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Research Article

The Improved Riccati Equation Method and Exact Solutions to mZK Equation

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We utilize the improved Riccati equation method to construct more general exact solutions to nonlinear equations. And we obtain the travelling wave solutions involving parameters, which are expressed by the hyperbolic functions, the trigonometric functions, and the rational functions. When the parameters are taken as special values, the method provides not only solitary wave solutions but also periodic waves solutions. The method appears to be easier and more convenient by means of a symbolic computation system. Of course, it is also effective to solve other nonlinear evolution equations in mathematical physics.

1. Introduction

More and more problems in the branches of modern mathematical physics are described in terms of suitable nonlinear models, and nonlinear physical phenomena are related to nonlinear equations, which are involved in many fields from physics to biology, chemistry, mechanics, and so forth. Nonlinear wave phenomena are very important in nonlinear sciences, in recent years, much effort has been spent on the construction of exact solutions of nonlinear partial differential equations. Many powerful and efficient methods have been presented to obtain the exact traveling wave solutions of nonlinear evolution equations, such as the Backlund transformation method [1, 2], Exp-function method [3, 4], homogeneous balance method [5, 6], tanh-function method [7, 8], the Jacobi elliptic function expansion [9, 10], and the G'/G-expansion method [11, 12]. A search of directly seeking for exact solutions of nonlinear equations has been more interesting because of the availability of symbolic computation, Mathematica or Maple. These computation systems are adequately utilized to perform some complicated and tedious algebraic and differential calculations on a computer. By using these methods and tools, one can successfully obtain exact solutions.

The ZK equation governs the behavior of weakly nonlinear ion-acoustic waves in plasma comprising cold ions and hot isothermal electrons in the presence of a uniform magnetic field. When the ion or electron plasma does not meet the Boltzmann distribution, Munro and Parkes derive the modified ZK equation (mZK equation), they also studied planar periodic two-dimensional long-wave perturbation wave solutions and the stability of traveling wave solutions in isolation [13, 14]. The mZK equation represents an anisotropic two-dimensional generalization of the KdV equation and can be derived in a magnetized plasma for small amplitude Alfven wave at a critical angle to undisturbed magnetic field. mZK equation is effectively applied to describe the evolution of various solitary waves in isothermal multicomponent magnetized plasma, similarly the description of stability of solitary waves of mZK equation has also appeared in [15]. The mZK equation has attracted the attention of many researchers in the past few years. For instance, from the mathematical point of view, local and global existence for the Cauchy problem was studied in [16–18].

The G'/G-expansion method was proposed originally by Wang et al., which is one of the most effective direct methods to obtain travelling wave solutions for a large number of nonlinear evolution equations. This useful method is widely employed by many authors [11, 12]. The key ideas of the G'/G-expansion method are that the travelling wave solutions of nonlinear evolution equations can be expressed by polynomials in G'/G, where G satisfies a second order linear differential equation, the degree of the polynomials can be determined by considering the homogeneous balance between the highest order partial derivatives and nonlinear terms appearing in nonlinear evolution equations considered, the coefficients of the polynomials can be obtained by solving a set of simultaneous algebraic equations resulted from the process of using the proposed method.

The paper is motivated by the desire to present a new method, named the improved Riccati equation method, so that it can be successfully applied to seeking the exact travelling wave solutions to the mZK equation. We will obtion two group values of coefficients regarding Riccati equation and nonlinear evolution equation. By contrast to both Riccati equation method and G'/G-expansion method, at this point, it is surely a meaningful improvement and innovation we have made to obtain much more abundant solutions. Following the description of the improved Riccati equation method, one can have access to exact solutions to nonlinear evolution equations smoothly.

2. Description of the Improved Riccati Equation Method

Step 1. We consider the nonlinear evolution equations in three independent variables x, y, t and dependent variable u:

$$N(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, u_y, u_{xy}, \ldots) = 0. \tag{2.1}$$

We seek their traveling wave solutions in the following form

$$u(x, y, t) = u(\xi), \quad \xi = kx + cy + dt, \tag{2.2}$$

where k, c, and d are arbitrary constants.

Equation (2.1) can be converted to an ordinary differential equation

$$N_1(u, u', u'', \ldots) = 0. \tag{2.3}$$

Step 2. In order to construct travelling wave solutions of nonlinear equations, it is reasonable to introduce the following ansatz

$$u(x, y, t) = u(\xi) = \sum_{i=-n}^{n} a_i f^i(\xi), \tag{2.4}$$

where a_i are constants to be determined later, the balancing number n is a positive integer which can be determined by balancing the highest order derivative terms with the highest power nonlinear terms in (2.3) and $f(\xi)$ satisfies the following elliptic equation:

$$f'(\xi) = p + rf(\xi) + qf^2(\xi), \tag{2.5}$$

where p, r, q are real parameters. And $f(\xi)$ can also be expanded to the following ansatz:

$$f(\xi) = \sum_{i=-m}^{m} b_i \left(\frac{G'}{G}\right)^m, \tag{2.6}$$

and $G(\xi)$ satisfies the following elliptic equations:

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \tag{2.7}$$

where b_i are constants to be determined later, λ, μ are real parameters. m is a positive integer which can be determined by balancing the highest order derivative terms with the highest power nonlinear terms in (2.5), and so we can get $m = 1$.

Step 3. We substitute (2.6) and (2.7) into (2.5), equating the coefficients of all powers of (G'/G) to zero, and we can get solutions of $f(\xi)$ with computerized symbolic computation.

Step 4. Then we substitute (2.4) and (2.5) into (2.3), equating the coefficients of all powers of $f(\xi)$ to zero, solving this set of algebraic equation with computerized symbolic computation, inserting these results and solutions of $f(\xi)$ into (2.4). Finally, setting $\xi = kx + cy + dt$, we obtain the exact travelling wave solutions of (2.1).

3. Applications

We consider the modified Zakharov-Kuznetsov (mZK) equations in the following form:

$$u_t + u^2 u_x + u_{xxx} + u_{xyy} = 0. \tag{3.1}$$

We also make the transformation $\xi = kx + sy - \omega t$, where ω, k, s are constants to be determined later. Then (3.1) is reduced to the following:

$$-\omega u + \frac{ku^3}{3} + \left(k^3 + ks^2\right)u'' = 0. \tag{3.2}$$

By balancing the highest order derivative terms and nonlinear terms in (3.2), so we get $n = 1$. Then we can suppose that (3.2) has the solutions in the form:

$$u(\xi) = a_{-1}f^{-1}(\xi) + a_0 + a_1 f(\xi). \tag{3.3}$$

On substituting (2.5) and (3.3) into (3.2), collecting all terms with the same powers of $f^i(\xi)$ and setting each coefficient of the polynomials to zero, solving the over-determined algebraic equations by Mathematica, we can obtain the following results.

Set 1.

$$\begin{gathered}\omega = -\frac{1}{2}k\left(4pq - r^2\right)\left(k^2 + s^2\right), \qquad a_{-1} = \pm p\sqrt{6(-k^2 - s^2)}, \\ a_0 = \pm\sqrt{\frac{3}{2}r^2(-k^2 - s^2)}, \qquad a_1 = 0, \qquad k^2 + s^2 \neq 0,\ k \neq 0,\ p \neq 0,\ r \neq 0.\end{gathered} \tag{3.4}$$

Set 2.

$$\begin{gathered}\omega = -\frac{1}{2}k\left(4pq - r^2\right)\left(k^2 + s^2\right), \qquad a_{-1} = 0, \qquad a_0 = \pm\sqrt{\frac{3}{2}r^2(-k^2 - s^2)}, \\ a_1 = \pm q\sqrt{6(-k^2 - s^2)}, \quad k^2 + s^2 \neq 0,\ k \neq 0,\ q \neq 0,\ r \neq 0.\end{gathered} \tag{3.5}$$

Similarly, we can also get the following result.

Case 1.

$$q \neq 0, \qquad p = \frac{r^2 - \lambda^2 + 4\mu}{4q}, \qquad b_0 = \frac{-\lambda - r}{2q}, \qquad b_1 = -\frac{1}{q}, \qquad b_{-1} = 0. \tag{3.6}$$

Case 2.

$$q \neq 0, \qquad p = \frac{r^2 - \lambda^2 + 4\mu}{4q}, \qquad b_0 = \frac{\lambda - r}{2q}, \qquad b_1 = 0, \qquad b_{-1} = p + \frac{\lambda^2 - r^2}{4q}. \tag{3.7}$$

Using Case 1, substituting Sets 1, 2, and the general solutions of (2.6) into formula (3.3), we have three types of travelling wave solutions as follows (c_1 and c_2 are arbitrary constants, $\xi = kx + sy + (1/2)k(4pq - r^2)(k^2 + s^2)t$).

When $r^2 - 4pq > 0$, we obtain the hyperbolic function solutions of (3.1)

$$\begin{aligned}
u_1(\xi) &= \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)} \\
&\times\left(-\frac{\lambda+r}{2q}-\frac{1}{q}\left(\frac{\sqrt{r^2-4pq}}{2}\frac{c_1\sinh\left(\sqrt{r^2-4pq}/2\right)\xi+c_2\cosh\left(\sqrt{r^2-4pq}/2\right)\xi}{c_2\sinh\left(\sqrt{r^2-4pq}/2\right)\xi+c_1\cosh\left(\sqrt{r^2-4pq}/2\right)\xi}-\frac{\lambda}{2}\right)\right)^{-1}, \\
u_2(\xi) &= \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm q\sqrt{6(-k^2-s^2)} \\
&\times\left(-\frac{\lambda+r}{2q}-\frac{1}{q}\left(\frac{\sqrt{r^2-4pq}}{2}\frac{c_1\sinh\left(\sqrt{r^2-4pq}/2\right)\xi+c_2\cosh\left(\sqrt{r^2-4pq}/2\right)\xi}{c_2\sinh\left(\sqrt{r^2-4pq}/2\right)\xi+c_1\cosh\left(\sqrt{r^2-4pq}/2\right)\xi}-\frac{\lambda}{2}\right)\right).
\end{aligned} \tag{3.8}$$

If $c_1 \neq 0, c_1^2 > c_2^2$, then $u(\xi)$ becomes the solitary wave solutions of (3.1) as follows:

$$\begin{aligned}
u_1(\xi) &= \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)} \\
&\quad\times\left(-\frac{\lambda+r}{2q}-\frac{1}{q}\left(\frac{\sqrt{r^2-4pq}}{2}\tanh\left(\frac{\sqrt{r^2-4pq}}{2}\xi+\xi_0\right)-\frac{\lambda}{2}\right)\right)^{-1} \\
u_2(\xi) &= \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm q\sqrt{6(-k^2-s^2)} \\
&\quad\times\left(-\frac{\lambda+r}{2q}-\frac{1}{q}\left(\frac{\sqrt{r^2-4pq}}{2}\tanh\left(\frac{\sqrt{r^2-4pq}}{2}\xi+\xi_0\right)-\frac{\lambda}{2}\right)\right),
\end{aligned} \tag{3.9}$$

where $\xi_0 = \tanh^{-1}(c_2/c_1)$.

If $c_2 \neq 0, c_2^2 > c_1^2$, then $u(\xi)$ becomes the solitary wave solutions of (3.1) as follows:

$$u_1(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)} \times\left(-\frac{\lambda+r}{2q}-\frac{1}{q}\left(\frac{\sqrt{r^2-4pq}}{2}\coth\left(\frac{\sqrt{r^2-4pq}}{2}\xi+\xi_0\right)-\frac{\lambda}{2}\right)\right)^{-1},$$
$$u_2(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm q\sqrt{6(-k^2-s^2)} \times\left(-\frac{\lambda+r}{2q}-\frac{1}{q}\left(\frac{\sqrt{r^2-4pq}}{2}\coth\left(\frac{\sqrt{r^2-4pq}}{2}\xi+\xi_0\right)-\frac{\lambda}{2}\right)\right), \tag{3.10}$$

where $\xi_0 = \tanh^{-1}(c_1/c_2)$.

When $r^2-4pq=0$, we get the rational function solutions of (3.1)

$$u_1(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)}\left(-\frac{\lambda+r}{2q}-\frac{1}{q}\left(\frac{c_2}{c_1+c_2\xi}-\frac{\lambda}{2}\right)\right)^{-1},$$
$$u_2(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm q\sqrt{6(-k^2-s^2)}\left(-\frac{\lambda+r}{2q}-\frac{1}{q}\left(\frac{c_2}{c_1+c_2\xi}-\frac{\lambda}{2}\right)\right). \tag{3.11}$$

When $r^2-4pq<0$, we obtain the trigonometric function solutions of (3.1)

$$u_1(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)} \times\left(-\frac{\lambda+r}{2q}-\frac{1}{q}\left(\frac{\sqrt{4pq-r^2}}{2}\frac{c_2\cos\left(\sqrt{4pq-r^2}/2\right)\xi-c_1\sin\left(\sqrt{4pq-r^2}/2\right)\xi}{c_1\cos\left(\sqrt{4pq-r^2}/2\right)\xi+c_2\sin\left(\sqrt{4pq-r^2}/2\right)\xi}-\frac{\lambda}{2}\right)\right)^{-1},$$
$$u_2(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm q\sqrt{6(-k^2-s^2)} \times\left(-\frac{\lambda+r}{2q}-\frac{1}{q}\left(\frac{\sqrt{4pq-r^2}}{2}\frac{c_2\cos\left(\sqrt{4pq-r^2}/2\right)\xi-c_1\sin\left(\sqrt{4pq-r^2}/2\right)\xi}{c_1\cos\left(\sqrt{4pq-r^2}/2\right)\xi+c_2\sin\left(\sqrt{4pq-r^2}/2\right)\xi}-\frac{\lambda}{2}\right)\right). \tag{3.12}$$

If $c_1 \neq 0, c_1^2 > c_2^2$, then $u(\xi)$ becomes the periodic wave solutions of (3.1) as follows:

$$u_1(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)} \times \left(-\frac{\lambda+r}{2q} - \frac{1}{q}\left(\frac{\sqrt{4pq-r^2}}{2}\tan\left(\xi_0 - \frac{\sqrt{4pq-r^2}}{2}\xi\right) - \frac{\lambda}{2}\right)\right)^{-1},$$
$$u_2(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm q\sqrt{6(-k^2-s^2)} \times \left(-\frac{\lambda+r}{2q} - \frac{1}{q}\left(\frac{\sqrt{4pq-r^2}}{2}\tan\left(\xi_0 - \frac{\sqrt{4pq-r^2}}{2}\xi\right) - \frac{\lambda}{2}\right)\right), \tag{3.13}$$

where $\xi_0 = \tan^{-1}(c_2/c_1)$.

If $c_2 \neq 0, c_2^2 > c_1^2$, then $u(\xi)$ becomes the periodic wave solutions of (3.1) as follows:

$$u_1(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)} \times \left(-\frac{\lambda+r}{2q} - \frac{1}{q}\left(\frac{\sqrt{4pq-r^2}}{2}\cot\left(\frac{\sqrt{4pq-r^2}}{2}\xi + \xi_0\right) - \frac{\lambda}{2}\right)\right)^{-1},$$
$$u_2(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm q\sqrt{6(-k^2-s^2)} \times \left(-\frac{\lambda+r}{2q} - \frac{1}{q}\left(\frac{\sqrt{4pq-r^2}}{2}\cot\left(\frac{\sqrt{4pq-r^2}}{2}\xi + \xi_0\right) - \frac{\lambda}{2}\right)\right), \tag{3.14}$$

where $\xi_0 = \tan^{-1}(c_1/c_2)$.

Using Case 2, substituting Sets 1, 2, and the general solutions of (2.6) into formula (3.3), we have three types of travelling wave solutions as follows (c_1 and c_2 are arbitrary constants, $\xi = kx + sy + (1/2)k(4pq - r^2)(k^2 + s^2)t$).

When $r^2 - 4pq > 0$, we obtain the hyperbolic function solutions of (3.1)

$$\begin{aligned}
&u_1(\xi)\\
&= \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)}\\
&\quad\times\left(\frac{\lambda-r}{2q}+\frac{\mu}{q}\left(\frac{\sqrt{r^2-4pq}}{2}\frac{c_1\sinh\left(\sqrt{r^2-4pq}/2\right)\xi+c_2\cosh\left(\sqrt{r^2-4pq}/2\right)\xi}{c_2\sinh\left(\sqrt{r^2-4pq}/2\right)\xi+c_1\cosh\left(\sqrt{r^2-4pq}/2\right)\xi}-\frac{\lambda}{2}\right)^{-1}\right)^{-1},\\
&u_2(\xi)\\
&= \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)}\\
&\quad\times\left(\frac{\lambda-r}{2q}+\frac{\mu}{q}\left(\frac{\sqrt{r^2-4pq}}{2}\frac{c_1\sinh\left(\sqrt{r^2-4pq}/2\right)\xi+c_2\cosh\left(\sqrt{r^2-4pq}/2\right)\xi}{c_2\sinh\left(\sqrt{r^2-4pq}/2\right)\xi+c_1\cosh\left(\sqrt{r^2-4pq}/2\right)\xi}-\frac{\lambda}{2}\right)^{-1}\right).
\end{aligned}\tag{3.15}$$

If $c_1 \neq 0, {c_1}^2 > {c_2}^2$, then $u(\xi)$ becomes the solitary wave solutions of (3.1) as follows:

$$\begin{aligned}
u_1(\xi) &= \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)}\\
&\quad\times\left(\frac{\lambda-r}{2q}+\frac{\mu}{q}\left(\frac{\sqrt{r^2-4pq}}{2}\tanh\left(\frac{\sqrt{r^2-4pq}}{2}\xi+\xi_0\right)-\frac{\lambda}{2}\right)^{-1}\right)^{-1},\\
u_2(\xi) &= \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm q\sqrt{6(-k^2-s^2)}\\
&\quad\times\left(\frac{\lambda-r}{2q}+\frac{\mu}{q}\left(\frac{\sqrt{r^2-4pq}}{2}\tanh\left(\frac{\sqrt{r^2-4pq}}{2}\xi+\xi_0\right)-\frac{\lambda}{2}\right)^{-1}\right),
\end{aligned}\tag{3.16}$$

where $\xi_0 = \tanh^{-1}(c_2/c_1)$.

If $c_2 \neq 0, c_2^2 > c_1^2$, then $u(\xi)$ becomes the solitary wave solutions of (3.1) as follows:

$$u_1(\xi) = \pm \sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)} \times \left(\frac{\lambda - r}{2q} + \frac{\mu}{q}\left(\frac{\sqrt{r^2-4pq}}{2}\coth\left(\frac{\sqrt{r^2-4pq}}{2}\xi + \xi_0\right) - \frac{\lambda}{2}\right)^{-1}\right)^{-1},$$

$$u_2(\xi) = \pm \sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm q\sqrt{6(-k^2-s^2)} \times \left(\frac{\lambda - r}{2q} + \frac{\mu}{q}\left(\frac{\sqrt{r^2-4pq}}{2}\coth\left(\frac{\sqrt{r^2-4pq}}{2}\xi + \xi_0\right) - \frac{\lambda}{2}\right)^{-1}\right), \tag{3.17}$$

where $\xi_0 = \tanh^{-1}(c_1/c_2)$.

When $r^2 - 4pq = 0$, we get the rational function solutions of (3.1)

$$u_1(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)}\left(\frac{\lambda - r}{2q} + \frac{\mu}{q}\left(\frac{c_2}{c_1 + c_2\xi} - \frac{\lambda}{2}\right)^{-1}\right)^{-1}, \tag{3.18}$$

$$u_2(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm q\sqrt{6(-k^2-s^2)}\left(\frac{\lambda - r}{2q} + \frac{\mu}{q}\left(\frac{c_2}{c_1 + c_2\xi} - \frac{\lambda}{2}\right)^{-1}\right). \tag{3.19}$$

When $r^2 - 4pq < 0$, we obtain the trigonometric function solutions of (3.1)

$$u_1(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)} \times \left(\frac{\lambda - r}{2q} + \frac{\mu}{q}\left(\frac{\sqrt{4pq-r^2}}{2}\frac{c_2\cos\left(\sqrt{4pq-r^2}/2\right)\xi - c_1\sin\left(\sqrt{4pq-r^2}/2\right)\xi}{c_1\cos\left(\sqrt{4pq-r^2}/2\right)\xi + c_2\sin\left(\sqrt{4pq-r^2}/2\right)\xi} - \frac{\lambda}{2}\right)^{-1}\right),$$

$$u_2(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm q\sqrt{6(-k^2-s^2)} \times \left(\frac{\lambda - r}{2q} + \frac{\mu}{q}\left(\frac{\sqrt{4pq-r^2}}{2}\frac{c_2\cos\left(\sqrt{4pq-r^2}/2\right)\xi - c_1\sin\left(\sqrt{4pq-r^2}/2\right)\xi}{c_1\cos\left(\sqrt{4pq-r^2}/2\right)\xi + c_2\sin\left(\sqrt{4pq-r^2}/2\right)\xi} - \frac{\lambda}{2}\right)^{-1}\right), \tag{3.20}$$

If $c_1 \neq 0, c_1^2 > c_2^2$, then $u(\xi)$ becomes the periodic wave solutions of (3.1) as follows:

$$u_1(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)} \times \left(\frac{\lambda-r}{2q}+\frac{\mu}{q}\left(\frac{\sqrt{4pq-r^2}}{2}\tan\left(\xi_0-\frac{\sqrt{4pq-r^2}}{2}\xi\right)-\frac{\lambda}{2}\right)^{-1}\right)^{-1},$$
$$u_2(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm q\sqrt{6(-k^2-s^2)} \times \left(\frac{\lambda-r}{2q}+\frac{\mu}{q}\left(\frac{\sqrt{4pq-r^2}}{2}\tan\left(\xi_0-\frac{\sqrt{4pq-r^2}}{2}\xi\right)-\frac{\lambda}{2}\right)^{-1}\right), \tag{3.21}$$

where $\xi_0 = \tan^{-1}(c_2/c_1)$.

If $c_2 \neq 0, c_2^2 > c_1^2$, then $u(\xi)$ becomes the periodic wave solutions of (3.1) as follows:

$$u_1(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm p\sqrt{6(-k^2-s^2)} \times \left(\frac{\lambda-r}{2q}+\frac{\mu}{q}\left(\frac{\sqrt{4pq-r^2}}{2}\cot\left(\frac{\sqrt{4pq-r^2}}{2}\xi+\xi_0\right)-\frac{\lambda}{2}\right)^{-1}\right)^{-1},$$
$$u_2(\xi) = \pm\sqrt{\frac{3}{2}r^2(-k^2-s^2)} \pm q\sqrt{6(-k^2-s^2)} \times \left(\frac{\lambda-r}{2q}+\frac{\mu}{q}\left(\frac{\sqrt{4pq-r^2}}{2}\cot\left(\frac{\sqrt{4pq-r^2}}{2}\xi+\xi_0\right)-\frac{\lambda}{2}\right)^{-1}\right), \tag{3.22}$$

where $\xi_0 = \tan^{-1}(c_1/c_2)$.

4. Conclusion

In summary, the improved Riccati equation method has been proposed and used to find out exact solutions of nonlinear equations with the aid of Mathmatica software. Our method allows us carry out the solution process of nonlinear wave equations more systematically and conveniently by computer algebra systems such as Maple and Mathematica. We have successfully obtained some travelling wave solutions of the mZK equations. When the parameters are taken as special values, the solitary wave solutions and periodic wave solutions are obtained. We surely believe that these solutions will be of great importance for analyzing the nonlinear phenomena arising in applied physical sciences. The work shows that the improved Riccati equation method is sufficient, effective and suitable for solving other nonlinear evolution equations, it deserves further applying and studying as well.

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