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Research Article

Positive Solutions of a Nonlinear Fourth-Order Dynamic Eigenvalue Problem on Time Scales

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Let $\mathbb{T}$ be a time scale and $a, b \in \mathbb{T}$, $a < \rho^2(b)$. We study the nonlinear fourth-order eigenvalue problem on $\mathbb{T}$, $u^{\Delta^4}(t) = \lambda h(t) f(u(t), u^{\Delta^2}(t))$, $t \in [a, \rho^2(b)]_{\mathbb{T}}$, $u(a) = u^{\Delta}(\sigma(b)) = u^{\Delta^2}(a) = u^{\Delta^3}(\rho(b)) = 0$ and obtain the existence and nonexistence of positive solutions when $0 < \lambda \le \lambda^*$ and $\lambda > \lambda^*$, respectively, for some λ^*. The main tools to prove the existence results are the Schauder fixed point theorem and the upper and lower solution method.

1. Introduction

The deformation of an elastic beam with one end fixed and the other end free can be described by the nonlinear fourth-order dynamic eigenvalue problem on $\mathbb{T}$

$$\begin{gathered} u^{\Delta^4}(t) = \lambda h(t) f\left(u(t), u^{\Delta^2}(t)\right), \quad t \in \left[a, \rho^2(b)\right]_{\mathbb{T}}, \\ u(a) = u^{\Delta}(\sigma(b)) = u^{\Delta^2}(a) = u^{\Delta^3}\left(\rho(b)\right) = 0, \end{gathered} \tag{1.1}$$

where $\mathbb{T}$ is a time scale, $\lambda > 0$ is a parameter, $a, b \in \mathbb{T}$, and $a < \rho^2(b)$.

Nonlinear dynamic eigenvalue problems of the above type have been studied by some authors, but most of them study only second-order dynamic equations. In 2000, Chyan and Henderson [1] obtained the existence of at least one positive solution for some λ to second-order case of the dynamic equation in problem (1.1) under conjugate boundary value condition and right focal boundary value condition, respectively. Anderson [2] discussed the same second-order dynamic equation under the Sturm-Liouville boundary value condition and directly generalized the result of [1]. Erbe et al. [3] then studied the general second-order Sturm-Liouville dynamic boundary value problem and obtained the existence, nonexistence, and multiplicity results of positive solutions. In 2005, Li and Liu [4] further studied the dependence of positive solutions on the parameter $\lambda > 0$ for the second-order dynamic equation under the right focal boundary value condition. Luo and Ma [5] in 2006 were concerned with the existence and multiplicity of nodal solutions and obtained eigenvalue intervals of the nonlinear second-order dynamic eigenvalue problem under conjugate boundary value condition by using bifurcation methods. In 2007, Sun et al. [6] obtained some sufficient conditions for the nonexistence and existence of at least one or two positive solutions for the p-Laplacian three-point dynamic eigenvalue problem with mixed derivatives by using the Krasnosel'skii's fixed point theorem in a cone. In 2009, Luo [7] derived the eigenvalue intervals in which there exist positive solutions of a singular second-order multipoint dynamic eigenvalue problem with mixed derivatives by making use of the fixed point index theory.

As for the nonlinear higher-order dynamic eigenvalue problems, few papers can be found in the literature to the best of our knowledge. L. Kong and Q. Kong [8], and Boey and Wong [9] discussed the even-order dynamic eigenvalue problem and the right focal eigenvalue problem, respectively, but their problems do not contain (1.1). Particularly for fourth-order problems and special case $\lambda \equiv 1$, Wang and Sun [10] studied the existence of positive solutions for dynamic equations under nonhomogeneous boundary-value conditions describing an elastic beam that is simply supported at its two ends. And both Karaca [11] and Pang and Bai [12] obtained the existence of a solution for two classes of fourth-order four-point problems on time scales by the Leray-Schauder fixed point theorem and the upper and lower solution method, respectively, but the problems they studied are different to (1.1).

This paper studies the relationship between the existence and nonexistence of positive solutions and the value of parameter $\lambda > 0$. We find the existence of a λ^* such that problem (1.1) has positive solutions for $0 < \lambda \leq \lambda^*$ and no positive solutions for $\lambda > \lambda^*$.

The rest of this paper is organized as follows: in Section 2, we firstly introduce the time scales concepts and notations and present some basic properties on time scales which are needed later. Next, Section 3 gives some preliminary results relevant to our discussion, and Section 4 is devoted to establish our main theorems.

2. Introduction to Time Scales

The calculus theory on time scales, which unifies continuous and discrete analysis, is now still an active area of research. We refer the reader to [13–16] and the references therein for introduction on this theory. For the convenience of readers, we present some necessary definitions and results here.

A time scale $\mathbb{T}$ is a nonempty closed subset of $\mathbb{R}$, assuming that $\mathbb{T}$ has the topology that it inherits from the standard topology on $\mathbb{R}$. Define the forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ by

$$\sigma(t) = \inf\{\tau > t \mid \tau \in \mathbb{T}\}, \qquad \rho(t) = \sup\{\tau < t \mid \tau \in \mathbb{T}\}. \tag{2.1}$$

Here we put $\inf \emptyset = \sup \mathbb{T}$, $\sup \emptyset = \inf \mathbb{T}$. Let $\mu(t) = \sigma(t) - t$, $t \in \mathbb{T}$ be the graininess function. And $\mathbb{T}^k$ which are derived from the time scale $\mathbb{T}$ is

$$\mathbb{T}^k := \{t \in \mathbb{T} : \ t \text{ is nonmaximal or } \rho(t) = t\}, \tag{2.2}$$

and $\mathbb{T}^{k^n} := (\mathbb{T}^{k^{n-1}})^k, n > 1, n \in \mathbb{N}$. Define interval I on $\mathbb{T}$ by $I_{\mathbb{T}} = I \cap \mathbb{T}$.

Definition 2.1. If $u : \mathbb{T} \to \mathbb{R}$ is a function and $t \in \mathbb{T}^k$, then the Δ-*derivative* of u at the point t is defined to be the number $u^{\Delta}(t)$ (provided it exists) with the property that for each $\varepsilon > 0$, there is a neighborhood U of t such that

$$\left|u(\sigma(t)) - u(s) - u^{\Delta}(t)(\sigma(t) - s)\right| \leqslant \varepsilon|\sigma(t) - s| \tag{2.3}$$

for all $s \in U$. The function u is called Δ-differentiable on $\mathbb{T}$ if $u^{\Delta}(t)$ exists for all $t \in \mathbb{T}^k$.

The second Δ-*derivative of* u *at* $t \in \mathbb{T}^{k^2}$, if it exists, is defined to be $u^{\Delta^2}(t) = u^{\Delta\Delta}(t) := (u^{\Delta})^{\Delta}(t)$. Similarly,

$$u^{\Delta^i}(t) := \left(u^{\Delta^{i-1}}\right)^{\Delta}(t), \quad i > 2, i \in \mathbb{N} \tag{2.4}$$

is called *the* ith Δ-*derivative of* u *at* $t \in \mathbb{T}^{k^i}$. We also define the function $u^{\sigma} := u \circ \sigma$.

Definition 2.2. If $U^{\Delta} = u$ holds on $\mathbb{T}^k$, we define *the Cauchy* Δ-*integral* by

$$\int_s^t u(\tau)\Delta\tau = U(t) - U(s), \quad s, t \in \mathbb{T}^k. \tag{2.5}$$

Lemma 2.3. *If* $a, b \in \mathbb{T}$, $a < \rho(b)$, $h : [a, b]_{\mathbb{T}} \to [0, +\infty)$ *is continuous and* $h(t) \not\equiv 0$ *on* $[a, b)_{\mathbb{T}}$, *then*

$$\int_a^b h(t)\Delta t > 0. \tag{2.6}$$

Proof. From [15, Theorems 1.28 (Viii) and 1.29], it is clear. □

Lemma 2.4 (See [14, Theorem 1.16]). *If the* Δ-*derivative of* u *exists at* $t \in \mathbb{T}^k$, *then*

$$u(\sigma(t)) = u(t) + \mu(t)u^{\Delta}(t). \tag{2.7}$$

Define the Banach space $C(\mathbb{T})$ to be the set of continuous functions $u : \mathbb{T} \to \mathbb{R}$ with the norm

$$\|u\|_{\infty} = \max\{|u(t)| \mid t \in \mathbb{T}\}. \tag{2.8}$$

For $i \in \mathbb{N}$, we define the Banach space $C^i_\Delta(\mathbb{T})$ to be the set of the ith Δ-differential functions $u : \mathbb{T} \to \mathbb{R}$ for which $u^{\Delta^i} \in C(\mathbb{T}^{k^i})$ with the norm

$$\|u\|_i = \max\left\{ \|u\|_\infty, \left\|u^\Delta\right\|_\infty, \dots, \left\|u^{\Delta^i}\right\|_\infty \right\}, \tag{2.9}$$

where

$$\left\|u^{\Delta^j}\right\|_\infty = \max\left\{ \left|u^{\Delta^j}(t)\right| \mid t \in \mathbb{T}^{k^j} \right\}, \quad j = 0, 1, \dots, i. \tag{2.10}$$

3. Preliminaries

Throughout this paper, we assume that both

$$\xi = \min\left\{ t \in \mathbb{T} \mid t \geq \frac{\sigma^2(b) + 3a}{4} \right\}, \qquad \omega = \max\left\{ t \in \mathbb{T} \mid t \leq \frac{3\sigma^2(b) + a}{4} \right\} \tag{3.1}$$

exist and $a < \xi < \rho(\omega) \leq \omega \leq \rho(b)$. So there exists a number $m > 0$ such that

$$\omega - \xi \geq m. \tag{3.2}$$

We also make the following assumptions:

$(H1)$ $h : [a, \sigma(b)]_\mathbb{T} \to [0, +\infty)$ is continuous and $h(t) \not\equiv 0$ on $[\xi, \omega)_\mathbb{T}$;

$(H2)$ $f : [0, +\infty) \times (-\infty, 0] \to [0, +\infty)$ is continuous. $f(u, w)$ is nondecreasing in u, nonincreasing in w and $f(0, 0) > 0$.

Set $v(t) = u^{\Delta^2}(t), t \in [a, b]_\mathbb{T}$. Then problem (1.1) is equivalent to the system

$$\begin{gathered} u^{\Delta^2}(t) = v(t), \quad t \in [a, b]_\mathbb{T}, \\ v^{\Delta^2}(t) = \lambda h(t) f(u(t), v(t)), \quad t \in \left[a, \rho^2(b)\right]_\mathbb{T}, \\ u(a) = u^\Delta(\sigma(b)) = 0, \\ v(a) = v^\Delta(\rho(b)) = 0. \end{gathered} \tag{3.3}$$

According to [14, Corollary 4.84 and Theorem 4.70], the Green's function of problems

$$\begin{gathered} u^{\Delta^2}(t) = 0, \quad t \in [a, b]_\mathbb{T}, \\ u(a) = u^\Delta(\sigma(b)) = 0, \\ v^{\Delta^2}(t) = 0, \quad t \in \left[a, \rho^2(b)\right]_\mathbb{T}, \\ v(a) = v^\Delta(\rho(b)) = 0, \end{gathered} \tag{3.4}$$

is of the same form

$$G(t,s)=\begin{cases} t-a, & t\leq s,\\ \sigma(s)-a, & \sigma(s)\leq t,\end{cases} \tag{3.5}$$

and the solution of system (3.3) is

$$\begin{aligned} u(t)&=-\int_a^{\sigma(b)}G(t,s)v(s)\Delta s, \quad t\in\left[a,\sigma^2(b)\right]_{\mathbb{T}},\\ v(t)&=-\lambda\int_a^{\rho(b)}G(t,s)h(s)f(u(s),v(s))\Delta s, \quad t\in[a,b]_{\mathbb{T}}.\end{aligned} \tag{3.6}$$

Therefore, the solution of problem (1.1) is

$$u(t)=\lambda\int_a^{\sigma(b)}\left[\int_a^{\rho(b)}G(t,s)G(s,j)h(j)f\left(u(j),u^{\Delta^2}(j)\right)\Delta j\right]\Delta s, \quad t\in\left[a,\sigma^2(b)\right]_{\mathbb{T}}. \tag{3.7}$$

Lemma 3.1. *Green's function* (3.5) *is of the following properties:*

$$0\leq G(t,s)\leq\min\{t-a,\sigma(s)-a\}, \quad (t,s)\in\left[a,\sigma^2(b)\right]_{\mathbb{T}}\times[a,\sigma(b)]_{\mathbb{T}}, \tag{3.8}$$

$$G(t,s)\geq\frac{1}{4}(\sigma(s)-a), \quad (t,s)\in[\xi,\omega]_{\mathbb{T}}\times[a,\sigma(b)]_{\mathbb{T}}, \tag{3.9}$$

$$G(t,s)\geq\frac{t-a}{\sigma^2(b)-a}G(x,s), \quad (t,s)\in\left[a,\sigma^2(b)\right]_{\mathbb{T}}\times[a,\sigma(b)]_{\mathbb{T}},\ x\in\left[a,\sigma^2(b)\right]_{\mathbb{T}}, \tag{3.10}$$

$$G(t,s)\geq\frac{1}{4}G(x,s), \quad (t,s)\in[\xi,\omega]_{\mathbb{T}}\times[a,\sigma(b)]_{\mathbb{T}},\ x\in\left[a,\sigma^2(b)\right]_{\mathbb{T}}, \tag{3.11}$$

$$G(t,s)\geq\frac{\sigma^2(b)-a}{4}, \quad (t,s)\in[\xi,\omega]_{\mathbb{T}}\times[\xi,\omega]_{\mathbb{T}}. \tag{3.12}$$

Proof. We here only give the proof of (3.10), and the others can be obtained easily. We divide the proof into the following four cases.

Case 1 ($t\leq s,\ x\leq s$). We have

$$G(t,s)=t-a\geq\frac{x-a}{\sigma^2(b)-a}(t-a)=\frac{t-a}{\sigma^2(b)-a}(x-a)=\frac{t-a}{\sigma^2(b)-a}G(x,s). \tag{3.13}$$

Case 2 ($t\leq s,\ \sigma(s)\leq x$). We have

$$G(t,s)=t-a\geq\frac{\sigma(s)-a}{\sigma^2(b)-a}(t-a)=\frac{t-a}{\sigma^2(b)-a}(\sigma(s)-a)=\frac{t-a}{\sigma^2(b)-a}G(x,s). \tag{3.14}$$

Case 3 ($x \leq s,\ \sigma(s) \leq t$). We have

$$G(t,s) = \sigma(s) - a \geq x - a \geq \frac{t-a}{\sigma^2(b)-a}(x-a) = \frac{t-a}{\sigma^2(b)-a}G(x,s). \tag{3.15}$$

Case 4 ($\sigma(s) \leq t,\ \sigma(s) \leq x$). We have

$$G(t,s) = \sigma(s) - a \geq \frac{t-a}{\sigma^2(b)-a}(\sigma(s)-a) = \frac{t-a}{\sigma^2(b)-a}G(x,s). \tag{3.16}$$

□

Define

$$E = \left\{u \in C^2_\Delta\left(\left[a,\sigma^2(b)\right]\right) \mid u(a) = u^\Delta(\sigma(b)) = u^{\Delta^2}(a) = u^{\Delta^3}(\rho(b)) = 0\right\}. \tag{3.17}$$

Lemma 3.2. *For $u \in E$, one has*

$$\|u\|_\infty \leq 2\left[\sigma^2(b)-a\right]\left\|u^\Delta\right\|_\infty, \tag{3.18}$$

$$\left\|u^\Delta\right\|_\infty \leq 2\left[\sigma^2(b)-a\right]\left\|u^{\Delta^2}\right\|_\infty, \tag{3.19}$$

where $\|u\|_\infty = \max\{|u(t)| \mid t \in [a,\sigma^2(b)]_{\mathbb{T}}\}$, $\|u^\Delta\|_\infty = \max\{|u^\Delta(t)| \mid t \in [a,\sigma(b)]_{\mathbb{T}}\}$, $\|u^{\Delta^2}\|_\infty = \max\{|u^{\Delta^2}(t)| \mid t \in [a,b]_{\mathbb{T}}\}$.

Proof. Firstly, we show that (3.18) holds.

For all $t \in [a,\sigma(b)]$, we have from $u(a) = 0$ that

$$\begin{aligned} |u(t)| &= \left|u(a) + \int_a^t u^\Delta(s)\Delta s\right| \\ &\leq \int_a^t \left|u^\Delta(s)\right|\Delta s \\ &\leq \left\|u^\Delta\right\|_\infty (t-a) \\ &\leq \left[\sigma^2(b)-a\right]\left\|u^\Delta\right\|_\infty. \end{aligned} \tag{3.20}$$

Combining this with

$$\begin{aligned} \left|u\left(\sigma^2(b)\right)\right| &= \left|u(\sigma(b)) + \mu(\sigma(b))u^\Delta(\sigma(b))\right| \\ &\leq \left[\sigma^2(b)-a\right]\left\|u^\Delta\right\|_\infty + \left[\sigma^2(b)-a\right]\left\|u^\Delta\right\|_\infty \\ &= 2\left[\sigma^2(b)-a\right]\left\|u^\Delta\right\|_\infty, \end{aligned} \tag{3.21}$$

we have $\|u\|_\infty \leq 2[\sigma^2(b)-a]\|u^\Delta\|_\infty$.

Secondly, we show that (3.19) holds.

For all $t \in [a,b]$, we have from $u^{\Delta}(\sigma(b)) = 0$ that

$$\begin{aligned} \left|u^{\Delta}(t)\right| &= \left|-\int_t^b u^{\Delta^2}(s)\Delta s + u^{\Delta}(b)\right| \\ &= \left|-\int_t^b u^{\Delta^2}(s)\Delta s + u^{\Delta}(\sigma(b)) - \mu(b)u^{\Delta^2}(b)\right| \\ &\le \left\|u^{\Delta^2}\right\|_\infty (b-t) + \left\|u^{\Delta^2}\right\|_\infty \mu(b) \\ &\le 2\left[\sigma^2(b) - a\right]\left\|u^{\Delta^2}\right\|_\infty. \end{aligned} \tag{3.22}$$

Combining this with

$$\left|u^{\Delta}(\sigma(b))\right| = 0 \le 2\left[\sigma^2(b) - a\right]\left\|u^{\Delta^2}\right\|_\infty, \tag{3.23}$$

we have $\|u^{\Delta}\|_\infty \le 2[\sigma^2(b) - a]\|u^{\Delta^2}\|_\infty$. □

Define

$$\begin{aligned} \|u\|_E &= \max\left\{ \|u\|_\infty,\ 2\left[\sigma^2(b) - a\right]\left\|u^{\Delta}\right\|_\infty,\ 4\left[\sigma^2(b) - a\right]^2\left\|u^{\Delta^2}\right\|_\infty \right\} \\ &= 4\left[\sigma^2(b) - a\right]^2\left\|u^{\Delta^2}\right\|_\infty. \end{aligned} \tag{3.24}$$

Then E is a Banach space under the norm $\|\cdot\|_E$. Set

$$(A_\lambda u)(t) = \lambda \int_a^{\sigma(b)} \left[\int_a^{\rho(b)} G(t,s)G(s,j)h(j)f\left(u(j), u^{\Delta^2}(j)\right)\Delta j\right]\Delta s, \quad t \in \left[a, \sigma^2(b)\right]_{\mathbb{T}}. \tag{3.25}$$

Then $A_\lambda : E \to E$. Since $A_\lambda :\ C^2_\Delta([a,\sigma^2(b)]) \to C^4_\Delta([a,\sigma^2(b)]) \hookrightarrow C^2_\Delta([a,\sigma^2(b)])$, we have that A_λ is completely continuous.

Lemma 3.3. *Suppose* $(H1)$ *and* $(H2)$ *hold, and* $u(t)$ *is a solution of problem* (1.1), *then*

$$u(t) \ge 0, \quad t \in \left[a, \sigma^2(b)\right]_{\mathbb{T}}, \qquad u^{\Delta^2}(t) \le 0, \quad t \in [a,b]_{\mathbb{T}}, \tag{3.26}$$

$$u(t) > 0, \quad u^{\Delta^2}(t) < 0, \quad t \in [\xi,\omega]_{\mathbb{T}}, \tag{3.27}$$

$$\min_{t\in[\xi,\omega]_{\mathbb{T}}} \left(-u^{\Delta^2}(t)\right) \ge \frac{1}{4}\left\|u^{\Delta^2}\right\|_\infty = \frac{1}{16[\sigma^2(b) - a]^2}\|u\|_E, \tag{3.28}$$

$$\min_{t\in[\xi,\omega]_{\mathbb{T}}} u(t) \ge \frac{1}{4}\|u\|_\infty, \tag{3.29}$$

$$\min_{t\in[\xi,\omega]_{\mathbb{T}}} u(t) \ge \frac{m}{64[\sigma^2(b) - a]}\|u\|_E. \tag{3.30}$$

Proof. By $\lambda > 0$, (3.7), (3.8) and the fact that h, f are nonnegative functions, (3.26) holds. For $t \in [\xi, \omega]_{\mathbb{T}}$, from (3.7), (3.26), (3.2) and (3.12), we have

$$\begin{aligned} u(t) &= \lambda \int_a^{\sigma(b)} \left[\int_a^{\rho(b)} G(t,s)G(s,j)h(j)f\left(u(j), u^{\Delta^2}(j)\right)\Delta j \right] \Delta s \\ &\geq \lambda f(0,0) \int_\xi^\omega \left[\int_\xi^\omega G(t,s)G(s,j)h(j)\Delta j \right] \Delta s \\ &\geq \lambda \frac{m\left[\sigma^2(b) - a\right]^2}{16} f(0,0) \int_\xi^\omega h(j)\Delta j \\ &> 0, \\ -u^{\Delta^2}(t) &= \lambda \int_a^{\rho(b)} G(t,s)h(s)f\left(u(s), u^{\Delta^2}(s)\right)\Delta s \\ &\geq \lambda f(0,0) \int_\xi^\omega G(t,s)h(s)\Delta s \\ &\geq \lambda \frac{\sigma^2(b) - a}{4} f(0,0) \int_\xi^\omega h(s)\Delta s \\ &> 0 \end{aligned} \tag{3.31}$$

Therefor (3.27) holds.

For all $t \in [\xi, \omega]_{\mathbb{T}}$, for all $x \in [a, \sigma^2(b)]_{\mathbb{T}}$, from (3.7), (3.11), and (3.12), we have

$$\begin{aligned} -u^{\Delta^2}(t) &= \lambda \int_a^{\rho(b)} G(t,s)h(s)f\left(u(s), u^{\Delta^2}(s)\right)\Delta s \\ &\geq \frac{1}{4}\lambda \int_a^{\rho(b)} G(x,s)h(s)f\left(u(s), u^{\Delta^2}(s)\right)\Delta s \\ &= \frac{1}{4}\left[-u^{\Delta^2}(x)\right], \\ u(t) &= \lambda \int_a^{\sigma(b)} \left[\int_a^{\rho(b)} G(t,s)G(s,j)h(j)f\left(u(j), u^{\Delta^2}(j)\right)\Delta j \right] \Delta s \\ &\geq \frac{1}{4}\lambda \int_a^{\sigma(b)} \left[\int_a^{\rho(b)} G(x,s)G(s,j)h(j)f\left(u(j), u^{\Delta^2}(j)\right)\Delta j \right] \Delta s \\ &= \frac{1}{4}u(x), \\ u(t) &= \lambda \int_a^{\sigma(b)} \left[\int_a^{\rho(b)} G(t,s)G(s,j)h(j)f\left(u(j), u^{\Delta^2}(j)\right)\Delta j \right] \Delta s \\ &\geq \frac{\sigma^2(b) - a}{4}\lambda \int_\xi^\omega \left[\int_a^{\rho(b)} G(s,j)h(j)f\left(u(j), u^{\Delta^2}(j)\right)\Delta j \right] \Delta s \\ &\geq \frac{\sigma^2(b) - a}{16}\lambda \int_\xi^\omega \left[\int_a^{\rho(b)} G(x,j)h(j)f\left(u(j), u^{\Delta^2}(j)\right)\Delta j \right] \Delta s \end{aligned}$$

$$\begin{aligned}
&\geq \frac{m\left[\sigma^2(b)-a\right]}{16}\left[-u^{\Delta^2}(x)\right] \\
&= \frac{m}{64\left[\sigma^2(b)-a\right]}4\left[\sigma^2(b)-a\right]^2\left[-u^{\Delta^2}(x)\right].
\end{aligned} \tag{3.32}$$

Thus (3.28), (3.29), and (3.30) hold. □

At the end of this section, we state a lemma of the upper and lower solution method, which is needed for some proofs in next section.

Lemma 3.4 (See [17, Theorem 3.3.8]). *Let P be a cone with nonempty interior in Banach space E, and $A : P \to P$ a completely continuous and increasing operator. Suppose the following conditions hold:*

(i) *there exist $x_0,\ y_0 \in P$, such that $x_0 \leq Ax_0,\ Ay_0 \leq y_0,\ x_0 \not\leq y_0$;*

(ii) *there exist $\phi \in P$ and a constant $\alpha > 0$, such that $Ax \geq \alpha\|Ax\|\phi$, for all $x \in P$;*

(iii) *y_0 is in the interior of P, and there exists $\beta > 0$, such that $\phi \geq \beta x_0$.*

Then A has at least one fixed point $x \neq 0$ in P.

4. The Main Result

Our main result is the following existence theorem.

Theorem 4.1. *Suppose $(H1)$ and $(H2)$ hold, and either $(H3)$ or $(H4)$ holds. Here*

$(H3)$ $\lim_{(u,w)\to(+\infty,-\infty)}(f(u,w)/u) = +\infty$;

$(H4)$ $\lim_{(u,w)\to(+\infty,-\infty)}(f(u,w)/(-w)) = +\infty$.

Then there exists $\lambda^* > 0$, such that problem (1.1) has at least one positive solution for $\lambda \in (0,\lambda^*]$, and has no positive solution for $\lambda \in (\lambda^*,+\infty)$.

To prove Theorem 4.1, first, we show that problem (1.1) has positive solutions for some λ small enough.

Theorem 4.2. *Suppose $(H1)$ and $(H2)$ hold. Then there exists $\lambda_* > 0$, such that problem (1.1) has at least one positive solution for $\lambda \in (0,\lambda_*]$.*

Proof. The fixed point of A_λ defined in (3.25) is the solution of problem (1.1), so it will be enough to find the fixed point of A_λ.

Set

$$\widetilde{M} = \max_{t\in[a,b]_{\mathbb{T}}}\int_a^{\rho(b)} G(t,s)h(s)\Delta s > 0. \tag{4.1}$$

Let

$$\lambda_* = \frac{1}{\widetilde{M} f\left(4[\sigma^2(b) - a]^2, -1\right)}. \tag{4.2}$$

For $\lambda \in (0, \lambda_*]$, we have

$$f\left(4\left[\sigma^2(b) - a\right]^2, -1\right) \leq \frac{1}{\widetilde{M}\lambda}. \tag{4.3}$$

Set

$$D_1 = \left\{ u \in E \mid \|u\|_E \leq 4\left[\sigma^2(b) - a\right]^2 \right\}. \tag{4.4}$$

If $u \in D_1$, then

$$\|u\|_\infty \leq \|u\|_E \leq 4\left[\sigma^2(b) - a\right]^2, \qquad \left\|u^{\Delta^2}\right\|_\infty = \frac{\|u\|_E}{4[\sigma^2(b) - a]^2} \leq 1. \tag{4.5}$$

Consequently, for $t \in [a, b]_{\mathbb{T}}$,

$$\begin{aligned} \left| (A_\lambda u)^{\Delta^2}(t) \right| &= \left| -\lambda \int_a^{\rho(b)} G(t,s)h(s)f\left(u(s), u^{\Delta^2}(s)\right)\Delta s \right| \\ &\leq \widetilde{M}\lambda f\left(\|u\|_\infty, -\left\|u^{\Delta^2}\right\|_\infty\right) \\ &\leq \widetilde{M}\lambda f\left(4\left[\sigma^2(b) - a\right]^2, -1\right) \\ &\leq 1. \end{aligned} \tag{4.6}$$

Then

$$\|A_\lambda u\|_E = 4\left[\sigma^2(b) - a\right]^2 \left\|(A_\lambda u)^{\Delta^2}\right\|_\infty \leq 4\left[\sigma^2(b) - a\right]^2, \tag{4.7}$$

that is, $A_\lambda : D_1 \to D_1$. By the Schauder fixed point theorem, A_λ has at least one fixed point u_λ in E satisfying $\|u_\lambda\|_E \leq 4[\sigma^2(b) - a]^2$. From Lemma 3.3, u_λ is a positive solution of problem (1.1). □

Next, we show that there exist no positive solution for some λ large enough.

Theorem 4.3. *Suppose that* $(H1)$ *and* $(H2)$, *hold, and either* $(H3)$ *or* $(H4)$ *holds. Then problem* (1.1) *has no positive solution for* $\lambda \gg 1$.

Proof. Suppose $u_\lambda \in E$ is a solution to problem (1.1) for some $\lambda > 0$. We divide our discussions into two cases.

Case 1 (($H1$), ($H2$) and ($H3$) hold). By ($H3$), for a fixed $M_1 > 0$, there is $H > 0$ such that

$$f(u,w) \geq M_1 u, \qquad u \geq \frac{mH}{64[\sigma^2(b)-a]}, \qquad w \leq \frac{-H}{16[\sigma^2(b)-a]^2}. \tag{4.8}$$

If $u \in E$ with $\|u_\lambda\|_E \geq H$, then from (3.30) and (3.28), we have

$$\begin{gathered}\min_{t\in[\xi,\omega]_{\mathbb{T}}} u_\lambda(t) \geq \frac{m}{64[\sigma^2(b)-a]}\|u_n\|_E \geq \frac{mH}{64[\sigma^2(b)-a]},\\ \min_{t\in[\xi,\omega]_{\mathbb{T}}}\left(-u_\lambda^{\Delta^2}(t)\right) \geq \frac{1}{4}\left\|u_\lambda^{\Delta^2}\right\|_\infty = \frac{\|u_\lambda\|_E}{16[\sigma^2(b)-a]^2} \geq \frac{H}{16[\sigma^2(b)-a]^2}.\end{gathered} \tag{4.9}$$

Further by (4.8),

$$f\left(u_\lambda(t), u_\lambda^{\Delta^2}(t)\right) \geq M_1 u_\lambda(t), \quad t \in [\xi,\omega]_{\mathbb{T}}. \tag{4.10}$$

If $u \in E$ with $\|u_\lambda\|_E < H$, then from (3.27), there is

$$f\left(u_\lambda(t), u_\lambda^{\Delta^2}(t)\right) \geq M_2 u_\lambda(t), \quad t \in [\xi,\omega]_{\mathbb{T}} \tag{4.11}$$

with

$$M_2 = \min_{t\in[\xi,\omega]_{\mathbb{T}}} \frac{f\left(u_\lambda(t), u_\lambda^{\Delta^2}(t)\right)}{u_\lambda(t)} > 0. \tag{4.12}$$

Set $\widehat{M} = \min\{M_1, M_2\}$, we have from (4.10) and (4.11) that

$$f\left(u_\lambda(t), u_\lambda^{\Delta^2}(t)\right) \geq \widehat{M} u_\lambda(t), \qquad t \in [\xi,\omega]_{\mathbb{T}}. \tag{4.13}$$

Combining this with (3.7), (4.13), (3.29), (3.12), and (3.2), we have for $t \in [\xi,\omega]_{\mathbb{T}}$,

$$\begin{aligned} u_\lambda(t) &= \lambda\int_a^{\sigma(b)}\left[\int_a^{\rho(b)} G(t,s)G(s,j)h(j)f\left(u_\lambda(j), u_\lambda^{\Delta^2}(j)\right)\Delta j\right]\Delta s \\ &\geq \lambda\widehat{M}\int_a^{\sigma(b)}\left[\int_\xi^\omega G(t,s)G(s,j)h(j)u_\lambda(j)\Delta j\right]\Delta s \\ &\geq \frac{\lambda\widehat{M}\|u_\lambda\|_\infty}{4}\int_\xi^\omega\left[\int_\xi^\omega G(t,s)G(s,j)h(j)\Delta j\right]\Delta s \\ &\geq \lambda\frac{\widehat{M}m[\sigma^2(b)-a]^2}{64}\|u_\lambda\|_\infty\int_\xi^\omega h(j)\Delta j. \end{aligned} \tag{4.14}$$

If we choose λ such that

$$\lambda \frac{\widehat{M} m\left[\sigma^2(b)-a\right]^2}{64} \int_{\xi}^{\omega} h(j)\Delta j > 1, \tag{4.15}$$

then

$$u_\lambda(t) > \|u_\lambda\|_\infty, \quad t \in [\xi,\omega]_{\mathbb{T}}, \tag{4.16}$$

which is a contradiction.

Case 2 (($H1$), ($H2$), and ($H4$) hold). Similar to Case 1, by ($H4$), there is also an $\widehat{M}$, such that

$$f\left(u_\lambda(t), u_\lambda^{\Delta^2}(t)\right) \geq \widehat{M}\left(-u_\lambda^{\Delta^2}(t)\right), \quad t \in [\xi,\omega]_{\mathbb{T}}. \tag{4.17}$$

Thus for $t \in [\xi,\omega]_{\mathbb{T}}$, we have from (3.7), (4.17), (3.28), and (3.12) that

$$\begin{aligned} -u_\lambda^{\Delta^2}(t) &= \lambda \int_a^{\rho(b)} G(t,s)h(s) f\left(u_\lambda(s), u_\lambda^{\Delta^2}(s)\right)\Delta s \\ &\geq \lambda \widehat{M} \int_{\xi}^{\omega} G(t,s)h(s)\left(-u_\lambda^{\Delta^2}(s)\right)\Delta s \\ &\geq \lambda \frac{\widehat{M}\left[\sigma^2(b)-a\right]}{16} \left\|u_\lambda^{\Delta^2}\right\|_\infty \int_{\xi}^{\omega} h(s)\Delta s. \end{aligned} \tag{4.18}$$

If we take λ such that

$$\lambda \frac{\widehat{M}\left[\sigma^2(b)-a\right]}{16} \int_{\xi}^{\omega} h(s)\Delta s > 1, \tag{4.19}$$

then

$$-u_\lambda^{\Delta^2}(t) > \left\|u_\lambda^{\Delta^2}\right\|_\infty, \quad t \in [\xi,\omega]_{\mathbb{T}}, \tag{4.20}$$

which is a contradiction.

Therefore, problem (1.1) has no positive solution for $\lambda \gg 1$. □

Define the set $B = \{\lambda > 0 :$ problem (1.1) has at least one positive solution in $E\}$.

Theorem 4.4. *One has*

$$\overline{\lambda} \in B \Longrightarrow \lambda \in B, \quad \forall \lambda \in \left(0, \overline{\lambda}\right]. \tag{4.21}$$

Proof. Let λ_* be defined as Theorem 4.2. For $\overline{\lambda} \leq \lambda_*$, the result holds from Theorem 4.2. So, we discuss the case that $\overline{\lambda} > \lambda_*$.

For $\lambda \in (0, \overline{\lambda}]$, three cases will be discussed.

(1) $\lambda \in (0, \lambda_*]$;

(2) $\lambda \in (\lambda_*, \overline{\lambda})$;

(3) $\lambda = \overline{\lambda}$.

Cases (1) and (3) are clear from Theorem 4.2 and the assumption $\overline{\lambda} \in B$, respectively. Now, we deal with Case (2).

Define

$$D_2 = \left\{ u \in E \mid u(t) \geq 0,\ t \in \left[a, \sigma^2(b)\right],\ u^{\Delta^2}(t) \leq 0,\ t \in [a, b] \right\}. \tag{4.22}$$

Then D_2 is a cone with nonempty interior in E. For $\lambda \in (\lambda_*, \overline{\lambda})$, let A_λ be defined as (3.25). Then $A_\lambda : D_2 \to D_2$ and A_λ is an increasing operator from $(H2)$. Set u_* and $\overline{u}$ as positive solutions of problem (1.1) at λ_* and $\overline{\lambda}$, respectively. Then

$$\begin{gathered}(A_\lambda u_*)(t) > (A_{\lambda_*} u_*)(t) = u_*(t), \\ (A_\lambda \overline{u})(t) < \left(A_{\overline{\lambda}} \overline{u}\right)(t) = \overline{u}(t).\end{gathered} \tag{4.23}$$

So $\overline{u}$ is an upper solution of the operator A_λ and u_* is a lower solution. If $u_* \leq \overline{u}$, then there exists a positive solution u satisfying $u_*(t) \leq u(t) \leq \overline{u}(t)$ for $t \in [a, \sigma^2(b)]_{\mathbb{T}}$ by the upper and lower solution method. If $u_* \not\leq \overline{u}$, we verify the conditions of Lemma 3.4.

Clearly, the condition (i) in Lemma 3.4 holds for $u_* = x_0,\ \overline{u} = y_0$.

For all $u \in D_2$, for all $t, z \in [a, \sigma^2(b)]_{\mathbb{T}},\ x \in [\xi, \omega]_{\mathbb{T}}$, we have from (3.10), (3.12), (3.11), and (3.2) that

$$\begin{aligned}(A_\lambda u)(t) &= \lambda \int_a^{\sigma(b)} \left[\int_a^{\rho(b)} G(t,s) G(s,j) h(j) f\left(u(j), u^{\Delta^2}(j)\right) \Delta j \right] \Delta s \\ &\geq \lambda \frac{t-a}{\sigma^2(b) - a} \int_a^{\sigma(b)} \left[\int_a^{\rho(b)} G(x,s) G(s,j) h(j) f\left(u(j), u^{\Delta^2}(j)\right) \Delta j \right] \Delta s \\ &\geq \lambda \frac{t-a}{4} \int_\xi^{\omega} \left[\int_a^{\rho(b)} G(s,j) h(j) f\left(u(j), u^{\Delta^2}(j)\right) \Delta j \right] \Delta s \\ &\geq \lambda \frac{t-a}{4} \int_\xi^{\omega} \left[\int_a^{\rho(b)} \frac{1}{4} G(z,j) h(j) f\left(u(j), u^{\Delta^2}(j)\right) \Delta j \right] \Delta s \\ &\geq \frac{m}{16}(t-a) \left[-(A_\lambda u)^{\Delta^2}(z) \right].\end{aligned} \tag{4.24}$$

Then

$$A_\lambda u \geq \frac{m}{16}\left\|(A_\lambda u)^{\Delta^2}\right\|_\infty (t-a) = \frac{m}{64[\sigma^2(b)-a]^2}\|A_\lambda u\|_E (t-a), \tag{4.25}$$

and the condition (ii) of Lemma 3.4 is satisfied for $\phi = t-a, \alpha = m/64[\sigma^2(b)-a]^2 > 0$.

From (3.26) and (3.27), we have that $\overline{u}$ is in the interior of D_2. From (3.8), there is

$$\begin{aligned} u_*(t) &= \lambda_* \int_a^{\sigma(b)} \left[\int_a^{\rho(b)} G(t,s)G(s,j)h(j)f\left(u_*(j), u_*^{\Delta^2}(j)\right)\Delta j\right]\Delta s \\ &\leq \lambda_*(t-a)\int_a^{\sigma(b)} \left[\int_a^{\rho(b)} G(s,j)h(j)f\left(u_*(j), u_*^{\Delta^2}(j)\right)\Delta j\right]\Delta s \\ &\leq \lambda_*(t-a)\left[\int_a^{\sigma(b)}(s-a)\Delta s\right]\left[\int_a^{\rho(b)} h(j)f\left(u_*(j), u_*^{\Delta^2}(j)\right)\Delta j\right]. \end{aligned} \tag{4.26}$$

This implies $\phi(t) = t-a \geq \beta u_*(t),\ t\in[a,\sigma^2(b)]_{\mathbb{T}}$ for

$$\beta = \left\{\lambda_*\left[\int_a^{\sigma(b)}(s-a)\Delta s\right]\left[\int_a^{\rho(b)} h(j)f\left(u_*(j), u_*^{\Delta^2}(j)\right)\Delta j\right]\right\}^{-1} > 0, \tag{4.27}$$

and the condition (iii) of Lemma 3.4 is satisfied.

By Lemmas 3.4 and 3.3, we get a positive solution u_λ in $D_2 \in E$. That is, $\lambda \in B$. □

Now, we give the proof of Theorem 4.1.

Proof of Theorem 4.1. From Theorems 4.2 and 4.3, $B \neq \emptyset$ is bounded. Thus, we can define $\lambda^* = \sup B$. Firstly, we show that $\lambda^* \in B$.

Choose a sequence $\{\lambda_n\}_{n=1}^\infty$, $\lambda_n \in B$ $(n = 1,2,\ldots)$ which belongs to a compact subinterval in $(0,\infty)$, and $\lambda_n \to \lambda^*(n\to\infty)$. Then there exists $N_1 > 0$, for $n \geq N_1$,

$$\lambda_n > \frac{\lambda^*}{2}. \tag{4.28}$$

Let $u_n \in E$ satisfy

$$A_{\lambda_n} u_n = u_n. \tag{4.29}$$

If $\{u_n\}_{n=1}^\infty$ is uniformly bounded, then there exists u^* such that $u_n \to u^*$ $(n\to\infty)$ and

$$u_n = A_{\lambda_n} u_n \longrightarrow A_{\lambda^*} u^* \quad (n \longrightarrow \infty). \tag{4.30}$$

Consequently,

$$u^* = A_{\lambda^*} u^*. \tag{4.31}$$

By Lemma 3.3, u^* is a positive solution of problem (1.1), and $\lambda^* \in B$.

Next, we will prove that $\{u_n\}_{n=1}^{\infty}$ is uniformly bounded and the discussions will be divided into two cases.

Case 1 (($H1$), ($H2$), and ($H3$) hold). From ($H3$), there exists $H_1 > 0$,

$$f(u, w) \geq M_1 u, \quad \text{for } u \geq \frac{mH_1}{64[\sigma^2(b) - a]}, \quad -w \geq \frac{H_1}{16[\sigma^2(b) - a]^2}, \tag{4.32}$$

where $M_1 > 0$ satisfies

$$\frac{M_1 [\sigma^2(b) - a]^2 m\lambda^*}{128} \int_{\xi}^{\omega} h(j)\Delta j > 1. \tag{4.33}$$

Suppose on the contrary that $\{u_n\}_{n=1}^{\infty}$ is unbounded. Then

$$\|u_n\|_E \longrightarrow \infty, \quad n \longrightarrow \infty, \tag{4.34}$$

which implies that there exists $N_2 > 0$, $\|u_n\|_E \geq H_1$ for $n \geq N_2$. From (3.30) and (3.28), we have

$$\begin{aligned} \min_{t \in [\xi, \omega]_{\mathbb{T}}} u_n(t) &\geq \frac{m}{64[\sigma^2(b) - a]} \|u_n\|_E \geq \frac{mH_1}{64[\sigma^2(b) - a]}, \\ \min_{t \in [\xi, \omega]_{\mathbb{T}}} \left(-u_n^{\Delta^2}(t)\right) &\geq \frac{1}{16[\sigma^2(b) - a]^2} \|u_n\|_E \geq \frac{H_1}{16[\sigma^2(b) - a]^2}. \end{aligned} \tag{4.35}$$

Subsequently, for $t \in [\xi, \omega]_{\mathbb{T}}$, $n \geq \max\{N_1, N_2\}$,

$$\begin{aligned} u_n(t) &= \lambda_n \int_a^{\sigma(b)} \left[\int_a^{\rho(b)} G(t,s)G(s,j)h(j) f\left(u_n(j), u_n^{\Delta^2}(j)\right)\Delta j\right]\Delta s \\ &\geq \lambda_n M_1 \int_a^{\sigma(b)} \left[\int_{\xi}^{\omega} G(t,s)G(s,j)h(j)u_n(j)\Delta j\right]\Delta s \\ &\geq \frac{\lambda_n M_1}{4} \|u_n\|_{\infty} \int_{\xi}^{\omega} \left[\int_{\xi}^{\omega} G(t,s)G(s,j)h(j)\Delta j\right]\Delta s \\ &\geq \frac{M_1 [\sigma^2(b) - a]^2 m\lambda^*}{128} \|u_n\|_{\infty} \int_{\xi}^{\omega} h(j)\Delta j \\ &> \|u_n\|_{\infty} \end{aligned} \tag{4.36}$$

by (4.32), (3.29), (3.12), (4.28), (3.2), and (4.33). This is a contradiction.

Case 2 (($H1$), ($H2$), and ($H4$) hold). From ($H4$), there exists $H_2 > 0$,

$$f(u,w) \geq M_2(-w), \quad \text{for } u \geq \frac{mH_2}{64[\sigma^2(b)-a]}, \quad -w \geq \frac{H_2}{16[\sigma^2(b)-a]^2}, \tag{4.37}$$

where $M_2 > 0$ satisfies

$$\frac{M_2[\sigma^2(b)-a]\lambda^*}{32}\int_{\xi}^{\omega} h(s)\Delta s > 1. \tag{4.38}$$

Suppose on the contrary that $\{u_n\}_{n=1}^{\infty}$ is unbounded. Then similar to Case 1, there exists $N_3 > 0$ such that $\|u_n\|_E \geq H_2$ for $n \geq N_3$. Thus for $t \in [\xi,\omega]_{\mathbb{T}}$, $n \geq \max\{N_1, N_3\}$, we have

$$\begin{aligned}
-u_n^{\Delta^2}(t) &= \lambda_n \int_a^{\rho(b)} G(t,s)h(s)f\left(u_n(s), u_n^{\Delta^2}(s)\right)\Delta s \\
&\geq \lambda_n M_2 \int_{\xi}^{\omega} G(t,s)h(s)\left(-u_n^{\Delta^2}(s)\right)\Delta s \\
&> \frac{M_2}{4}\frac{\lambda^*}{2}\left\|u_n^{\Delta^2}\right\|_{\infty}\frac{\sigma^2(b)-a}{4}\int_{\xi}^{\omega} h(s)\Delta s \\
&> \left\|u_n^{\Delta^2}\right\|_{\infty}
\end{aligned} \tag{4.39}$$

by (4.37), (3.28), (3.12), (4.28) and (4.38). This is also a contradiction.

According to the definition of λ^* and B, and Theorem 4.4, we complete the proof. □

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