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Research Article

Event-Triggered State Estimation for a Class of Delayed Recurrent Neural Networks with Sampled-Data Information

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The paper investigates the state estimation problem for a class of recurrent neural networks with sampled-data information and time-varying delays. The main purpose is to estimate the neuron states through output sampled measurement; a novel event-triggered scheme is proposed, which can lead to a significant reduction of the information communication burden in the network; the feature of this scheme is that whether or not the sampled data should be transmitted is determined by the current sampled data and the error between the current sampled data and the latest transmitted data. By using a delayed-input approach, the error dynamic system is equivalent to a dynamic system with two different time-varying delays. Based on the Lyapunov-krasovskii functional approach, a state estimator of the considered neural networks can be achieved by solving some linear matrix inequalities, which can be easily facilitated by using the standard numerical software. Finally, a numerical example is provided to show the effectiveness of the proposed event-triggered scheme.

1. Introduction

The research of neural networks has been paid much attention during the past few years, due to its potential application in various fields, such as image processing, pattern recognition, and associative memory [1–5]. As a special class of nonlinear dynamical systems, the dynamic behavior of recurrent neural networks has been one of the most important issues. In particular, the analysis problems of stability and synchronization of recurrent neural networks have received great attention and a number of profound results have been proposed [6–12].

In many application, such as signal processing and control engineering, for large-scale neural networks, it is quite common that only partial information can be accessible from the network outputs. Therefore, it is of great significance to estimate the neuron states through available output measurements of the networks and then utilizes the estimated neuron states to achieve certain design objectives; note that state estimation problem for neural networks has been hot reach topics that have drawn considerable attention, and many profound results have been available in the literature [13–25]. The authors in [13] studied the problem of state estimation for a class of delayed neural networks; the traditional monotonicity and smoothness assumption on the activation function had been removed. The design problem of state estimator for a class of neural networks with constant delays was investigated in [14], where a delay-dependent criterion for existence of the estimator was proposed. As an extension, The authors in [14, 15] further discussed state estimation for neural networks with time-varying delays. In practice, sometimes a neural network has finite state modes and modes may switch from one to another at different times. On the other hand, discrete-time neural networks could be more suitable to model digitally transmitted signals in dynamical way; based on the above reason, The authors in [16] investigated state estimation problem for a new class of discrete-time neural networks with Markovian jumping parameters and mode dependent mixed time-delays, where he discrete and distributed delays were mode-dependent. Different from the stuelies in [16, 17] which considered state estimation for Markovian jumping delayed continuous-time recurrent neural networks, where only matrix parameters were mode-dependent. Similar to [16], for continuous-time recurrent neural networks with discrete and distributed delays, state estimation was also investigated in [18]. In [19, 20], synchronization and state estimation had been studied for discrete-time complex networks with distributed delays; it was noticed that in [20], a novel notion of bounded H_∞ synchronization had been first defined to characterize the transient performance of synchronization. Some robust state estimation problems for uncertain neural networks with time-varying delays had been investigated in [21–23], where the parameter uncertainties are assumed to be norm bounded; some sufficient conditions were presented to guarantee the existence of the desired state estimator. Taking into account the stochastic properties of time-varying delays, the authors in [24] discussed state estimation problem for a class of discrete-time stochastic neural networks with random delays; sufficient delay-distribution-dependent conditions were established in terms of linear matrix inequalities (LMIs) that guarantee the existence of the state estimator.

The sampled-data control theory had attracted much attention due to its effectiveness in engineering applications. Especially, a new approach to deal with the sampled-data control problems had been proposed in [26], where the sampling period had been converted into time-varying delay. As its extension, the authors in [27] investigated the sampled-data state estimation problem for a class of recurrent neural networks with time-varying delays, where the sampled measurements had been used to estimate the neuron states. Using a similar approach, where the sampled-data synchronization control problem was investigated in [28] for a class of general delayed complex networks, the sampled-data feedback controllers were designed in terms of the solution to certain linear matrix inequalities. But in the above references, the sampling rate for each signal is the same; but in the actual system, it may be varying from sample to sample owing to unpredictable perturbations; this factor was considered in [29], the problem of robust H_∞ control was investigated for sampled-data systems with probabilistic sampling, where two different sampling periods were considered whose occurrence probabilities were given constants and satisfied Bernoulli distribution. In [30], stochastic sampled-data approach was used for studying the problem of distributed H_∞ filtering in sensor networks, by converting the sampling periods into bounded time-delays, the design problem of H_∞ filters amounted to solving the H_∞ filtering problem for a class of stochastic nonlinear systems with multiple bounded time delays. In [31], the sampled-data synchronization control problem was addressed, where the sampling period was time varying and switched between two different values in a random way. It is worth noting that most of the above results were involved the traditional approach of sampling at pre-specified time instances, which was called time-triggered sampling; this sampling method may lead to an inherently periodic transmission and produce many useless messages if the current sampled signal had not significantly changed in contrast to the previous sampled signal, which led to a conservative usage of the communication resources. Recently, event-triggered scheme provided an effective approach of determining; its main property was that the signal was sampled and only some functions of the system state or output measurement exceeded threshold. Compared with periodic sampling method, the event-triggered scheme could reduce the burden of the communication and also preserve the desired properties of the ideal continuous state feedback system, such as stability and convergence. The utilization on event-triggered scheme could be found in many literatures such as [32–37]. The event-triggered H_∞ control design was investigated in [32] for networked control systems with uncertainties and transmission delays, and a novel event-triggered scheme was proposed. The study in [33] was concerned with the control problem of event-triggered networked systems with both state and control input quantizations. In [34], the problems of exponential stability and L_2-gain analysis of event-triggered networked control systems were studied, where the event-triggered conditions were proposed in the sensor side and controller side. In [35–37], the consensus problems for multiagent systems were investigated by event-triggered control, where different trigger functions were proposed. Unfortunately, as far as we know, up to now, no theoretical results are given for state estimation of recurrent neural networks with time-varying delays based on event-triggered scheme. The purpose of our study is to fill the gap.

Motivated by the above discussion, the paper is concerned with the sampled-data state estimation problem for a class of recurrent neural networks with time-varying delays. The main purpose is to estimate the neuron states through output sampled measurement, and a novel event-triggered scheme is proposed, which can lead to a significant reduction of the information communication burden in the network. By using a delayed-input approach, the error dynamics system is equivalent to a dynamic system with two different time-varying delays. Based on constructing a Lyapunov-Krasovskii functional and employing some analysis techniques, a state estimator of the considered neural networks can be achieved by solving some linear matrix inequalities, which can be easily facilitated by using the standard numerical software. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

The main contributions of this paper are highlighted as follows.

(1) The novel event-triggered scheme is proposed, compared with a time-triggered periodic communication scheme, since the proposed communication scheme only depends on the state at the sampled instant and the state error between the current sampled instant and the latest transmitted state. Therefore, the number of the transmitted state signals through the network could be reduced apparently.

(2) Sufficient conditions obtained are in the form of linear matrix inequalities which can be readily solved by using the LMI toolbox in Matlab, and the solvability of derived conditions dependents on not only trigger parameters and sampling period but also the size of the delay.

Notation 1. The notation used here is fair standard except where otherwise stated. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of real $n\times m$ matrices. The superscript T represents the transpose of the matrix (or vector). I denotes the identity matrix of compatible dimensions. The asterisk represents the symmetric block in one symmetric matrix. $\text{diag}\{\cdots\}$ stands for a block-diagonal matrix. The notation $X \geq 0$ $(X > 0)$ means that X is positive semi-definite (positive definite). $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. If they are not explicitly specified, arguments of a function or a matrix will be omitted in the analysis when no confusion can arise.

2. Preliminaries

Consider a class of recurrent neural networks with time-varying delays as follows:

$$\begin{aligned} \dot{x}(t) &= -Ax(t) + W_0 g(x(t)) + W_1 g(x(t-\tau(t))) \\ y(t) &= Cx(t), \end{aligned} \tag{2.1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in \mathbb{R}^n$ is the state vector associated with n neurons; $A = \text{diag}\{a_1, a_2, \ldots, a_n\}$ is a positive diagonal matrix; $W_0 \in \mathbb{R}^{n\times n}$ and $W_1 \in \mathbb{R}^{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively; $\tau(t) \in [0, \tau]$ is the time-varying bound delay; $C \in \mathbb{R}^{m\times n}$ is a constant matrix; $y(t) = (y_1(t), y_2(t), \ldots, y_m(t))^T \in \mathbb{R}^m$ denotes the output vector; $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T \in \mathbb{R}^n$ represents the neuron activation function with $g(0) = 0$.

In this paper, the measurement output is sampled before it enters the estimator; based on the sampling technique and zero-order hold, the actual output can be described as

$$\overline{y}(t) = y(t_k) = Cx(t_k), \quad t \in [t_k, t_{k+1}), \tag{2.2}$$

where $\overline{y}(t) \in \mathbb{R}^m$ is the actual output of the estimator, and t_k denotes the sampling instant satisfying $\lim_{k\to\infty} t_k = \infty$.

Remark 2.1. In practical systems, periodic sampling mechanism may often lead to sending many unnecessary signals through the networks, which will increase the load of network transmission and waste the network bandwidth; therefore, it is significant to introduce a mechanism to determine which sampled signal should be sent out or not. As stated in [32, 33], the event-trigger sampling scheme is effective way because they can reduce the traffic and the power consumption.

The sampled data $y(t_{k+j})$ is transmitted (or released) by the event generator only when the current sampled value $y(t_{k+j})$ and the previously transmitted one $y(t_k)$ satisfy the following judgement algorithm:

$$\left[y(t_{k+j}) - y(t_k)\right]^T W \left[y(t_{k+j}) - y(t_k)\right] < \sigma y^T(t_{k+j}) W y(t_{k+j}), \tag{2.3}$$

where $W \in \mathbb{R}^{m\times m}$ is a positive matrix, and $\sigma \in [0, 1)$ is a positive scalar. The sampled state $y(t_{k+j})$ satisfying the inequality (2.3) will not be transmitted; only the one that exceeds the threshold in (2.3) will be sent to the estimator. Specially, when $\sigma = 0$, the inequality (2.3) is not satisfied for almost all the sampled state $y(t_{k+j})$, and the event-triggered scheme reduces to a periodic release scheme.

Remark 2.2. From event-triggered algorithm (2.3), it is easily seen that all the released signals are subsequences of the sampled data, that is, the set of the release instants $\{t_0, t_1, t_2 \ldots\} \subseteq \{0, 1, 2, \ldots\}$. The amount of $\{t_0, t_1, t_2 \ldots\}$ depends on not only the value of σ but also the variation of the system output.

Suppose that the time-varying delay in network communication is $d_k \in [0, d]$ ($k = 1, 2, \ldots, +\infty$), the output $\overline{y}(t)$ in (2.2) can be rewritten as

$$\overline{y}(t) = y(t_k) = Cx(t_k), \quad t \in [t_k + d_k, t_{k+1} + d_{k+1}). \tag{2.4}$$

Substituting the output (2.4) into the judgement algorithm (2.3), we can obtain

$$\left[x(t_{k+j}) - x(t_k)\right]^T C^T W C \left[x(t_{k+j}) - x(t_k)\right] < \sigma x^T(t_{k+j}) C^T W C x(t_{k+j}). \tag{2.5}$$

For technical convenience, similar to [32, 33], consider the following two intervals:

$$[t_k + d_k, t_k + h + d), \qquad [t_k + lh + d, t_k + lh + h + d), \tag{2.6}$$

where l is a positive integer and h is a sampling period.

(1) if $t_k + h + d > t_{k+1} + d_{k+1}$, define a function $d(t)$ as follows:

$$d(t) = t - t_k, \quad t \in [t_k + d_k, t_{k+1} + d_{k+1}). \tag{2.7}$$

It can easily be obtained that the following inequality holds:

$$d_k \leq d(t) \leq t_{k+1} - t_k + d_{k+1} \leq h + d. \tag{2.8}$$

(2) if $t_k + h + d < t_{k+1} + d_{k+1}$, there exists a positive integer m, such that

$$t_k + mh + d < t_{k+1} + d_{k+1} < t_k + mh + h + d. \tag{2.9}$$

It can be easily shown that

$$[t_k + d_k, t_{k+1} + d_{k+1}) = I_1 \cup I_2 \cup I_3, \tag{2.10}$$

where

$$
\begin{aligned}
I_1 &= [t_k + d_k, t_k + h + d) \\
I_2 &= \bigcup_{m=1}^{l-1} \{ I_2^m \} \\
I_2^m &= [t_k + mh + d, t_k + mh + h + d) \\
I_3 &= [t_k + lh + d, t_{k+1} + d_{k+1}).
\end{aligned}
\tag{2.11}
$$

Define a function $d(t)$ as

$$
d(t) = \begin{cases} t - t_k & t \in I_1 \\ t - t_k - mh & t \in I_2^m \ (m = 1, 2, \ldots, l-1) \\ t - t_k - lh & t \in I_3. \end{cases}
\tag{2.12}
$$

From the definition of $d(t)$ defined in (2.12), we can derive

$$
\begin{aligned}
&0 \le d_k \le d(t) < h + d, \quad t \in I_1 \\
&0 \le d_k \le d \le d(t) < h + d, \quad t \in I_2^m \ (m = 1, 2, \ldots, l-1) \\
&0 \le d_k \le d \le d(t) < h + d, \quad t \in I_3.
\end{aligned}
\tag{2.13}
$$

From (2.13), it can be derived that $0 \le d(t) < d_M$, where $d_M = h + d$. For $t \in [t_k + d_k, t_{k+1} + d_{k+1})$, we define

$$
e_k(t) = \begin{cases} 0 & t \in I_1 \\ x(t_k + mh) - x(t_k) & t \in I_2^m \ (m = 1, 2, \ldots, l-1) \\ x(t_k + lh) - x(t_k) & t \in I_3. \end{cases}
\tag{2.14}
$$

Combining the above definitions of $d(t)$ and $e_k(t)$, the algorithm (2.5) can be rewritten as

$$
e_k^T(t) C^T W C e_k(t) < \sigma x^T(t - d(t)) C^T W C x(t - d(t)).
\tag{2.15}
$$

Based on the available sampled measurement $\overline{y}(t)$, the following state estimator is adopted:

$$
\dot{\hat{x}}(t) = -A\hat{x}(t) + K\left(\overline{y}(t) - C\hat{x}(t)\right),
\tag{2.16}
$$

where K is feedback gain matrix to be designed, and $\hat{x}(t) = (\hat{x}_1(t), \hat{x}_2(t), \ldots, \hat{x}_n(t))^T \in \mathbb{R}^n$ is estimator state vector.

Setting $e(t) = x(t) - \hat{x}(t)$, the estimation error dynamics can be obtained from (2.1) and (2.16), and it follows that

$$\dot{e}(t) = -(A + KC)e(t) + KCx(t) - KCx(t - d(t)) + KCe_k(t) + W_0 g(x(t)) + W_1 g(x(t - \tau(t))). \tag{2.17}$$

Let $\overline{x}(t) = (x^T(t), e^T(t))^T$, we can get the following augmented system from (2.1) and (2.17)

$$\dot{\overline{x}}(t) = \overline{A}\overline{x}(t) + \overline{B}\overline{x}(t - d(t)) + \overline{W}_0 g(H\overline{x}(t)) + \overline{W}_1 g(H\overline{x}(t - \tau(t))) + \overline{C}e_k(t), \tag{2.18}$$

where

$$\overline{A} = \begin{bmatrix} -A & 0 \\ KC & -A - KC \end{bmatrix} \quad \overline{B} = \begin{bmatrix} 0 & 0 \\ -KC & 0 \end{bmatrix} \quad \overline{W}_0 = \begin{bmatrix} W_0 \\ W_0 \end{bmatrix}$$
$$\overline{W}_1 = \begin{bmatrix} W_1 \\ W_1 \end{bmatrix} \quad H^T = \begin{bmatrix} I \\ 0 \end{bmatrix} \quad \overline{C} = \begin{bmatrix} 0 \\ KC \end{bmatrix}. \tag{2.19}$$

Before giving the main results, the following assumption, definition, and lemma are essential in establishing our main results.

Assumption 2.3 (see, [27]). The activation function $g(\cdot)$ satisfies the following sector-bounded condition:

$$[g(x) - U_1 x]^T [g(x) - U_2 x] \leq 0, \tag{2.20}$$

where U_1 and U_2 are two real constant matrices with $U_2 - U_1 \geq 0$.

Definition 2.4 (see, [27]). The augmented system (2.18) is exponentially stable, if there exist two constants $\alpha > 0$ and $\beta > 0$, such that

$$\|\overline{x}(t)\|^2 \leq \alpha e^{-\beta t} \sup_{-r \leq \theta \leq 0} \|\phi(\theta)\|^2, \tag{2.21}$$

where $\phi(\cdot)$ is in the initial function system (2.18) as $\phi(t) = \overline{x}(t)$, $t \in [-r, 0]$.

Lemma 2.5 (see, [38, 39]). *Suppose $\tau(t) \in [\tau_m, \tau_M]$, Q_i $(i = 1,2,3)$ are some constant matrices with appropriate dimensions, then*

$$Q_1 + (\tau_M - \tau(t))Q_2 + (\tau(t) - \tau_m)Q_3 < 0, \tag{2.22}$$

if the following inequalities hold

$$\begin{aligned} &Q_1 + (\tau_M - \tau_m)Q_2 < 0 \\ &Q_1 + (\tau_M - \tau_m)Q_3 < 0. \end{aligned} \tag{2.23}$$

Lemma 2.6 (see, [40]). *For any constant positive matrix $T \in \mathbb{R}^{n\times n}$, scalar $\tau_1 \le \tau(t) < \tau_2$ and vector function $\dot{x}(t) : [-\tau_2, \tau_1] \to \mathbb{R}^n$ such that the following integration is well defined, then it holds that*

$$-(\tau_2 - \tau_1)\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(v)T\dot{x}(v)dv \le \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \\ x(t-\tau_2) \end{bmatrix}^T \begin{bmatrix} -T & T & 0 \\ * & -2T & T \\ * & * & -T \end{bmatrix} \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \\ x(t-\tau_2) \end{bmatrix}. \tag{2.24}$$

3. Main Results

In this section, we design a sampled-date estimator with form (2.18) for recurrent neural networks with time-varying delay based event-triggered control.

The system (2.18) can be rewritten as

$$\dot{\overline{x}}(t) = \mathcal{A}\xi(t), \tag{3.1}$$

where

$$\begin{gathered} \xi(t) = \left[\overline{x}^T(t), \overline{x}^T(t-d(t)), \overline{x}^T(t-d_M), \overline{x}^T(t-\tau(t)), \overline{x}^T(t-\tau), g^T(H\overline{x}(t)), g^T(H\overline{x}(t-\tau(t))), e_k^T(t)\right]^T \\ \mathcal{A} = \left[\overline{A}, \overline{B}, 0, 0, 0, \overline{W}_0, \overline{W}_1, \overline{C}\right]. \end{gathered} \tag{3.2}$$

Theorem 3.1. *Suppose that Assumption 2.3 holds, for given estimator gain matrix K, the augmented system (3.1) is exponentially stable, if there exist some positive definite matrices $P > 0$, $Q_i > 0$, $R_i > 0$ and S_i, T_i $(i = 1,2)$ with appropriate dimension, and two positive scalars $\alpha > 0$, $\beta > 0$, such that the following linear matrix inequalities hold:*

$$\Pi_i = \begin{bmatrix} \Pi & \Phi_1 & \Phi_2 & \Phi_3^{(i)} \\ * & -d_M R_1 & 0 & 0 \\ * & * & -\tau R_2 & 0 \\ * & * & * & -\tau R_2 \end{bmatrix} < 0 \quad (i = 1,2), \tag{3.3}$$

where

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & 0 & \Pi_{13} & 0 & \Pi_{14} & P\overline{W}_1 & P\overline{C} \\ * & \Pi_{22} & \frac{1}{d_M}R_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & \Pi_{45} & 0 & -\beta\overline{U}_2 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 & 0 \\ * & * & * & * & * & -\alpha I & 0 & 0 \\ * & * & * & * & * & * & -\beta I & 0 \\ * & * & * & * & * & * & * & -C^T W C \end{bmatrix}$$

$$\Phi_1 = \left[d_M R_1\overline{A} \;\; d_M R_1\overline{B} \;\; 0 \;\; 0 \;\; 0 \;\; d_M R_1\overline{W}_0 \;\; d_M R_1\overline{W}_1 \;\; d_M R_1\overline{C}\right]^T$$

$$\Phi_2 = \left[\tau R_2\overline{A} \;\; \tau R_2\overline{B} \;\; 0 \;\; 0 \;\; 0 \;\; \tau R_2\overline{W}_0 \;\; \tau R_2\overline{W}_1 \;\; \tau R_2\overline{C}\right]^T$$

$$\Phi_3^{(1)} = \left[\tau S_1^T \;\; 0 \;\; 0 \;\; \tau S_2^T \;\; 0 \;\; 0 \;\; 0 \;\; 0\right]^T$$

$$\Phi_3^{(2)} = \left[0 \;\; 0 \;\; 0 \;\; \tau T_1^T \;\; \tau T_2^T \;\; 0 \;\; 0 \;\; 0\right]^T$$

$$\Pi_{11} = P\overline{A} + \overline{A}^T P + Q_1 + Q_2 - \frac{1}{d_M}R_1 + S_1 + S_1^T - \alpha\overline{U}_1$$

$$\Pi_{12} = P\overline{B} + \frac{1}{d_M}R_1$$

$$\Pi_{13} = S_2^T - S_1$$

$$\Pi_{14} = P\overline{W}_0 - \alpha\overline{U}_2$$

$$\Pi_{22} = -\frac{2}{d_M}R_1 + \sigma\Omega$$

$$\Pi_{33} = -Q_1 - \frac{1}{d_M}R_1$$

$$\Pi_{44} = -S_2 - S_2^T + T_1 + T_1^T - \beta\overline{U}_1$$

$$\Pi_{45} = -T_1 + T_2^T$$

$$\Pi_{55} = -Q_2 - T_2 - T_2^T$$

$$\Omega = \begin{bmatrix} C^T W C & 0 \\ 0 & 0 \end{bmatrix}. \tag{3.4}$$

Proof. Construct the following Lyapunov-Krasovskii functional candidate:

$$V(t,\overline{x}(t)) = V_1(t,\overline{x}(t)) + V_2(t,\overline{x}(t)) + V_3(t,\overline{x}(t)) + V_4(t,\overline{x}(t)), \tag{3.5}$$

where

$$\begin{aligned}
V_1(t,\overline{x}(t)) &= \overline{x}^T(t)P\overline{x}(t)\\
V_2(t,\overline{x}(t)) &= \int_{t-d_M}^{t}\overline{x}^T(s)Q_1\overline{x}(s)ds + \int_{t-\tau}^{t}\overline{x}^T(s)Q_2\overline{x}(s)ds\\
V_3(t,\overline{x}(t)) &= \int_{t-d_M}^{t}\int_{\theta}^{t}\dot{\overline{x}}^T(s)R_1\dot{\overline{x}}(s)ds\,d\theta\\
V_4(t,\overline{x}(t)) &= \int_{t-\tau}^{t}\int_{\theta}^{t}\dot{\overline{x}}^T(s)R_2\dot{\overline{x}}(s)ds\,d\theta,
\end{aligned} \tag{3.6}$$

and $P > 0$, $Q_i > 0$ and $R_i > 0$ $(i = 1,2)$ are matrices to be determined.

The derivative of $V_i(t,\overline{x}(t))$ $(i = 1,2,3,4)$ along the trajectory of system (3.1) can be shown as follows:

$$\dot{V}_1(t,\overline{x}(t)) = 2\overline{x}^T(t)P\mathcal{A}\xi(t) \tag{3.7}$$

$$\dot{V}_2(t,\overline{x}(t)) = \overline{x}^T(t)(Q_1+Q_2)\overline{x}(t) - \overline{x}^T(t-d_M)Q_1\overline{x}(t-d_M) - \overline{x}^T(t-\tau)Q_2\overline{x}(t-\tau) \tag{3.8}$$

$$\begin{aligned}
\dot{V}_3(t,\overline{x}(t)) &= d_M\dot{\overline{x}}^T(t)R_1\dot{\overline{x}}(t) - \int_{t-d_M}^{t}\dot{\overline{x}}^T(s)R_1\dot{\overline{x}}(s)ds\\
&= d_M\xi^T(t)\mathcal{A}^TR_1\mathcal{A}\xi(t) - \int_{t-d_M}^{t}\dot{\overline{x}}^T(s)R_1\dot{\overline{x}}(s)ds
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
\dot{V}_4(t,\overline{x}(t)) &= \tau\dot{\overline{x}}^T(t)R_2\dot{\overline{x}}(t) - \int_{t-\tau}^{t}\dot{\overline{x}}^T(s)R_2\dot{\overline{x}}(s)ds\\
&= \tau\xi^T(t)\mathcal{A}^TR_2\mathcal{A}\xi(t) - \int_{t-\tau}^{t}\dot{\overline{x}}^T(s)R_2\dot{\overline{x}}(s)ds.
\end{aligned} \tag{3.10}$$

Noting that (3.9), it follows from Lemma 2.6 that

$$-\int_{t-d_M}^{t}\dot{\overline{x}}^T(s)R_1\dot{\overline{x}}(s)ds \le \frac{1}{d_M}\begin{bmatrix}\overline{x}(t)\\ \overline{x}(t-d(t))\\ \overline{x}(t-d_M)\end{bmatrix}^T\begin{bmatrix}-R_1 & R_1 & 0\\ * & -2R_1 & R_1\\ * & * & -R_1\end{bmatrix}\begin{bmatrix}\overline{x}(t)\\ \overline{x}(t-d(t))\\ \overline{x}(t-d_M)\end{bmatrix}. \tag{3.11}$$

Employing the free matrix method [38, 39], it is easily derived that

$$\begin{aligned}
2\xi^T(t)S\left[\overline{x}(t) - \overline{x}(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot{\overline{x}}(s)ds\right] &= 0,\\
2\xi^T(t)T\left[\overline{x}(t-\tau(t)) - \overline{x}(t-\tau) - \int_{t-\tau}^{t-\tau(t)}\dot{\overline{x}}(s)ds\right] &= 0,
\end{aligned} \tag{3.12}$$

where

$$S = \begin{bmatrix} S_1^T & 0 & 0 & S_2^T & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$
$$T = \begin{bmatrix} 0 & 0 & 0 & T_1^T & T_2^T & 0 & 0 & 0 \end{bmatrix}^T. \tag{3.13}$$

It follows that from (3.12) that

$$-2\xi^T(t)S\int_{t-\tau(t)}^{t}\dot{\overline{x}}(s)ds \le \tau(t)\xi^T(t)SR_2^{-1}S^T\xi(t) + \int_{t-\tau(t)}^{t}\dot{\overline{x}}^T(s)R_2\dot{\overline{x}}(s)ds$$
$$-2\xi^T(t)T\int_{t-\tau}^{t-\tau(t)}\dot{\overline{x}}(s)ds \le (\tau-\tau(t))\xi^T(t)TR_2^{-1}T^T\xi(t) + \int_{t-\tau}^{t-\tau(t)}\dot{\overline{x}}^T(s)R_2\dot{\overline{x}}(s)ds. \tag{3.14}$$

By Assumption 2.3, the following inequality holds:

$$\begin{bmatrix} \overline{x}(t) \\ g(H\overline{x}(t)) \end{bmatrix}^T \begin{bmatrix} \overline{U}_1 & \overline{U}_2 \\ \overline{U}_2^T & I \end{bmatrix} \begin{bmatrix} \overline{x}(t) \\ g(H\overline{x}(t)) \end{bmatrix} \le 0, \tag{3.15}$$

where

$$\overline{U}_1 = H^T\hat{U}_1H, \qquad \overline{U}_2 = H^T\hat{U}_2$$
$$\hat{U}_1 = \frac{U_1^TU_2 + U_2^TU_1}{2}, \qquad \hat{U}_2 = \frac{U_1^T + U_2^T}{2}. \tag{3.16}$$

For all $\alpha, \beta > 0$, it can be derived from (3.15) that

$$-\alpha\begin{bmatrix} \overline{x}(t) \\ g(H\overline{x}(t)) \end{bmatrix}^T \begin{bmatrix} \overline{U}_1 & \overline{U}_2 \\ \overline{U}_2^T & I \end{bmatrix} \begin{bmatrix} \overline{x}(t) \\ g(H\overline{x}(t)) \end{bmatrix} \ge 0$$
$$-\beta\begin{bmatrix} \overline{x}(t-\tau(t)) \\ g(H\overline{x}(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} \overline{U}_1 & \overline{U}_2 \\ \overline{U}_2^T & I \end{bmatrix} \begin{bmatrix} \overline{x}(t-\tau(t)) \\ g(H\overline{x}(t-\tau(t))) \end{bmatrix} \ge 0. \tag{3.17}$$

Then, (2.15) can be rewritten as

$$\sigma\overline{x}^T(t-d(t))\Omega\overline{x}(t-d(t)) - \begin{bmatrix} e_k^T(t)C^TWCe_k(t) & 0 \\ 0 & 0 \end{bmatrix} > 0, \tag{3.18}$$

where

$$\Omega = \begin{bmatrix} C^TWC & 0 \\ 0 & 0 \end{bmatrix}. \tag{3.19}$$

It follows from (3.7)–(3.18) that

$$\begin{aligned}
\dot{V}(t,\overline{x}(t)) \leq{} & 2\overline{x}^T(t)P\mathcal{A}\xi(t) + \overline{x}^T(t)(Q_1+Q_2)\overline{x}(t) - \overline{x}^T(t-d_M)Q_1\overline{x}(t-d_M) \\
& - \overline{x}^T(t-\tau)Q_2\overline{x}(t-\tau) + d_M\xi^T(t)\mathcal{A}^T R_1\mathcal{A}\xi(t) + \tau\xi^T(t)\mathcal{A}^T R_2\mathcal{A}\xi(t) \\
& + 2\xi^T(t)S(\overline{x}(t) - \overline{x}(t-\tau(t))) + 2\xi^T(t)T(\overline{x}(t-\tau(t)) - \overline{x}(t-\tau)) \\
& + \frac{1}{d_M}\begin{bmatrix}\overline{x}(t)\\ \overline{x}(t-d(t))\\ \overline{x}(t-d_M)\end{bmatrix}^T \begin{bmatrix}-R_1 & R_1 & 0\\ * & -2R_1 & R_1\\ * & * & -R_1\end{bmatrix}\begin{bmatrix}\overline{x}(t)\\ \overline{x}(t-d(t))\\ \overline{x}(t-d_M)\end{bmatrix} \\
& - \alpha\begin{bmatrix}\overline{x}(t)\\ g(H\overline{x}(t))\end{bmatrix}^T\begin{bmatrix}\overline{U}_1 & \overline{U}_2\\ \overline{U}_2^T & I\end{bmatrix}\begin{bmatrix}\overline{x}(t)\\ g(H\overline{x}(t))\end{bmatrix} \\
& - \beta\begin{bmatrix}\overline{x}(t-\tau(t))\\ g(H\overline{x}(t-\tau(t)))\end{bmatrix}^T\begin{bmatrix}\overline{U}_1 & \overline{U}_2\\ \overline{U}_2^T & I\end{bmatrix}\begin{bmatrix}\overline{x}(t-\tau(t))\\ g(H\overline{x}(t-\tau(t)))\end{bmatrix} \\
& + \sigma\overline{x}^T(t-d(t))\Omega\overline{x}(t-d(t)) - \begin{bmatrix}e_k^T(t)C^T W C e_k(t) & 0\\ 0 & 0\end{bmatrix} \\
& + \tau(t)\xi^T(t)SR_2^{-1}S^T\xi(t) + (\tau-\tau(t))\xi^T(t)TR_2^{-1}T^T\xi(t) \\
={} & \xi^T(t)\left(\Pi + d_M\mathcal{A}^T R_1\mathcal{A} + \tau\mathcal{A}^T R_2\mathcal{A}\right)\xi(t) + \tau(t)\xi^T(t)SR_2^{-1}S^T\xi(t) \\
& + (\tau-\tau(t))\xi^T(t)TR_2^{-1}T^T\xi(t).
\end{aligned} \tag{3.20}$$

By using Schur complement and Lemma 2.5, it can be seen that (3.3) is equivalent to

$$\Pi + d_M\mathcal{A}^T R_1\mathcal{A} + \tau\mathcal{A}^T R_2\mathcal{A} + \tau(t)SR_2^{-1}S^T + (\tau-\tau(t))TR_2^{-1}T^T < 0 \tag{3.21}$$

which implies $\dot{V}(t,\overline{x}(t)) < -\varepsilon\|\overline{x}(t)\|^2$; then similar to [41], we can obtain the exponential stability of system (3.1). The proof is completed. □

Remark 3.2. From Theorem 3.1, it can be seen that the trigger parameters σ, W and the upper bound of time delay τ are involved in (3.3); for given σ, the corresponding trigger parameter W and the upper bound of τ can be obtained by using LMI toolbox in Matlab. From the simulation example, it can be derived that the larger the σ, the small the τ; the larger average release period, which means the load of network transmission will be reduced.

Remark 3.3. When the estimator gain matrix K is given, the conditions (3.3) are in the form of linear matrix inequalities, which can be readily solved by using the standard numerical software. The conditions (3.3) are not linear matrix inequalities when the estimator gain matrix K is a matrix variable to be designed, and thus Theorem 3.1 cannot be used to design K directly, a design method will be provided in the following Theorem.

After establishing analysis results in Theorem 3.1, the design problem of state estimator is to be considered and the following results can be readily derived from Theorem 3.1.

Theorem 3.4. *Suppose that Assumption 2.3 holds, the augmented system* (3.1) *is exponentially stable, if there exist* $P = \mathrm{diag}\{\mathbf{P}_1, \mathbf{P}_2\} > 0$*,* $Q_i = \mathrm{diag}\{\mathbf{Q}_i, \mathbf{Q}_i\} > 0$*,* $R_i = \mathrm{diag}\{\mathbf{R}_i, \mathbf{R}_i\} > 0$ *and* $S_i = \mathrm{diag}\{\mathbf{S}_i, \mathbf{S}_i\}$*,* $T_i = \mathrm{diag}\{\mathbf{T}_i, \mathbf{T}_i\}$ $(i = 1, 2)$ *and* V *with appropriate dimension, and two positive scalars* $\alpha > 0$*,* $\beta > 0$*, such that the following linear matrix inequalities hold:*

$$\overline{\Pi}_i = \begin{bmatrix} \overline{\Pi} & \overline{\Phi}_1 & \overline{\Phi}_2 & \overline{\Phi}_3^{(i)} \\ * & \overline{\Phi}_4 & 0 & 0 \\ * & * & \overline{\Phi}_5 & 0 \\ * & * & * & -\tau R_2 \end{bmatrix} < 0 \quad (i = 1, 2), \tag{3.22}$$

where

$$\begin{gathered} \overline{\Pi} = \begin{bmatrix} \overline{\Pi}_{11} & \overline{\Pi}_{12} & 0 & \overline{\Pi}_{13} & 0 & \overline{\Pi}_{14} & \overline{\Pi}_{15} & \overline{\Pi}_{16} \\ * & \overline{\Pi}_{22} & \overline{\Pi}_{23} & 0 & 0 & 0 & 0 & 0 \\ * & * & \overline{\Pi}_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \overline{\Pi}_{44} & \overline{\Pi}_{45} & 0 & \overline{\Pi}_{46} & 0 \\ * & * & * & * & \overline{\Pi}_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \overline{\Pi}_{66} & 0 & 0 \\ * & * & * & * & * & * & \overline{\Pi}_{77} & 0 \\ * & * & * & * & * & * & * & \overline{\Pi}_{88} \end{bmatrix} \\ \overline{\Phi}_1 = \begin{bmatrix} \overline{\Pi}_{17}^T & \overline{\Pi}_{24}^T & 0 & 0 & 0 & \overline{\Pi}_{67}^T & \overline{\Pi}_{78}^T & \overline{\Pi}_{89}^T \end{bmatrix}^T \\ \overline{\Phi}_2 = \begin{bmatrix} \overline{\Pi}_{18}^T & \overline{\Pi}_{25}^T & 0 & 0 & 0 & \overline{\Pi}_{68}^T & \overline{\Pi}_{79}^T & \overline{\Pi}_{8,10}^T \end{bmatrix}^T \\ \overline{\Phi}_3^{(1)} = \begin{bmatrix} \overline{\Pi}_{19}^T & 0 & 0 & \overline{\Pi}_{47}^T & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ \Phi_3^{(2)} = \begin{bmatrix} 0 & 0 & 0 & \widehat{\Pi}_{19}^T & \widehat{\Pi}_{47}^T & 0 & 0 & 0 \end{bmatrix}^T \\ \overline{\Phi}_4 = \begin{bmatrix} 2d_M\mathbf{P}_1 + d_M\mathbf{R}_1 & 0 \\ 0 & 2d_M\mathbf{P}_2 + d_M\mathbf{R}_1 \end{bmatrix} \\ \overline{\Phi}_5 = \begin{bmatrix} 2\tau\mathbf{P}_1 + \tau\mathbf{R}_2 & 0 \\ 0 & 2\tau\mathbf{P}_2 + \tau\mathbf{R}_2 \end{bmatrix} \\ \overline{\Pi}_{11} = \begin{bmatrix} -\mathbf{P}_1A - A^T\mathbf{P}_1 - \alpha\widehat{U}_1 & C^TV^T \\ VC & -\mathbf{P}_2A - A^T\mathbf{P}_2 - VC - C^TV^T \end{bmatrix} \\ + \begin{bmatrix} \mathbf{Q}_1 + \mathbf{Q}_2 - \dfrac{1}{d_M}\mathbf{R}_1 + \mathbf{S}_1 + \mathbf{S}_1^T & 0 \\ 0 & \mathbf{Q}_1 + \mathbf{Q}_2 - \dfrac{1}{d_M}\mathbf{R}_1 + \mathbf{S}_1 + \mathbf{S}_1^T \end{bmatrix} \end{gathered} \tag{3.23}$$

$$\overline{\Pi}_{12} = \begin{bmatrix} \frac{1}{d_M}\mathbf{R}_1 & 0 \\ VC & \frac{1}{d_M}\mathbf{R}_1 \end{bmatrix}, \qquad \overline{\Pi}_{13} = \begin{bmatrix} \mathbf{S}_2^T - \mathbf{S}_1 & 0 \\ 0 & \mathbf{S}_2^T - \mathbf{S}_1 \end{bmatrix}$$

$$\overline{\Pi}_{14} = \begin{bmatrix} \mathbf{P}_1 W_0 - \alpha\widehat{U}_2 \\ \mathbf{P}_2 W_0 \end{bmatrix}, \qquad \overline{\Pi}_{15} = \begin{bmatrix} \mathbf{P}_1 W_1 \\ \mathbf{P}_2 W_1 \end{bmatrix}$$

$$\overline{\Pi}_{16} = \begin{bmatrix} 0 \\ VC \end{bmatrix}, \qquad \overline{\Pi}_{17} = \begin{bmatrix} -d_M A^T \mathbf{P}_1 & d_M C^T V^T \\ 0 & -d_M A^T \mathbf{P}_2 - d_M C^T V^T \end{bmatrix}$$

$$\overline{\Pi}_{18} = \begin{bmatrix} -\tau A^T \mathbf{P}_1 & \tau C^T V^T \\ 0 & -\tau A^T \mathbf{P}_2 - \tau C^T V^T \end{bmatrix}, \qquad \overline{\Pi}_{19} = \begin{bmatrix} \tau \mathbf{S}_1 & 0 \\ 0 & \tau \mathbf{S}_1 \end{bmatrix}$$

$$\widehat{\Pi}_{19} = \begin{bmatrix} \tau \mathbf{T}_1 & 0 \\ 0 & \tau \mathbf{T}_1 \end{bmatrix}, \qquad \overline{\Pi}_{22} = \begin{bmatrix} -\frac{2}{d_M}\mathbf{R}_1 + \sigma C^T W C & 0 \\ 0 & -\frac{2}{d_M}\mathbf{R}_1 \end{bmatrix}$$

$$\overline{\Pi}_{23} = \begin{bmatrix} \frac{1}{d_M}\mathbf{R}_1 & 0 \\ 0 & \frac{1}{d_M}\mathbf{R}_1 \end{bmatrix}, \qquad \overline{\Pi}_{24} = \begin{bmatrix} 0 & -d_M C^T V^T \\ 0 & 0 \end{bmatrix}$$

$$\overline{\Pi}_{25} = \begin{bmatrix} 0 & \tau C^T V^T \\ 0 & 0 \end{bmatrix}, \qquad \overline{\Pi}_{33} = \begin{bmatrix} -\mathbf{Q}_1 - \frac{1}{d_M}\mathbf{R}_1 & 0 \\ 0 & -\mathbf{Q}_1 - \frac{1}{d_M}\mathbf{R}_1 \end{bmatrix}$$

$$\overline{\Pi}_{44} = \begin{bmatrix} -\mathbf{S}_2 - \mathbf{S}_2^T + \mathbf{T}_1 + \mathbf{T}_1^T - \beta\widehat{U}_1 & 0 \\ 0 & -\mathbf{S}_2 - \mathbf{S}_2^T + \mathbf{T}_1 + \mathbf{T}_1^T \end{bmatrix}, \qquad \overline{\Pi}_{45} = \begin{bmatrix} \mathbf{T}_2^T - \mathbf{T}_1 & 0 \\ 0 & \mathbf{T}_2^T - \mathbf{T}_1 \end{bmatrix}$$

$$\overline{\Pi}_{46} = \begin{bmatrix} -\beta\widehat{U}_2 \\ 0 \end{bmatrix}, \qquad \overline{\Pi}_{47} = \begin{bmatrix} \tau \mathbf{S}_2 & 0 \\ 0 & \tau \mathbf{S}_2 \end{bmatrix}, \qquad \widehat{\Pi}_{47} = \begin{bmatrix} \tau \mathbf{T}_2 & 0 \\ 0 & \tau \mathbf{T}_2 \end{bmatrix}$$

$$\overline{\Pi}_{55} = \begin{bmatrix} -\mathbf{Q}_2 - \mathbf{T}_2 - \mathbf{T}_2^T & 0 \\ 0 & -\mathbf{Q}_2 - \mathbf{T}_2 - \mathbf{T}_2^T \end{bmatrix}, \qquad \overline{\Pi}_{66} = -\alpha I$$

$$\overline{\Pi}_{67} = \begin{bmatrix} d_M W_0^T \mathbf{P}_1 & d_M W_0^T \mathbf{P}_2 \end{bmatrix}, \qquad \overline{\Pi}_{68} = \begin{bmatrix} \tau W_0^T \mathbf{P}_1 & \tau W_0^T \mathbf{P}_2 \end{bmatrix}, \qquad \overline{\Pi}_{77} = -\beta I$$

$$\overline{\Pi}_{78} = \begin{bmatrix} d_M W_1^T \mathbf{P}_1 & d_M W_1^T \mathbf{P}_2 \end{bmatrix}, \qquad \overline{\Pi}_{79} = \begin{bmatrix} \tau W_1^T \mathbf{P}_1 & \tau W_1^T \mathbf{P}_2 \end{bmatrix}, \qquad \overline{\Pi}_{88} = -C^T W C$$

$$\overline{\Pi}_{89} = \begin{bmatrix} 0 & -d_M C^T V^T \end{bmatrix}, \qquad \overline{\Pi}_{8,10} = \begin{bmatrix} 0 & -\tau C^T V^T \end{bmatrix}, \tag{3.24}$$

then the desired estimator gain matrix is given as $K = \mathbf{P}_2^{-1} V$.

Proof. By using Schur complement in Theorem 3.1, $\Pi_i < 0$ $(i = 1, 2)$ can be rewritten as

$$\begin{gathered} \Pi + d_M \mathcal{A}^T R_1 \mathcal{A} + \tau \mathcal{A}^T R_2 \mathcal{A} + \tau S R_2^{-1} S^T < 0 \\ \Pi + d_M \mathcal{A}^T R_1 \mathcal{A} + \tau \mathcal{A}^T R_2 \mathcal{A} + \tau T R_2^{-1} T^T < 0. \end{gathered} \tag{3.25}$$

By using Lemma 2.5, (3.25) are equivalent to the following matrix inequalities

$$\begin{bmatrix} \overline{\Pi} & \widehat{\Phi}_1 & \widehat{\Phi}_2 & \Phi_3^{(i)} \\ * & -d_M R_1^{-1} & 0 & 0 \\ * & * & -\tau R_2^{-1} & 0 \\ * & * & * & -\tau R_2 \end{bmatrix} < 0 \quad (i = 1, 2), \tag{3.26}$$

where

$$\overline{\Pi} = \begin{bmatrix} \overline{\Pi}_{11} & \overline{\Pi}_{12} & 0 & \overline{\Pi}_{13} & 0 & \overline{\Pi}_{14} & \overline{\Pi}_{15} & \overline{\Pi}_{16} \\ * & \overline{\Pi}_{22} & \overline{\Pi}_{23} & 0 & 0 & 0 & 0 & 0 \\ * & * & \overline{\Pi}_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \overline{\Pi}_{44} & \overline{\Pi}_{45} & 0 & \overline{\Pi}_{46} & 0 \\ * & * & * & * & \overline{\Pi}_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \overline{\Pi}_{66} & 0 & 0 \\ * & * & * & * & * & * & \overline{\Pi}_{77} & 0 \\ * & * & * & * & * & * & * & \overline{\Pi}_{88} \end{bmatrix}$$

$$\widehat{\Phi}_1 = \left[d_M\overline{A} \;\; d_M\overline{B} \;\; 0 \;\; 0 \;\; 0 \;\; d_M\overline{W}_0 \;\; d_M\overline{W}_1 \;\; d_M\overline{C} \right]^T$$
$$\widehat{\Phi}_2 = \left[\tau\overline{A} \;\; \tau\overline{B} \;\; 0 \;\; 0 \;\; 0 \;\; \tau\overline{W}_0 \;\; \tau\overline{W}_1 \;\; \tau\overline{C} \right]^T. \tag{3.27}$$

Then performing a congruence transformation of diag$\{I, P, P, I\}$ to (3.26), it can be derived that

$$\begin{bmatrix} \overline{\Pi} & \widetilde{\Phi}_1 & \widetilde{\Phi}_2 & \Phi_3^{(i)} \\ * & -d_M P R_1^{-1} P & 0 & 0 \\ * & * & -\tau P R_2^{-1} P & 0 \\ * & * & * & -\tau R_2 \end{bmatrix} < 0 \quad (i = 1, 2), \tag{3.28}$$

where

$$\widetilde{\Phi}_1 = \left[d_M P\overline{A} \;\; d_M P\overline{B} \;\; 0 \;\; 0 \;\; 0 \;\; d_M P\overline{W}_0 \;\; d_M P\overline{W}_1 \;\; d_M P\overline{C} \right]^T$$
$$\widetilde{\Phi}_2 = \left[\tau P\overline{A} \;\; \tau P\overline{B} \;\; 0 \;\; 0 \;\; 0 \;\; \tau P\overline{W}_0 \;\; \tau P\overline{W}_1 \;\; \tau P\overline{C} \right]^T. \tag{3.29}$$

Setting $P_2 K = V$ in (3.28) and considering the following inequality:

$$-P R_i^{-1} P \leq -2P + R_i \quad (i = 1, 2). \tag{3.30}$$

By using (3.30), we can obtain

$$\begin{bmatrix} \Pi & \tilde{\Phi}_1 & \tilde{\Phi}_2 & \Phi_3^{(i)} \\ * & -d_M P R_1^{-1} P & 0 & 0 \\ * & * & -\tau P R_2^{-1} P & 0 \\ * & * & * & -\tau R_2 \end{bmatrix} < \begin{bmatrix} \Pi & \tilde{\Phi}_1 & \tilde{\Phi}_2 & \Phi_3^{(i)} \\ * & -2d_M P + d_M R_1 & 0 & 0 \\ * & * & -2\tau P + \tau R_2 & 0 \\ * & * & * & -\tau R_2 \end{bmatrix}. \tag{3.31}$$

Substitute $\overline{A}$, $\overline{B}$, $\overline{W}_0$, H,$\overline{W}_1$, $\overline{C}$, P, Q_i, R_i, S_i, T_i $(i = 1, 2)$ into the right of (3.31), combining (3.22), we can obtain

$$\begin{bmatrix} \Pi & \tilde{\Phi}_1 & \tilde{\Phi}_2 & \Phi_3^{(i)} \\ * & -2d_M P + d_M R_1 & 0 & 0 \\ * & * & -2\tau P + \tau R_2 & 0 \\ * & * & * & -\tau R_2 \end{bmatrix} < 0 \quad (i = 1, 2). \tag{3.32}$$

The rest of the proof follows directly from Theorem 3.1. □

Remark 3.5. When the estimator gain matrix K is a matrix variable to be designed, in order to transform the conditions (3.3) to linear matrix inequalities, and meanwhile reduce the computational complexity (i.e., reduce the number of matrix variables), in Theorem 3.4, matrix variables in Theorem 3.1 are replaced by some diagonal matrices. Then setting $P_2 K = V$, we can obtain (3.22), which is in the form of linear matrix inequalities, which are easy to be verified by LMI toolbox.

Remark 3.6. It is noticed that $d_M = h + d$, if d_M is solved, we can select a sampling period $h < d_M$. For given d, the maximal allowable sampling period $h_{\max}$ can be obtained by the following two-step procedure.

(1) For given τ and d, setting $h_{\max} = h_0$ and step size STEP $=$ STEP$_0$, where h_0 and STEP$_0$ are two specified positive constants.

(2) If LMIs (3.22) are feasible, set $h_{\max} = h_0 + \text{STEP}_0$ and return to step (2): otherwise, h is the maximal allowable sampling period.

4. Numerical Results

In this section, a numerical example is given to verify the effectiveness of the proposed control techniques for estimation of recurrent neural networks with time-varying delays.

Example 4.1. Consider recurrent neural networks (2.1) with the following parameters

$$A = \begin{bmatrix} 1.5 & 0 \\ 0 & 2 \end{bmatrix}, \qquad W_0 = \begin{bmatrix} 0.3 & -0.4 \\ -0.4 & 0.3 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.3 \end{bmatrix}, \qquad C = \begin{bmatrix} 0.9 & 0.8 \\ 0.7 & 0.5 \end{bmatrix}. \tag{4.1}$$

The neuron activation function is described as follows:

$$g(x) = \begin{bmatrix} 0.5x_1(t) - \tanh(0.2x_1(t)) + 0.2x_2(t) \\ 0.95x_2(t) - \tanh(0.75x_2(t)) \end{bmatrix}. \tag{4.2}$$

Table 1: $d_M = 0.01$.

σ	0	0.01	0.1	0.2	0.3	0.4	0.5
τ	1.2134	1.1966	1.1572	1.1570	1.1569	1.1569	1.1569

Table 2: $\tau = 1$, $d = 0.01$.

σ	0	0.01	0.1	0.15	0.2	0.3	0.99
$h_{\max}$	0.2244	0.2106	0.1998	0.1998	0.1998	0.1998	0.1998

It is easy to verify that the nonlinear function $f(\cdot)$ satisfies Assumption 2.3; by some simple calculations, we can obtain

$$U_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix} \qquad U_2 = \begin{bmatrix} 0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}. \tag{4.3}$$

Setting $d_M = 0.01$ and $\sigma = 0.1$, by applying Theorem 3.4, it can be obtained the maximum allowable delay $\tau = 1.1572$. More detailed calculation results for different values of σ are given in Table 1. It can be shown that the larger σ, the smaller τ. For given $\tau = 1$ and $d = 0.01$, based on Remark 3.6, we can obtain the maximal allowable sampling period $h_{\max}$, which are shown in Table 2. For given $\tau = 1$, $\sigma = 0.1$ and $d_M = 0.01$, by using LMI Toolbox in LMIs (3.22), the feasible solution can be obtained as follows:

$$\begin{gathered}
\mathbf{P}_1 = \begin{bmatrix} 5.4676 & -0.1329 \\ -0.1329 & 5.4000 \end{bmatrix}, \quad \mathbf{P}_2 = \begin{bmatrix} 5.0204 & -0.1155 \\ -0.1155 & 4.2212 \end{bmatrix}, \quad \mathbf{Q}_1 = \begin{bmatrix} 2.7863 & -0.0955 \\ -0.0955 & 2.0982 \end{bmatrix} \\
\mathbf{Q}_2 = \begin{bmatrix} 3.2539 & -0.0133 \\ -0.0133 & 2.8910 \end{bmatrix}, \quad \mathbf{R}_1 = \begin{bmatrix} 0.0222 & 0.0010 \\ 0.0010 & 0.0234 \end{bmatrix}, \quad \mathbf{R}_2 = \begin{bmatrix} 2.2237 & -0.0246 \\ -0.0246 & 1.4752 \end{bmatrix} \\
\mathbf{S}_1 = \begin{bmatrix} -0.5503 & 0.0195 \\ 0.0478 & -0.4156 \end{bmatrix}, \quad \mathbf{S}_2 = \begin{bmatrix} 1.0185 & 0.0674 \\ 0.1378 & 1.1920 \end{bmatrix}, \quad \mathbf{T}_1 = \begin{bmatrix} -1.0844 & -0.0634 \\ -0.0902 & -1.2378 \end{bmatrix} \\
\mathbf{T}_2 = \begin{bmatrix} 0.3511 & -0.0221 \\ -0.0195 & 0.3384 \end{bmatrix}, \quad V = \begin{bmatrix} -0.0792 & -0.1282 \\ -0.1755 & -0.0863 \end{bmatrix}, \quad \alpha = 6.4604, \ \beta = 5.7723.
\end{gathered} \tag{4.4}$$

Then the triggered matrix and the desired estimator can be obtained as follows:

$$W = \begin{bmatrix} 4.6153 & -2.7354 \\ -2.7354 & 6.5131 \end{bmatrix}, \qquad K = \begin{bmatrix} -0.0165 & -0.0308 \\ -0.0354 & -0.0214 \end{bmatrix}. \tag{4.5}$$

For giving the sampling period $h = 0.005$, Table 3 gives the relation of the trigger parameter σ, trigger times, the average release period, and the percentage of data transmissions; it can be seen that the larger the σ, the smaller trigger times; the larger average release period, the smaller percentage of data transmission, which are reasonable results. In the following, we provide some simulation results: when $\sigma = 0$, the time varying delay $\tau(t)$ obeys uniform distribution over $[0, 1]$, and the curves of the error dynamics of the neural networks $e_i(t)$ $(i = 1, 2)$ are depicted in Figure 1, from which we can see the errors converge to zero asymptotically. If setting $\sigma = 0.1$, The response of the error dynamics for the delayed

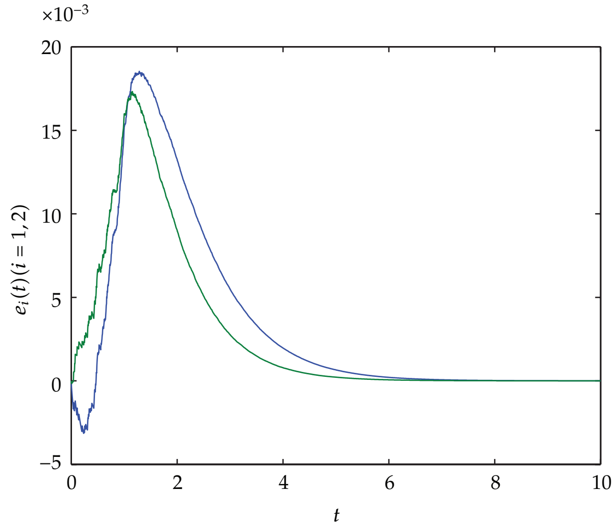

Figure 1: The error curves $e_i(t)$ $(i = 1, 2)$ with trigger parameter $\sigma = 0$ (time-triggered scheme).

Table 3: $h = 0.005$, $d_M = 0.01$, $\tau = 0.1$, $t = 10$.

σ	0	0.01	0.1
Trigger times	2000	188	74
Trigger matrix W	$\begin{bmatrix} 0.7582 & -0.2843 \\ -0.2843 & 0.9490 \end{bmatrix}$	$\begin{bmatrix} 0.7504 & -0.2881 \\ -0.2881 & 0.9444 \end{bmatrix}$	$\begin{bmatrix} 4.6153 & -2.7354 \\ -2.7354 & 6.5131 \end{bmatrix}$
Estimator matrix K	$\begin{bmatrix} -0.0070 & -0.0553 \\ -0.0398 & -0.0257 \end{bmatrix}$	$\begin{bmatrix} -0.0068 & -0.0552 \\ -0.0394 & -0.0255 \end{bmatrix}$	$\begin{bmatrix} -0.0165 & -0.0308 \\ -0.0354 & -0.0214 \end{bmatrix}$
Average release period	0.0050	0.0531	0.1348
Data transmission	100%	9.42%	3.71%

neural networks (2.17) which converge to zero asymptotically in the mean square is given in Figure 2. Figure 3 shows the event-triggered release instants and intervals. It can be seen from Figures 1 and 2 that the simulation results are almost the same, but the percentage of data transmission under even-triggered scheme used much small number than time-triggering scheme. To make this clear, seen the computation results lists in Table 2, from which we can see that data transmission rate with even-triggered scheme ($\sigma = 0.1$) is only 3.71% of sampled measurement output with time-triggered scheme ($\sigma = 0$); from these results, we can draw a conclusion that event-triggered scheme has advantage over the time-triggered scheme in improving the resource utilization.

5. Conclusions

This paper has provided a novel event-triggered scheme to investigate the sampled-data state estimation problem for a class of recurrent neural networks with time-varying delays. This scheme can lead to a significant reduction of the information communication burden in the

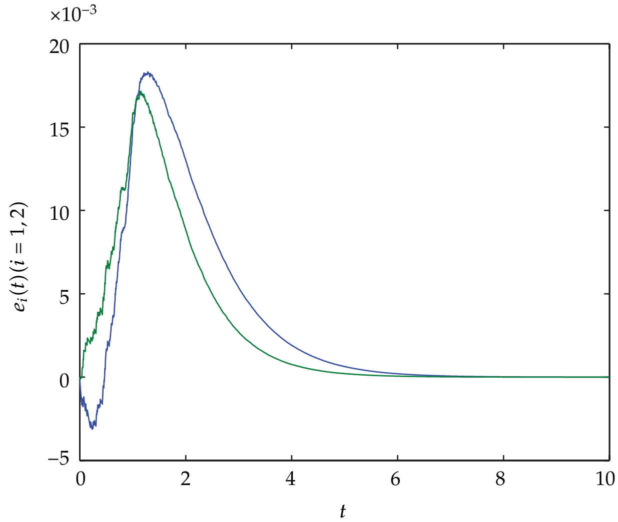

Figure 2: The error curves $e_i(t)$ $(i = 1, 2)$ with trigger parameter $\sigma = 0.1$ (event-triggered scheme).

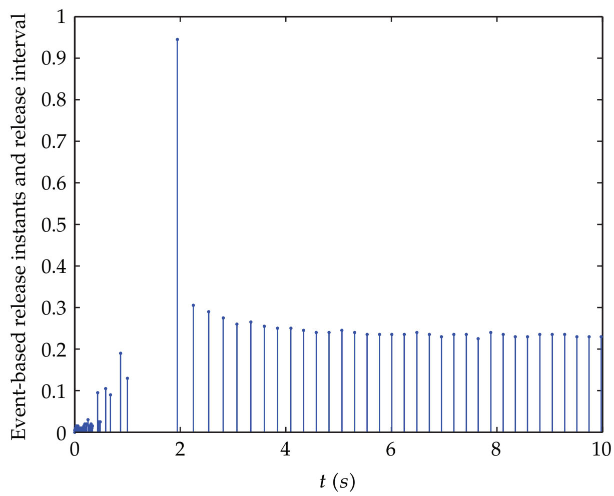

Figure 3: Release instants and release interval by event-triggered scheme.

network. By using a delayed-input approach, the error dynamics system is equivalently to a dynamic system with two different time-varying delays. Based on the Lyapunov-krasovskii functional approach, a state estimator of the considered neural networks can be achieved by solving some linear matrix inequalities, which can be readily solved by using the standard numerical software. Finally, an illustrative example is exploited to show the effectiveness of the event-triggered scheme.

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