and $w_n(t)$ to be the solutions of the following differential equations, respectively:

$$v_n'(t) = -\delta\left[v_n(t) - z_n\right], \quad t \ge 3\tau,$$
$$v_n(\theta) = z_{n-1} < N^*, \quad \theta \in [2\tau, 3\tau],$$
$$w_n'(t) = -\delta\left[w_n(t) - y_n\right], \quad t \ge 3\tau,$$
$$w_n(\theta) = w_{n-1} < N^*, \quad \theta \in [2\tau, 3\tau]. \tag{34}$$

Repeating the above procedure, we have the following relation:

$$z_1 < z_2 < \cdots < z_n \le \underline{N} \le \overline{N} \le y_n < \cdots < y_2 < y_1. \tag{35}$$

By (28) and (29), and taking limits on both sides of (35), we have

$$N^* = \lim_{n\to\infty} z_n \le \underline{N} \le \overline{N} \le \lim_{n\to\infty} y_n = N^*, \tag{36}$$

which implies

$$\lim_{t\to\infty} N(t,x) = N^*, \quad \text{uniformly in } x. \tag{37}$$

Case 2 ($N^* = y_0$). Similarly, let $y_k = N^*$ and z_k be the same as in the proof of Case 1; we can also get (35). Hence, the proof of Theorem 6 is complete. □

Remark 7. Our main results are also valid when N does not depend on a spatial variable $x \in \Omega$ in (4).

4. Numerical Simulations and Discussion

In this section, we will give some numerical simulations to verify our main results in Section 3 and present several interesting phenomena by simulations that we cannot give a theoretical proof. We just consider the case $n = 2$ in (4).

4.1. Numerical Simulations. Different parameters will be used for simulations, and some data come from [20]. Figure 1 corresponds to the case with $\delta = 0.4$, $p_1 = 0.1$, $p_2 = 0.15$, $a = 0.1$, $\tau_1 = 12$, and $\tau_2 = 15$, and under the above conditions, we have $0 < (p_1 + p_2)/\delta = 0.625 < 1$. We choose the initial condition $\psi(\theta, x) = 1$, $(\theta, x) \in [-15, 0]\times[0,1]$, and the solution $N(t,x)$ is decreasing and almost zero at time 160.

Figure 2 corresponds to the case with $\delta = 0.1$, $p_1 = 0.1$, $p_2 = 0.15$, $a = 0.2$, $\tau_1 = 12$, and $\tau_2 = 15$, and under the above

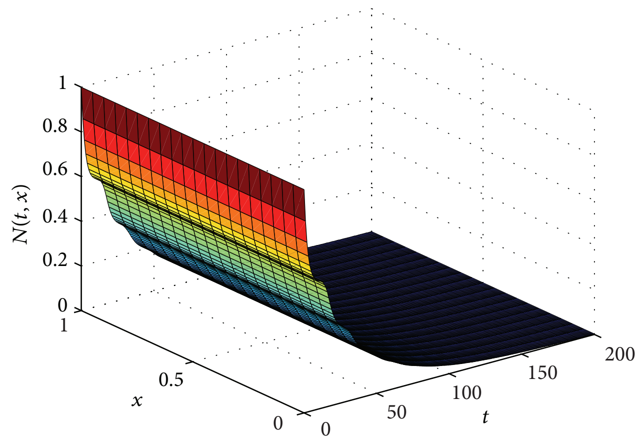

Figure 1: Parameters: $\delta = 0.4$, $p_1 = 0.1$, $p_2 = 0.15$, $a = 0.1$, $\tau_1 = 12$, and $\tau_2 = 15$. Initial condition is $\psi(\theta, x) = 1$, $(\theta, x) \in [-15, 0] \times [0, 1]$.

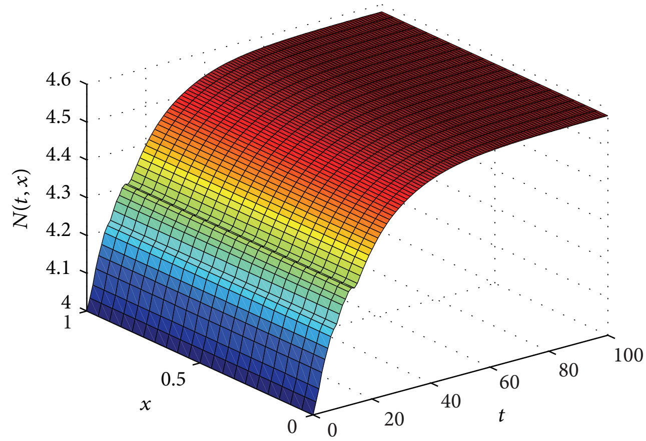

Figure 2: Parameters: $\delta = 0.1$, $p_1 = 0.1$, $p_2 = 0.15$, $a = 0.2$, $\tau_1 = 12$, $\tau_2 = 15$, and $N^* = 4.58145$. Initial condition is $\psi(\theta, x) = 4 + \sin\theta$, $(\theta, x) \in [-15, 0] \times [0, 1]$.

conditions, we have $1 < (p_1 + p_2)/\delta = 2.5 < e$ and $N^* = 4.58145$. Choose the initial condition $\psi(\theta, x) = 4 + \sin\theta$, $(\theta, x) \in [-15, 0] \times [0, 1]$. From Figure 2, we can observe that the solution $N(t, x)$ oscillates around 13 and 14 days; however, $N(t, x)$ tends to N^* as time t tends to 100 days. Therefore, Figures 1 and 2 support our main results (Theorems 5 and 6).

4.2. Discussion. In Section 3, we obtain two main results under the conditions $\sum_{i=1}^{n} (p_i/\delta) \le 1$ and $1 < \sum_{i=1}^{n} (p_i/\delta) \le e$, which are independent of the delays τ_i, $i = 1, 2, \ldots, n$. A natural problem is what will happen when $\sum_{i=1}^{n} (p_i/\delta) > e$ and the delays τ_i, $i = 1, 2, \ldots, n$ are changed.

It is similar to Theorem 3 in Luo and Liu [16]; we present the following open problems.

Open Problem 1. If $\sum_{i=1}^{n} (p_i/\delta) > e$ and $aN^*(e^{\delta\tau} - 1) \le 1$, then every nontrivial solution $N(t, x)$ of (4)–(6) satisfies

$$\lim_{t \to \infty} N(t, x) = N^*, \quad \text{uniformly in } x. \tag{38}$$

Figure 3 corresponds to the case with $\delta = 0.01$, $p_1 = 0.5$, $p_2 = 0.5$, $a = 0.2$, $\tau_1 = 12$, $\tau_2 = 15$, and $N^* = 23.0259$, and initial condition is $\psi(\theta, x) = 10 + \sin\theta$, $(\theta, x) \in [-15, 0] \times [0, 1]$. Under the above conditions, we have $(p_1 + p_2)/\delta = 100 > e$ and $aN^*(e^{\delta\tau} - 1) = 0.745274 < 1$. Sufficient conditions are dependent on coefficients and delay for the global attractivity of equilibria N^*, and Figure 3 shows that the Open Problem 1 is right, but we cannot prove that.

From Figure 4, we have $((p_1 + p_2)/\delta) = 5 > e$ and $aN^*(e^{\delta\tau} - 1) = 30.717 > 1$. The condition is not satisfied, but N^* is still globally attractive.

From Figure 5, we have $((p_1 + p_2)/\delta) = 50 > e$ and $aN^*(e^{\delta\tau} - 1) = 13.6204 > 1$. The condition is not satisfied, but the global attractivity N^* is not true. Moreover, Figure 5 shows that there is a periodic solution, which is very interesting. We guess that the reason is that the system brings Hopf bifurcation as the parameters change. Therefore, we state the following open problem.

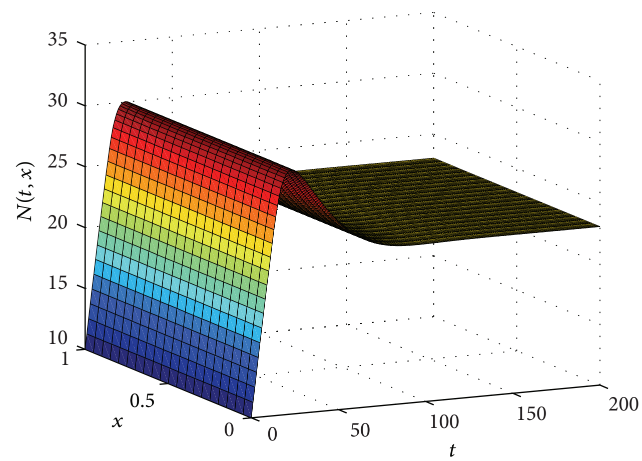

Figure 3: Parameters: $\delta = 0.01$, $p_1 = 0.5$, $p_2 = 0.5$, $a = 0.2$, $\tau_1 = 12$, $\tau_2 = 15$, and $N^* = 23.0259$. Initial condition is $\psi(\theta, x) = 10 + \sin\theta$, $(\theta, x) \in [-15, 0] \times [0, 1]$.

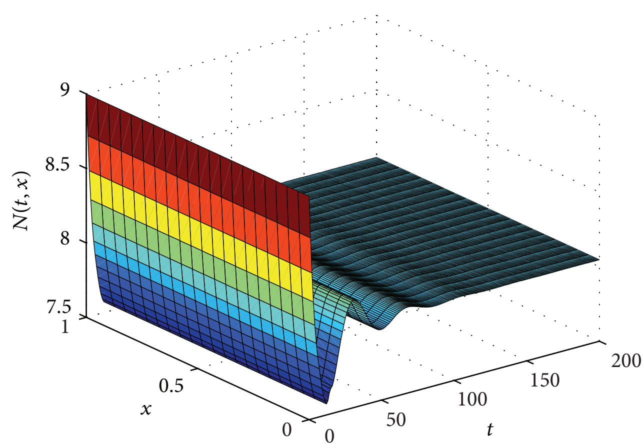

Figure 4: Parameters: $\delta = 0.2$, $p_1 = 0.5$, $p_2 = 0.5$, $a = 0.2$, $\tau_1 = 12$, $\tau_2 = 15$, and $N^* = 8.04719$. Initial condition is $\psi(\theta, x) = 9 + \sin\theta$, $(\theta, x) \in [-15, 0] \times [0, 1]$.

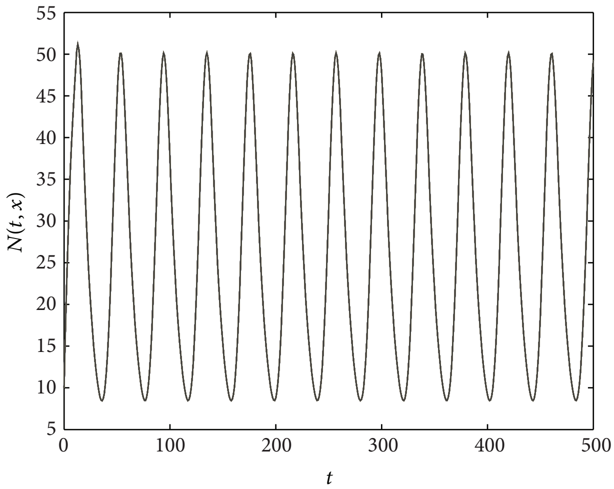

Figure 5: Parameters: $\delta = 0.1$, $p_1 = 3$, $p_2 = 2$, $a = 0.2$, $\tau_1 = 12$, $\tau_2 = 15$, and $N^* = 19.5601$. Initial condition is $\psi(\theta, x) = 10 + \sin\theta$, $(\theta, x) \in [-15, 0] \times [0, 1]$.

Open Problem 2. Under suitable conditions, the systems (4)–(6) will lead to Hopf bifurcation.

Remark 8. Now, we have not intensively studied these two problems. Because the nonmonotonicity of the nonlinear term in (4) makes it very difficult for us to solve Open Problem 1, and we cannot prove Open Problem 2 because of multiple delays.

Acknowledgment

Project was supported by Hunan Provincial Natural Science Foundation of China (12jj4012) and Research Project of National University of Defense Technology (JC12-02-01).

References

[1] A. J. Nicholson, "Competition for food amongst Lucilia Cuprina larvae," in *Proceedings of the 8th International Congress of Entomology*, vol. 15, pp. 277–281, Stockholm, Sweden, 1948.

[2] A. J. Nicholson, "An outline of the dynamics of animal populations," *Australian Journal of Zoology*, vol. 2, pp. 9–65, 1954.

[3] W. S. C. Gurney, S. P. Blythe, and R. M. Nisbet, "Nicholson's blowflies revisited," *Nature*, vol. 287, pp. 17–21, 1980.

[4] L. Berezansky, E. Braverman, and L. Idels, "Nicholson's blowflies differential equations revisited: main results and open problems," *Applied Mathematical Modelling*, vol. 34, no. 6, pp. 1405–1417, 2010.

[5] G. Karakostas, C. G. Philos, and Y. G. Sficas, "Stable steady state of some population models," *Journal of Dynamics and Differential Equations*, vol. 4, no. 1, pp. 161–190, 1992.

[6] M. R. S. Kulenović and G. Ladas, "Linearized oscillations in population dynamics," *Bulletin of Mathematical Biology*, vol. 49, no. 5, pp. 615–627, 1987.

[7] M. R. S. Kulenović, G. Ladas, and Y. G. Sficas, "Global attractivity in Nicholson's blowflies," *Applicable Analysis*, vol. 43, no. 1-2, pp. 109–124, 1992.

[8] J. Li, "Global attractivity in Nicholson's blowflies," *Applied Mathematics B*, vol. 11, no. 4, pp. 425–434, 1996.

[9] J. W.-H. So and J. S. Yu, "Global attractivity and uniform persistence in Nicholson's blowflies," *Differential Equations and Dynamical Systems*, vol. 2, no. 1, pp. 11–18, 1994.

[10] Y. Yang and J. W. H. So, "Dynamics for the diffusive Nicholson blowflies equation," in *Proceedings of the International Conference on Dynamical Systems and Differential Equations*, vol. II, Springfield, USA, 1996.

[11] C.-K. Lin and M. Mei, "On travelling wavefronts of Nicholson's blowflies equation with diffusion," *Proceedings of the Royal Society of Edinburgh A*, vol. 140, no. 1, pp. 135–152, 2010.

[12] S. H. Saker, "Oscillation of continuous and discrete diffusive delay Nicholson's blowflies models," *Applied Mathematics and Computation*, vol. 167, no. 1, pp. 179–197, 2005.

[13] X. Wang and Z. Li, "Dynamics for a type of general reaction-diffusion model," *Nonlinear Analysis. Theory, Methods & Applications A*, vol. 67, no. 9, pp. 2699–2711, 2007.

[14] T. Yi and X. Zou, "Global attractivity of the diffusive Nicholson blowflies equation with Neumann boundary condition: a non-monotone case," *Journal of Differential Equations*, vol. 245, no. 11, pp. 3376–3388, 2008.

[15] I. Györi and G. Ladas, *Oscillation Theory of Delay Differential Equations and Applications*, Clarendon Press, New York, NY, USA, 1991.

[16] J. W. Luo and K. Y. Liu, "Global attractivity of a generalized Nicholson blowfly model," *Hunan Daxue Xuebao*, vol. 23, no. 4, pp. 13–17, 1996.

[17] X. Wang and Z. X. Li, "Oscillations for a diffusive Nicholson blowflies equation with several arguments," *Applied Mathematics A*, vol. 20, no. 3, pp. 265–274, 2005.

[18] R. Redlinger, "On Volterra's population equation with diffusion," *SIAM Journal on Mathematical Analysis*, vol. 16, no. 1, pp. 135–142, 1985.

[19] R. Redlinger, "Existence theorems for semilinear parabolic systems with functionals," *Nonlinear Analysis. Theory, Methods & Applications A*, vol. 8, no. 6, pp. 667–682, 1984.

[20] J. W.-H. So, J. Wu, and Y. Yang, "Numerical steady state and Hopf bifurcation analysis on the diffusive Nicholson's blowflies equation," *Applied Mathematics and Computation*, vol. 111, no. 1, pp. 33–51, 2000.