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Research Article

Exponential Convergence for Cellular Neural Networks with Time-Varying Delays in the Leakage Terms

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We consider a class of cellular neural networks with time-varying delays in the leakage terms. By applying Lyapunov functional method and differential inequality techniques, we establish new results to ensure that all solutions of the networks converge exponentially to zero point.

1. Introduction

It is well known that the delayed cellular neural networks (CNNs) have been successfully applied to signal and image processing, pattern recognition, and optimization (see [1]). Hence, they have been the object of intensive analysis by numerous authors in the past decades. In particular, extensive results on the problem of the existence and stability of the equilibrium point for CNNs are given out in many works in the literature. We refer the reader to [2–6] and the references cited therein. Recently, to consider CNNs with the incorporation of time delays in the leakage terms, Gopalsamy [7] and Wang et al. [8] investigated a class of CNNS described by

$$\begin{aligned} x_i'(t) = & -c_i(t)x_i\left(t-\eta_i(t)\right)+\sum_{j=1}^{n}a_{ij}(t)f_j\left(x_j\left(t-\tau_{ij}(t)\right)\right) \\ & +\sum_{j=1}^{n}b_{ij}(t)\int_0^{\infty}K_{ij}(u)g_j\left(x_j(t-u)\right)du+I_i(t), \quad i=1,2,\ldots,n, \end{aligned} \tag{1.1}$$

in which n corresponds to the number of units in a neural network, $x_i(t)$ corresponds to the state vector of the ith unit at the time t, and $c_i(t)$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs at the time t. $a_{ij}(t)$ and $b_{ij}(t)$ are the connection weights at the time t, $\eta_i(t) \geq 0$ and $\tau_{ij}(t) \geq 0$ denote the leakage delay and transmission delay, respectively, $I_i(t)$ denotes the external bias on the ith unit at the time t, f_j and g_j are activation functions of signal transmission, and $i, j = 1, 2, \ldots, n$.

Suppose that the following conditions

(H_0) c_i and η_i are constants, where $i = 1, 2, \ldots, n$,

(H_0^*) for each $j \in \{1, 2, \ldots, n\}$, there exists a nonnegative constant $\widetilde{L}_j$ such that

$$\left|f_j(u) - f_j(v)\right| \leq \widetilde{L}_j |u - v|, \quad \forall u, v \in R, \tag{1.2}$$

are satisfied. Avoiding the continuously distributed delay terms, the authors of [7, 8] obtained that all solutions of system (1.1) converge to the equilibrium point or the periodic solution. However, to the best of our knowledge, few authors have considered the convergence behavior for all solutions of system (1.1) without the assumptions (H_0) and (H_0^*). Thus, it is worthwhile to continue to investigate the convergence behavior of system (1.1) in this case.

The main purpose of this paper is to give the new criteria for the convergence behavior for all solutions of system (1.1). By applying Lyapunov functional method and differential inequality techniques, without assuming (H_0) and (H_0^*), we derive some new sufficient conditions ensuring that all solutions of system (1.1) converge exponentially to zero point. Moreover, an example is also provided to illustrate the effectiveness of our results.

Throughout this paper, for $i, j = 1, 2, \ldots, n$, it will be assumed that c_i, I_i, η_i, a_{ij}, b_{ij}, τ_{ij} : $R \to R$ and K_{ij} : $[0, +\infty) \to R$ are continuous functions, and there exist constants c_i^+, η_i^+, a_{ij}^+, b_{ij}^+ and τ_{ij}^+ such that

$$\begin{gathered} c_i^+ = \sup_{t \in R} c_i(t), \qquad \eta_i^+ = \sup_{t \in R} \eta_i(t), \qquad a_{ij}^+ = \sup_{t \in R} \left|a_{ij}(t)\right|, \\ b_{ij}^+ = \sup_{t \in R} \left|b_{ij}(t)\right|, \qquad \tau_{ij}^+ = \sup_{t \in R} \tau_{ij}(t). \end{gathered} \tag{1.3}$$

We also assume that the following conditions (H_1), (H_2), and (H_3) hold:

(H_1) for each $i, j \in \{1, 2, \ldots, n\}$, there exist nonnegative constants $\widetilde{L}_j$ and L_j such that

$$\left|f_j(u)\right| \leq \widetilde{L}_j |u|, \quad \left|g_j(u)\right| \leq L_j |u|, \quad \forall u \in R, \tag{1.4}$$

(H_2) for all $t > 0$ and $i, j \in \{1, 2, \dots, n\}$, there exist constants $\eta > 0,\ \lambda > 0$ and $\xi_i > 0$ such that

$$\begin{aligned}
&\int_0^{\infty} \left|K_{ij}(u)\right| e^{\lambda u} du < +\infty, \\
-\eta > &- \left[c_i(t) - \lambda e^{-\lambda \eta_i(t)} - \eta_i(t) c_i(t) \left(\lambda + c_i^+ e^{\lambda \eta_i^+}\right)\right] e^{\lambda \eta_i(t)} \xi_i \\
&+ \sum_{j=1}^{n} \widetilde{L}_j \left(\left|a_{ij}(t)\right| e^{\lambda \tau_{ij}(t)} + a_{ij}^+ \eta_i(t) c_i(t) e^{\lambda \eta_i(t)} e^{\lambda \tau_{ij}^+}\right) \xi_j \\
&+ \sum_{j=1}^{n} L_j \int_0^{\infty} \left|K_{ij}(u)\right| e^{\lambda u} du \left(\left|b_{ij}(t)\right| + b_{ij}^+ \eta_i(t) c_i(t) e^{\lambda \eta_i(t)}\right) \xi_j;
\end{aligned} \tag{1.5}$$

(H_3) $I_i(t) = O(e^{-\lambda t})\ (t \to \pm\infty),\ i = 1, 2, \dots, n.$

The initial conditions associated with system (1.1) are of the form

$$x_i(s) = \varphi_i(s), \quad s \in (-\infty, 0],\ i = 1, 2, \dots, n, \tag{1.6}$$

where $\varphi_i(\cdot)$ denotes real-valued-bounded continuous function defined on $(-\infty, 0]$.

2. Main Results

Theorem 2.1. *Let (H_1), (H_2), and (H_3) hold. Then, for every solution $Z(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ of system (1.1) with any initial value $\varphi = (\varphi_1(t), \varphi_2(t), \dots, \varphi_n(t))^T$, there exists a positive constant K such that*

$$|x_i(t)| \le K \xi_i e^{-\lambda t} \quad \forall t > 0,\ i = 1, 2, \dots, n. \tag{2.1}$$

Proof. Let $Z(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ be a solution of system (1.1) with any initial value $\varphi = (\varphi_1(t), \varphi_2(t), \dots, \varphi_n(t))^T$, and let

$$X_i(t) = e^{\lambda t} x_i(t), \qquad i = 1, 2, \dots, n. \tag{2.2}$$

In view of (1.1), we have

$$X_i'(t) = \lambda X_i(t) + e^{\lambda t}\Bigg[-c_i(t)x_i\big(t-\eta_i(t)\big) + \sum_{j=1}^{n} a_{ij}(t) f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) + \sum_{j=1}^{n} b_{ij}(t)\int_0^{\infty} K_{ij}(u) g_j\big(x_j(t-u)\big)du + I_i(t)\Bigg]$$
$$= \lambda X_i(t) - c_i(t)e^{\lambda\eta_i(t)}X_i\big(t-\eta_i(t)\big) + e^{\lambda t}\Bigg[\sum_{j=1}^{n} a_{ij}(t) f_j\Big(e^{-\lambda(t-\tau_{ij}(t))}X_j\big(t-\tau_{ij}(t)\big)\Big) + \sum_{j=1}^{n} b_{ij}(t)\int_0^{\infty} K_{ij}(u) g_j\Big(e^{-\lambda(t-u)}X_j(t-u)\Big)du + I_i(t)\Bigg], \quad i = 1, 2, \ldots, n. \tag{2.3}$$

Let

$$M = \max_{i=1,2,\ldots,n}\sup_{s\le 0}\Big\{e^{\lambda s}\big|\varphi_i(s)\big|\Big\}. \tag{2.4}$$

From (1.3), (H_2), and (H_3), we can choose a positive constant K such that

$$K\xi_i > M, \qquad \eta > \frac{\big[\eta_i(t)c_i(t)e^{\lambda\eta_i(t)} + 1\big]\sup_{t\in R}\big|I_i(t)e^{\lambda t}\big|}{K}, \quad \forall t > 0,\ i = 1, 2, \ldots, n. \tag{2.5}$$

Then, it is easy to see that

$$|X_i(t)| \le M < K\xi_i \quad \forall t \le 0,\ i = 1, 2, \ldots, n. \tag{2.6}$$

We now claim that

$$|X_i(t)| < K\xi_i \quad \forall t > 0,\ i = 1, 2, \ldots, n. \tag{2.7}$$

If this is not valid, then, one of the following two cases must occur:

(1) there exist $i \in \{1, 2, \ldots, n\}$ and $t^* > 0$ such that

$$X_i(t^*) = K\xi_i, \quad \big|X_j(t)\big| < K\xi_j \quad \forall t < t^*,\ j = 1, 2, \ldots, n, \tag{2.8}$$

(2) there exist $i \in \{1, 2, \ldots, n\}$ and $t^{**} > 0$ such that

$$X_i(t^{**}) = -K\xi_i, \quad \big|X_j(t)\big| < K\xi_j \quad \forall t < t^{**},\ j = 1, 2, \ldots, n. \tag{2.9}$$

Now, we consider two cases.

Case i. If (2.8) holds. Then, from (2.3), (2.5), and (H_1)–(H_3), we have

$$\begin{aligned}
0 &\le X_i'(t^*)\\
&= \lambda X_i(t^*) - c_i(t^*)e^{\lambda\eta_i(t^*)}X_i\left(t^* - \eta_i(t^*)\right)\\
&\quad + e^{\lambda t^*}\left[\sum_{j=1}^{n} a_{ij}(t^*)f_j\left(e^{-\lambda(t^*-\tau_{ij}(t^*))}X_j\left(t^*-\tau_{ij}(t^*)\right)\right)\right.\\
&\qquad\left. + \sum_{j=1}^{n} b_{ij}(t^*)\int_0^{\infty} K_{ij}(u)g_j\left(e^{-\lambda(t^*-u)}X_j(t^*-u)\right)du + I_i(t^*)\right]\\
&= \lambda X_i(t^*) - c_i(t^*)e^{\lambda\eta_i(t^*)}X_i(t^*) + c_i(t^*)e^{\lambda\eta_i(t^*)}\left[X_i(t^*) - X_i\left(t^*-\eta_i(t^*)\right)\right]\\
&\quad + e^{\lambda t^*}\left[\sum_{j=1}^{n} a_{ij}(t^*)f_j\left(e^{-\lambda(t^*-\tau_{ij}(t^*))}X_j\left(t^*-\tau_{ij}(t^*)\right)\right)\right.\\
&\qquad\left. + \sum_{j=1}^{n} b_{ij}(t^*)\int_0^{\infty} K_{ij}(u)g_j\left(e^{-\lambda(t^*-u)}X_j(t^*-u)\right)du + I_i(t^*)\right]\\
&= -\left[c_i(t^*)e^{\lambda\eta_i(t^*)} - \lambda\right]X_i(t^*) + c_i(t^*)e^{\lambda\eta_i(t^*)}\int_{t^*-\eta_i(t^*)}^{t^*} X_i'(s)ds\\
&\quad + e^{\lambda t^*}\left[\sum_{j=1}^{n} a_{ij}(t^*)f_j\left(e^{-\lambda(t^*-\tau_{ij}(t^*))}X_j\left(t^*-\tau_{ij}(t^*)\right)\right)\right.\\
&\qquad\left. + \sum_{j=1}^{n} b_{ij}(t^*)\int_0^{\infty} K_{ij}(u)g_j\left(e^{-\lambda(t^*-u)}X_j(t^*-u)\right)du + I_i(t^*)\right]\\
&= -\left[c_i(t^*)e^{\lambda\eta_i(t^*)} - \lambda\right]X_i(t^*)\\
&\quad + c_i(t^*)e^{\lambda\eta_i(t^*)}\int_{t^*-\eta_i(t^*)}^{t^*}\left[\lambda X_i(s) - c_i(s)e^{\lambda\eta_i(s)}X_i\left(s-\eta_i(s)\right)\right.\\
&\qquad + e^{\lambda s}\left(\sum_{j=1}^{n} a_{ij}(s)f_j\left(e^{-\lambda(s-\tau_{ij}(s))}X_j\left(s-\tau_{ij}(s)\right)\right)\right.\\
&\qquad\left.\left. + \sum_{j=1}^{n} b_{ij}(s)\int_0^{\infty} K_{ij}(u)g_j\left(e^{-\lambda(s-u)}X_j(s-u)\right)du + I_i(s)\right)\right]ds\\
&\quad + e^{\lambda t^*}\left[\sum_{j=1}^{n} a_{ij}(t^*)f_j\left(e^{-\lambda(t^*-\tau_{ij}(t^*))}X_j\left(t^*-\tau_{ij}(t^*)\right)\right)\right.\\
&\qquad\left. + \sum_{j=1}^{n} b_{ij}(t^*)\int_0^{\infty} K_{ij}(u)g_j\left(e^{-\lambda(t^*-u)}X_j(t^*-u)\right)du + I_i(t^*)\right]
\end{aligned}$$

$$
\begin{aligned}
&\le -\left[c_i(t^*)e^{\lambda\eta_i(t^*)} - \lambda - \eta_i(t^*)c_i(t^*)e^{\lambda\eta_i(t^*)}\left(\lambda + c_i^+ e^{\lambda\eta_i^+}\right)\right]\xi_i K \\
&\quad + \sum_{j=1}^{n}\widetilde{L}_j\left(\left|a_{ij}(t^*)\right|e^{\lambda\tau_{ij}(t^*)} + a_{ij}^+\eta_i(t^*)c_i(t^*)e^{\lambda\eta_i(t^*)}e^{\lambda\tau_{ij}^+}\right)\xi_j K \\
&\quad + \sum_{j=1}^{n}L_j\int_0^{\infty}\left|K_{ij}(u)\right|e^{\lambda u}du\left(\left|b_{ij}(t^*)\right| + b_{ij}^+\eta_i(t^*)c_i(t^*)e^{\lambda\eta_i(t^*)}\right)\xi_j K \\
&\quad + \left[\eta_i(t^*)c_i(t^*)e^{\lambda\eta_i(t^*)} + 1\right]\sup\nolimits_{t\in R}\left|I_i(t)e^{\lambda t}\right| \\
&= \Bigg\{ -\left[c_i(t^*) - \lambda e^{-\lambda\eta_i(t^*)} - \eta_i(t^*)c_i(t^*)\left(\lambda + c_i^+ e^{\lambda\eta_i^+}\right)\right]e^{\lambda\eta_i(t^*)}\xi_i \\
&\qquad + \sum_{j=1}^{n}\widetilde{L}_j\left(\left|a_{ij}(t^*)\right|e^{\lambda\tau_{ij}(t^*)} + a_{ij}^+\eta_i(t^*)c_i(t^*)e^{\lambda\eta_i(t^*)}e^{\lambda\tau_{ij}^+}\right)\xi_j \\
&\qquad + \sum_{j=1}^{n}L_j\int_0^{\infty}\left|K_{ij}(u)\right|e^{\lambda u}du\left(\left|b_{ij}(t^*)\right| + b_{ij}^+\eta_i(t^*)c_i(t^*)e^{\lambda\eta_i(t^*)}\right)\xi_j \\
&\qquad + \frac{\left[\eta_i(t^*)c_i(t^*)e^{\lambda\eta_i(t^*)} + 1\right]\sup_{t\in R}\left|I_i(t)e^{\lambda t}\right|}{K}\Bigg\}K \\
&< \left\{-\eta + \frac{\left[\eta_i(t^*)c_i(t^*)e^{\lambda\eta_i(t^*)} + 1\right]\sup_{t\in R}\left|I_i(t)e^{\lambda t}\right|}{K}\right\}K \\
&< 0.
\end{aligned}
\tag{2.10}
$$

This contradiction implies that (2.8) does not hold.

Case ii. If (2.9) holds. Then, from (2.3), (2.5), and (H_1)–(H_3), we get

$$
\begin{aligned}
0 &\ge X_i'(t^{**}) \\
&= -\left[c_i(t^{**})e^{\lambda\eta_i(t^{**})} - \lambda\right]X_i(t^{**}) \\
&\quad + c_i(t^{**})e^{\lambda\eta_i(t^{**})}\int_{t^{**}-\eta_i(t^{**})}^{t^{**}}\Bigg[\lambda X_i(s) - c_i(s)e^{\lambda\eta_i(s)}X_i\left(s-\eta_i(s)\right) \\
&\qquad + e^{\lambda s}\Bigg(\sum_{j=1}^{n}a_{ij}(s)f_j\left(e^{-\lambda(s-\tau_{ij}(s))}X_j\left(s-\tau_{ij}(s)\right)\right) \\
&\qquad + \sum_{j=1}^{n}b_{ij}(s)\int_0^{\infty}K_{ij}(u)g_j\left(e^{-\lambda(s-u)}X_j(s-u)\right)du + I_i(s)\Bigg)\Bigg]ds
\end{aligned}
$$

$$
\begin{aligned}
&\quad + e^{\lambda t^{**}}\Bigg[\sum_{j=1}^{n} a_{ij}(t^{**}) f_j\left(e^{-\lambda(t^{**}-\tau_{ij}(t^{**}))} X_j(t^{**}-\tau_{ij}(t^{**}))\right) \\
&\qquad + \sum_{j=1}^{n} b_{ij}(t^{**}) \int_0^\infty K_{ij}(u) g_j\left(e^{-\lambda(t^{**}-u)} X_j(t^{**}-u)\right) du + I_i(t^{**})\Bigg] \\
&\geq -\left[c_i(t^{**}) e^{\lambda\eta_i(t^{**})} - \lambda - \eta_i(t^{**}) c_i(t^{**}) e^{\lambda\eta_i(t^{**})}\left(\lambda + c_i^+ e^{\lambda\eta_i^+}\right)\right](-\xi_i K) \\
&\quad + \sum_{j=1}^{n} \widetilde{L}_j\left(\left|a_{ij}(t^{**})\right| e^{\lambda\tau_{ij}(t^{**})} + a_{ij}^+ \eta_i(t^{**}) c_i(t^{**}) e^{\lambda\eta_i(t^{**})} e^{\lambda\tau_{ij}^+}\right)(-\xi_j K) \\
&\quad + \sum_{j=1}^{n} L_j \int_0^\infty \left|K_{ij}(u)\right| e^{\lambda u} du\left(\left|b_{ij}(t^{**})\right| + b_{ij}^+ \eta_i(t^{**}) c_i(t^{**}) e^{\lambda\eta_i(t^{**})}\right)(-\xi_j K) \\
&\quad - \left[\eta_i(t^{**}) c_i(t^{**}) e^{\lambda\eta_i(t^{**})} + |I_i(t^{**})|\right] e^{\lambda t^{**}} \\
&= \Bigg\{ -\left[c_i(t^{**}) - \lambda e^{-\lambda\eta_i(t^{**})} - \eta_i(t^{**}) c_i(t^{**})\left(\lambda + c_i^+ e^{\lambda\eta_i^+}\right)\right] e^{\lambda\eta_i(t^{**})} \xi_i \\
&\qquad + \sum_{j=1}^{n} \widetilde{L}_j\left(\left|a_{ij}(t^{**})\right| e^{\lambda\tau_{ij}(t^{**})} + a_{ij}^+ \eta_i(t^{**}) c_i(t^{**}) e^{\lambda\eta_i(t^{**})} e^{\lambda\tau_{ij}^+}\right)\xi_j \\
&\qquad + \sum_{j=1}^{n} L_j \int_0^\infty \left|K_{ij}(u)\right| e^{\lambda u} du\left(\left|b_{ij}(t^{**})\right| + b_{ij}^+ \eta_i(t^{**}) c_i(t^{**}) e^{\lambda\eta_i(t^{**})}\right)\xi_j \\
&\qquad + \frac{[\eta_i(t^{**}) c_i(t^{**}) e^{\lambda\eta_i(t^{**})} + 1] \sup_{t\in R}\left|I_i(t) e^{\lambda t}\right|}{K}\Bigg\}(-K) \\
&> \left\{-\eta + \frac{[\eta_i(t^{**}) c_i(t^{**}) e^{\lambda\eta_i(t^{**})} + 1] \sup_{t\in R}\left|I_i(t) e^{\lambda t}\right|}{K}\right\}(-K) \\
&> 0,
\end{aligned}
\tag{2.11}
$$

which is a contradiction and yields that (2.9) does not hold.

Consequently, we can obtain that (2.7) is true. Thus,

$$
|x_i(t)| \leq K\xi_i e^{-\lambda t} \quad \forall t > 0,\ i = 1, 2, \ldots, n. \tag{2.12}
$$

This implies that the proof of Theorem 2.1 is now completed. □

3. An Example

Example 3.1. Consider the following CNNs with time-varying delays in the leakage terms:

$$\begin{aligned}
x_1'(t) = &-\left(20-\frac{(1+|t|)\sin^2 t}{1+2|t|}\right)x_1\left(t-\frac{1+|\sin t|}{2000}\right)+\frac{|t|^3\sin t}{1+4|t|^3}f_1\left(x_1\left(t-2\sin^2 t\right)\right)\\
&+\frac{|t|^5\sin t}{1+36|t|^5}f_2\left(x_2\left(t-3\sin^2 t\right)\right)+\frac{|t|^7\sin t}{1+4|t|^7}\int_0^\infty e^{-u}g_1(x_1(t-u))du\\
&+\frac{t^2\sin t}{1+36t^2}\int_0^\infty e^{-u}g_2(x_2(t-u))du+e^{-3t}\sin t,\\
x_2'(t) = &-\left(40-\frac{\left(1+|t|^7\right)\cos^2 t}{1+2|t|^7}\right)x_2\left(t-\frac{1+|\cos t|}{2000}\right)+\frac{t^5\cos t}{1+2|t|^5}f_1\left(x_1\left(t-2\sin^2 t\right)\right)\\
&+\frac{t\cos t}{1+5|t|}f_2\left(x_2\left(t-5\sin^2 t\right)\right)+\frac{|t|^3\cos t}{1+6|t|^3}\int_0^\infty e^{-u}g_1(x_1(t-u))du\\
&+\frac{(1+|t|)\cos t}{7+7|t|}\int_0^\infty e^{-u}g_2(x_2(t-u))du+e^{-t}\sin t,
\end{aligned} \tag{3.1}$$

where $f_1(x)=f_2(x)=x\cos(x^3)$, $g_1(x)=g_2(x)=x\sin(x^2)$.

Noting that

$$\begin{gathered}
18\le c_1(t)=20-\frac{(1+|t|)\sin^2 t}{1+2|t|}\le 20,\qquad 38\le c_2(t)=40-\frac{\left(1+|t|^7\right)\cos^2 t}{1+2|t|^7}\le 40,\\
\eta_1(t)=\frac{1+|\sin t|}{2000}\le\frac{1}{1000},\qquad \eta_2(t)=\frac{1+|\cos t|}{2000}\le\frac{1}{1000},\\
a_{11}(t)=\frac{|t|^3\sin t}{1+4|t|^3},\qquad b_{11}(t)=\frac{|t|^7\sin t}{1+4|t|^7},\qquad a_{12}(t)=\frac{|t|^5\sin t}{1+36|t|^5},\\
b_{12}(t)=\frac{t^2\sin t}{1+36t^2},\qquad a_{21}(t)=\frac{t^5\cos t}{1+2|t|^5},\qquad b_{21}(t)=\frac{|t|^3\cos t}{1+6|t|^3},\\
a_{22}(t)=\frac{t\cos t}{1+5|t|},\qquad b_{22}(t)=\frac{(1+|t|)\cos t}{7+7|t|},\qquad \tau_{11}(t)=\tau_{21}(t)=2\sin^2 t,\\
\tau_{12}(t)=3\sin^2 t,\qquad \tau_{22}(t)=5\sin^2 t,\qquad L_i=\widetilde{L}_i=1,\qquad K_{ij}(u)=e^{-u},\quad i,\ j=1,2.
\end{gathered} \tag{3.2}$$

Define a continuous function $\Gamma_i(\omega)$ by setting

$$\begin{aligned}
\Gamma_i(\omega) = & -\left[c_i(t) - \omega e^{-\omega\eta_i(t)} - \eta_i(t)c_i(t)\left(\omega + c_i^+ e^{\omega\eta_i^+}\right)\right]e^{\omega\eta_i(t)} \\
& + \sum_{j=1}^{2}\widetilde{L}_j\left(\left|a_{ij}(t)\right|e^{\omega\tau_{ij}(t)} + a_{ij}^+\eta_i(t)c_i(t)e^{\omega\eta_i(t)}e^{\omega\tau_{ij}^+}\right) \\
& + \sum_{j=1}^{2}L_j\int_0^\infty\left|K_{ij}(u)\right|e^{\omega u}du\left(\left|b_{ij}(t)\right| + b_{ij}^+\eta_i(t)c_i(t)e^{\omega\eta_i(t)}\right), \quad \text{where } t > 0,\ i = 1,2.
\end{aligned} \tag{3.3}$$

Then, we obtain

$$\begin{aligned}
\Gamma_i(0) = & -\left[c_i(t) - \eta_i(t)c_i(t)c_i^+\right] + \sum_{j=1}^{2}\widetilde{L}_j\left(\left|a_{ij}(t)\right| + a_{ij}^+\eta_i(t)c_i(t)\right) \\
& + \sum_{j=1}^{2}L_j\int_0^\infty\left|K_{ij}(u)\right|du\left(\left|b_{ij}(t)\right| + b_{ij}^+\eta_i(t)c_i(t)\right), \quad i = 1,2.
\end{aligned} \tag{3.4}$$

Therefore,

$$\begin{aligned}
\Gamma_1(0) \le & -\left(18 - \frac{1}{1000}\times 20 \times 20\right) + 2\left[\left(\frac{1}{4} + \frac{1}{4}\times\frac{1}{1000}\times 20\right) + \left(\frac{1}{36} + \frac{1}{36}\times\frac{1}{1000}\times 20\right)\right], \\
< & -10, \\
\Gamma_2(0) \le & -\left(38 - \frac{1}{1000}\times 40 \times 40\right) + \left[\left(\frac{1}{2} + \frac{1}{2}\times\frac{1}{1000}\times 40\right) + \left(\frac{1}{5} + \frac{1}{5}\times\frac{1}{1000}\times 40\right)\right] \\
& + \left[\left(\frac{1}{6} + \frac{1}{6}\times\frac{1}{1000}\times 40\right) + \left(\frac{1}{7} + \frac{1}{7}\times\frac{1}{1000}\times 40\right)\right] \\
< & -20,
\end{aligned} \tag{3.5}$$

which, together with the continuity of $\Gamma_i(\omega)$, implies that we can choose positive constants $\lambda > 0$ and $\eta > 0$ such that for all $t > 0$, there holds

$$\begin{aligned}
& -\eta > \Gamma_i(\lambda) \\
& \quad = -\left[c_i(t) - \lambda - \eta_i(t)c_i(t)\left(\lambda + c_i^+ e^{\lambda\eta_i^+}\right)\right]e^{\lambda\eta_i(t)}\xi_i
\end{aligned}$$

$$+\sum_{j=1}^{2}\widetilde{L}_j\Big(\left|a_{ij}(t)\right|e^{\lambda\tau_{ij}(t)}+a_{ij}^{+}\eta_i(t)c_i(t)e^{\lambda\eta_i(t)}e^{\lambda\tau_{ij}^{+}}\Big)\xi_j$$
$$+\sum_{j=1}^{2}L_j\int_0^{\infty}\left|K_{ij}(u)\right|e^{\lambda u}du\Big(\left|b_{ij}(t)\right|+b_{ij}^{+}\eta_i(t)c_i(t)e^{\lambda\eta_i(t)}\Big)\xi_j,\quad \text{where } \xi_i=1,\ i=1,2. \tag{3.6}$$

This yields that system (3.1) satisfied (H_1), (H_2), and (H_3). Hence, from Theorem 2.1, all solutions of system (3.1) converge exponentially to the zero point $(0,0,\ldots,0)^T$.

Remark 3.2. Since $f_1(x)=f_2(x)=x\cos(x^3)$, $g_1(x)=g_2(x)=x\sin(x^2)$, and CNNs (3.1) are a very simple form of CNNs with time-varying delays in the leakage terms, it is clear that the conditions (H_0) and (H_0^*) are not satisfied. Therefore, all the results in [7–9] and the references therein cannot be applicable to prove that all solutions of system (3.1) converge exponentially to the zero point.

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