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Research Article

Logarithmically Improved Regularity Criteria for a Fluid System with the Linear Soret Effect

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We consider the 3D fluid system with the linear Soret effect. We obtain a logarithmically improved regularity criterion in the BMO space.

1. Introduction and Main Results

In this paper, we consider the regularity of the following fluid system with the linear Soret effect:

$$\begin{gathered}
u_t + u \cdot \nabla u + \nabla P = (\theta + \phi) e_3, \quad (x,t) \in \mathbb{R}^3 \times (0,\infty), \\
\theta_t + u \cdot \nabla \theta - \Delta \theta = 0, \\
\phi_t + u \cdot \nabla \phi - \Delta \phi = \Delta \theta, \\
\nabla \cdot u = 0, \\
u(x,0) = u_0, \quad \theta(x,0) = \theta_0, \quad \phi(x,0) = \phi_0,
\end{gathered} \tag{1.1}$$

where $u = (u^1(x,t), u^2(x,t), u^3(x,t))$ denotes the fluid velocity vector field, $P = P(x,t)$ is the scalar pressure, $\theta(x,t)$ is the scalar temperature, and ϕ is the concentration field, $e_3 = (0,0,1)^T$, while u_0, θ_0, and ϕ_0 are the given initial velocity, initial temperature, and initial concentration, respectively, with $\nabla \cdot u_0 = 0$. The term $\Delta\theta$ in (1.1) is the linear Soret effect (see page 102 in [1], [2–4]).

The question of global existence or blow-up in finite time of smooth solutions for the 3D incompressible Euler or Navier-Stokes equations has been one of the most outstanding open problems in applied analysis, as well as that for the 3D incompressible magnetohydrodynamics (MHD) equations. This challenging problem has attracted significant attention. In the absence of the global well-posedness, the development of blow-up or non-blow-up theory is of major importance for both theoretical and practical purposes.

When $\theta = \phi = 0$, (1.1) is the well-known Euler system, Beale-Kato-Majda [5] and Kozono-Taniuchi [6] showed the following regularity criteria, respectively:

$$\begin{gathered} \nabla \times u \in L^1\left(0,T;L^\infty\left(\mathbb{R}^3\right)\right), \\ \nabla \times u \in L^1(0,T;\mathrm{BMO}). \end{gathered} \tag{1.2}$$

When $\phi = 0$, (1.1) is the well-known Boussinesq system with zero viscosity, Fan and Zhou [7] proved the following blow-up criterion:

$$\nabla \times u \in L^1\left(0,T;\dot{B}^0_{\infty,\infty}\left(\mathbb{R}^3\right)\right). \tag{1.3}$$

Here $\dot{B}^0_{\infty,\infty}$ denotes the homogeneous Besov space.

Recently, Chan and Vasseur [8] and Zhou and Lei [9] proved some logarithmically improved regularity criterion for the 3D Navier-Stokes equations. Qiu et al. [10] obtained Serrin-type blow-up criteria of smooth solution for the 3D viscous Boussinesq equations. They showed that smooth solution $(u(\cdot,t),\theta(\cdot,t))$ for $0 \leqslant t < T$ remains smooth at time $t = T$, provided that the following condition holds:

$$\int_0^T \frac{\|\nabla \times u\|_{\mathrm{BMO}}}{\sqrt{\ln(e + \|\nabla \times u\|_{\mathrm{BMO}})}} dt < \infty. \tag{1.4}$$

Motivated by the previous results on the regularity criteria of the fluid dynamics equations, the purpose of this paper is to establish a logarithmically improved regularity criterion in terms of the vorticity field for (1.1) in the BMO space, which is defined in Section 2.

Now we state our main results as follows.

Theorem 1.1. *Let $T > 0$, and $(u_0(x),\theta_0(x),\phi_0(x)) \in H^3(\mathbb{R}^3)$ with $\operatorname{div} u_0 = 0$ in $\mathbb{R}^3$. Assume that (u,θ,ϕ) is a local smooth solution of* (1.1). *Then the solution (u,θ,ϕ) can be smoothly extended after time $t = T$ provided that* (1.4) *is satisfied.*

Remark 1.2. In the process of proof in [7, 10], they used the important fact that

$$\|\theta\|_{L^\infty} \leqslant \|\theta_0\|_{L^\infty}. \tag{1.5}$$

Due to the linear Soret effect, we cannot prove

$$\|\phi\|_{L^\infty} \leqslant C. \tag{1.6}$$

Thus the method in [7, 10] cannot be used here. We need new interpolation inequalities.

Remark 1.3. In [10], authors researched the regularity for the 3D viscous Boussinesq equations under condition (1.4). However, when $\phi = 0$, (1.1) is the well-known Boussinesq system with zero viscosity, so our results improve result of [10].

2. Preliminaries and Lemmas

First, we recall some definitions and lemmas, which play an important role in studying the regularity of solution to partial differential equations.

Definition 2.1. BMO denotes the space of functions of bounded mean oscillation of John and Nirenberg associated with the norm

$$\|f\|_{\mathrm{BMO}} \triangleq \sup_{x\in\mathbb{R}^3, R>0} \frac{1}{|B_R(x)|}\int_{B_R(x)}\left|f(y)-\frac{1}{|B_R(x)|}\int_{B_R(x)} f(z)dz\right|dy. \tag{2.1}$$

In order to prove Theorem 1.1, we need the following Gagliardo-Nirenberg inequality.

Lemma 2.2. *There exists a uniform positive constant $C > 0$ such that*

$$\left\|\nabla^i f\right\|_{L^{2m/i}} \leqslant C\|f\|_{L^\infty}^{(1-i/m)}\|\nabla^m f\|_{L^2}^{i/m}, \quad 0 \leqslant i \leqslant m \tag{2.2}$$

holds for all $f \in L^\infty(\mathbb{R}^n) \cap H^m(\mathbb{R}^n)$.

Lemma 2.3 (see [9]). *There exists a uniform positive constant $C > 0$ such that*

$$\|\nabla f\|_{L^\infty} \leqslant C\left(1+\|f\|_{L^2}+\|\nabla\times f\|_{BMO}\sqrt{\ln\left(e+\|f\|_{H^3}\right)}\right). \tag{2.3}$$

Lemma 2.4 (see [11]). *The following calculus inequality holds:*

$$\|\Lambda^s(fg)-f\Lambda^s g\|_{L^p} \leqslant \left(\|\nabla f\|_{L^{p_1}}\left\|\Lambda^{s-1}g\right\|_{L^{q_1}}+\|\Lambda^s f\|_{L^{p_2}}\|g\|_{L^{q_2}}\right), \tag{2.4}$$

with $s > 0$, $\Lambda^s = (-\Delta)^{s/2}$ and $1/p = 1/p_1 + 1/q_1 = 1/p_2 + 1/q_2$.

Lemma 2.5 (see [12]). *In three-dimensional space, the following inequalities*

$$\begin{gathered}\|\nabla f\|_{L^2} \leqslant C\|f\|_{L^2}^{2/3}\left\|\nabla^3 f\right\|_{L^2}^{1/3}, \\ \|f\|_{L^\infty} \leqslant C\|f\|_{L^2}^{1/4}\left\|\nabla^2 f\right\|_{L^2}^{3/4}, \\ \|f\|_{L^4} \leqslant C\|f\|_{L^2}^{3/4}\left\|\nabla^3 f\right\|_{L^2}^{1/4}.\end{gathered} \tag{2.5}$$

hold.

3. Proofs of the Main Results

In this section, we prove Theorem 1.1.

Proof of Theorem 1.1. Multiplying both the sides of the second equation of (1.1) by θ and the third equation of (1.1) by ϕ, respectively, and integrating by parts over $\mathbb{R}^3$, we obtain

$$\frac{1}{2}\frac{d}{dt}\|\theta\|_{L^2}^2 + \int_{\mathbb{R}^3} |\nabla\theta|^2 dx = 0, \tag{3.1}$$

$$\frac{1}{2}\frac{d}{dt}\|\phi\|_{L^2}^2 + \int_{\mathbb{R}^3} |\nabla\phi|^2 dx \leqslant \frac{1}{2}\int_{\mathbb{R}^3} |\nabla\phi|^2 dx + \frac{1}{2}\int_{\mathbb{R}^3} |\nabla\phi|^2 dx. \tag{3.2}$$

Combining (3.1) with (3.2), and using Gronwall's inequality, we infer that

$$\begin{aligned} \|\theta\|_{L^\infty(0,T;L^2)} + \|\theta\|_{L^2(0,T;H^1)} &\leqslant C, \\ \|\phi\|_{L^\infty(0,T;L^2)} + \|\phi\|_{L^2(0,T;H^1)} &\leqslant C. \end{aligned} \tag{3.3}$$

Multiplying both the sides of the first equation of (1.1) by u, and integrating by parts over $\mathbb{R}^3$, by (3.3), we get

$$\frac{1}{2}\frac{d}{dt}\|u\|_{L^2}^2 = \int_{\mathbb{R}^3} (\theta+\phi)e_3 \cdot u\, dx \leqslant \|\theta+\phi\|_{L^2}\|u\|_{L^2} \leqslant C\|u\|_{L^2}, \tag{3.4}$$

which implies

$$\|u\|_{L^\infty(0,T;L^2)} \leqslant C. \tag{3.5}$$

Next we go to estimate L^2-norm of ∇u, $\nabla\theta$ and $\nabla\phi$. Multiplying the first equation, the second equation and the third equation of (1.1) by $-\Delta u$, $-\Delta\theta$, and $-\Delta\phi$, respectively, then integrating by parts over $\mathbb{R}^3$, we deduce that

$$\begin{aligned} \frac{1}{2}\frac{d}{dt}\|\nabla u\|_{L^2}^2 &= -\sum_{i,j}\int_{\mathbb{R}^3} \partial_j u_i \cdot \partial_i u \cdot \partial_j u\, dx + \int_{\mathbb{R}^3} \nabla((\theta+\phi)e_3)\nabla u\, dx \\ &\leqslant C\|\nabla u\|_{L^\infty}\|\nabla u\|_{L^2}^2 + (\|\nabla\theta\|_{L^2} + \|\nabla\phi\|_{L^2})\|\nabla u\|_{L^2}, \end{aligned} \tag{3.6}$$

$$\frac{1}{2}\frac{d}{dt}\|\nabla\theta\|_{L^2}^2 + \|\Delta\theta\|_{L^2}^2 = -\sum_{i,j}\int_{\mathbb{R}^3} \partial_j u_i \partial_i\theta \partial_j\theta\, dx \leqslant C\|\nabla u\|_{L^\infty}\|\nabla\theta\|_{L^2}^2, \tag{3.7}$$

$$\begin{aligned} \frac{1}{2}\frac{d}{dt}\|\nabla\phi\|_{L^2}^2 + \|\Delta\phi\|_{L^2}^2 &= -\sum_{i,j}\int_{\mathbb{R}^3} \partial_j u_i \partial_i\phi \partial_j\phi\, dx - \int_{\mathbb{R}^3} \Delta\theta\Delta\phi dx \\ &\leqslant C\|\nabla u\|_{L^\infty}\|\nabla\phi\|_{L^2}^2 + \frac{1}{2}\left(\|\Delta\theta\|_{L^2}^2 + \|\Delta\phi\|_{L^2}^2\right). \end{aligned} \tag{3.8}$$

Now combining (3.6), (3.7) with (3.8) and using Gronwall's inequality, we obtain

$$\begin{aligned}&\|\nabla u(\cdot,t)\|_{L^2}^2+\|\nabla\theta(\cdot,t)\|_{L^2}^2+\left\|\nabla\phi(\cdot,t)\right\|_{L^2}^2\\&\quad\leqslant\left(\|\nabla u(t_1)\|_{L^2}^2+\|\nabla\theta(t_1)\|_{L^2}^2+\left\|\nabla\phi(t_1)\right\|_{L^2}^2\right)\exp^{C\int_{t_1}^t\|\nabla u\|_{L^\infty}ds}.\end{aligned}\tag{3.9}$$

Noting (1.4), one concludes that for any small constant $\varepsilon>0$, there exists $T^*<T$ such that

$$\int_{T^*}^T\frac{\|\nabla\times u\|_{\mathrm{BMO}}}{\sqrt{\ln(e+\|\nabla\times u\|_{\mathrm{BMO}})}}dt\leqslant\varepsilon.\tag{3.10}$$

Set

$$A(t)=\sup_{T^*\leqslant\tau\leqslant t}\left(\left\|\Lambda^3u(\cdot,\tau)\right\|_{L^2}^2+\left\|\Lambda^3\theta(\cdot,\tau)\right\|_{L^2}^2+\left\|\Lambda^3\phi(\cdot,\tau)\right\|_{L^2}^2\right),\tag{3.11}$$

where $\Lambda^s=(-\Delta)^{s/2}$.

It follows from (3.9), (3.10), and Lemma 2.3 that

$$\begin{aligned}&\|\nabla u(\cdot,t)\|_{L^2}^2+\|\nabla\theta(\cdot,t)\|_{L^2}^2+\left\|\nabla\phi(\cdot,t)\right\|_{L^2}^2\\&\quad\leqslant C_1\exp^{C\int_{T^*}^t\|\nabla u\|_{L^\infty}ds}\\&\quad\leqslant C_1\exp^{C_0\varepsilon\ln(e+A(t))}\\&\quad\leqslant C_1(e+A(t))^{C_0\varepsilon},\quad T^*\leqslant t\leqslant T,\end{aligned}\tag{3.12}$$

where C_1 depends on $\|\nabla u(T^*)\|_{L^2}^2+\|\nabla\theta(T^*)\|_{L^2}^2+\|\nabla\phi(T^*)\|_{L^2}^2$, and C_0 is an absolute positive constant.

Finally we go to estimate for H^3-norm of u, θ, and ϕ. Applying the operation Λ^3 on both the sides of the first equation of (1.1), then multiplying Λ^3u, and integrating by parts over $\mathbb{R}^3$, by (2.4), (3.12), Hölder's inequality, and Young's inequality, we have

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^3}\left|\Lambda^3u\right|^2&\leqslant-\int_{\mathbb{R}^3}\left[\Lambda^3(u\cdot\nabla u)-u\cdot\nabla\Lambda^3u\right]\Lambda^3u\,dx+\int_{\mathbb{R}^3}\Lambda^3(\theta+\phi)e_3\cdot\Lambda^3u\,dx\\&\leqslant C\|\nabla u\|_{L^\infty}\left\|\Lambda^3u\right\|_{L^2}^2+\left(\left\|\Lambda^3\theta\right\|_{L^2}+\left\|\Lambda^3\phi\right\|_{L^2}\right)\left\|\Lambda^3u\right\|_{L^2}\\&\leqslant C\|\nabla u\|_{L^\infty}\left\|\Lambda^3u\right\|_{L^2}^2+\left\|\Lambda^3\theta\right\|_{L^2}^2+\left\|\Lambda^3\phi\right\|_{L^2}^2+\left\|\Lambda^3u\right\|_{L^2}^2\\&\leqslant C(\|\nabla u\|_{L^\infty}+1)(e+A(t)).\end{aligned}\tag{3.13}$$

Similarly, by (2.4), Hölder's inequality, and Young's inequality, we obtain

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^3}\left|\Lambda^3\theta\right|^2+\int_{\mathbb{R}^3}\left|\Lambda^4\theta\right|^2 dx\\
&\quad=-\int_{\mathbb{R}^3}\left[\Lambda^2(u\cdot\nabla\theta)-u\cdot\nabla\Lambda^2\theta\right]\Lambda^4\theta\,dx\\
&\quad\leqslant C\|\nabla u\|_{L^\infty}\|\Delta\theta\|_{L^2}\left\|\Lambda^4\theta\right\|_{L^2}+C\|\Delta u\|_{L^4}\|\nabla\theta\|_{L^4}\left\|\Lambda^4\theta\right\|_{L^2}\\
&\quad\equiv A_2+A_3.
\end{aligned}\tag{3.14}$$

Combining Lemma 2.5, Young's inequality with (3.12), we deduce

$$\begin{aligned}
A_2&\leqslant C\|\nabla u\|_{L^\infty}\|\nabla\theta\|_{L^2}^{2/3}\left\|\Lambda^4\theta\right\|_{L^2}^{4/3}\\
&\leqslant\frac{1}{4}\left\|\Lambda^4\theta\right\|_{L^2}^2+C\|\nabla u\|_{L^\infty}^3\|\nabla\theta\|_{L^2}^2\\
&\leqslant\frac{1}{4}\left\|\Lambda^4\theta\right\|_{L^2}^2+C\|\nabla u\|_{L^\infty}\|\nabla u\|_{L^2}^{1/2}\left\|\Lambda^3u\right\|_{L^2}^{3/2}\|\nabla\theta\|_{L^2}^2\\
&\leqslant\frac{1}{4}\left\|\Lambda^4\theta\right\|_{L^2}^2+C\|\nabla u\|_{L^\infty}(e+A(t))^{(5/4)C_0\varepsilon}A^{3/4}(t).
\end{aligned}\tag{3.15}$$

Using Lemmas 2.2 and 2.5, Young's inequality, and (3.12), we obtain

$$\begin{aligned}
A_3&\leqslant C\|\nabla u\|_{L^\infty}^{1/2}\left\|\Lambda^3u\right\|_{L^2}^{1/2}\|\nabla\theta\|_{L^2}^{3/4}\left\|\Lambda^4\theta\right\|_{L^2}^{5/4}\\
&\leqslant\frac{1}{4}\left\|\Lambda^4\theta\right\|_{L^2}^2+C\|\nabla u\|_{L^\infty}^{4/3}\left\|\Lambda^3u\right\|_{L^2}^{4/3}\|\nabla\theta\|_{L^2}^2\\
&\leqslant\frac{1}{4}\left\|\Lambda^4\theta\right\|_{L^2}^2+C\|\nabla u\|_{L^\infty}\|\nabla u\|_{L^2}^{1/12}\left\|\Lambda^3u\right\|_{L^2}^{19/12}\|\nabla\theta\|_{L^2}^2\\
&\leqslant\frac{1}{4}\left\|\Lambda^4\theta\right\|_{L^2}^2+C\|\nabla u\|_{L^\infty}(e+A(t))^{(25/24)C_0\varepsilon}A^{19/24}(t).
\end{aligned}\tag{3.16}$$

Inserting the above estimates A_2 and A_3 into (3.14), we get

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^3}\left|\Lambda^3\theta\right|^2+\frac{1}{2}\int_{\mathbb{R}^3}\left|\Lambda^4\theta\right|^2dx\\
&\quad\leqslant C\|\nabla u\|_{L^\infty}(e+A(t))^{(5/4)C_0\varepsilon}A^{3/4}(t)+C\|\nabla u\|_{L^\infty}(e+A(t))^{(25/24)C_0\varepsilon}A^{19/24}(t)\\
&\quad\leqslant C\|\nabla u\|_{L^\infty}(e+A(t))\\
&\quad\leqslant C(\|\nabla u\|_{L^\infty}+1)(e+A(t)),
\end{aligned}\tag{3.17}$$

provided that $C_0\varepsilon\leqslant 1/5$, which can be achieved by the absolute continuous property of integral (1.4).

Similarly, for the ϕ-equation, we have

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^3}\left|\Lambda^3\phi\right|^2+\frac{1}{4}\int_{\mathbb{R}^3}\left|\Lambda^4\phi\right|^2dx\\
&\quad\leqslant C\|\nabla u\|_{L^\infty}(e+A(t))^{(5/4)C_0\varepsilon}A^{3/4}(t)+C\|\nabla u\|_{L^\infty}(e+A(t))^{(25/24)C_0\varepsilon}A^{19/24}(t)+\frac{1}{2}\int_{\mathbb{R}^3}\left|\Lambda^4\theta\right|^2dx\\
&\quad\leqslant C\|\nabla u\|_{L^\infty}(e+A(t))+\frac{1}{2}\int_{\mathbb{R}^3}\left|\Lambda^4\theta\right|^2dx\\
&\quad\leqslant C(\|\nabla u\|_{L^\infty}+1)(e+A(t))+\frac{1}{2}\int_{\mathbb{R}^3}\left|\Lambda^4\theta\right|^2dx.
\end{aligned}\tag{3.18}$$

Combining (3.13), (3.17) with (3.18), we easily conclude that

$$\begin{aligned}
&\frac{d}{dt}\left(\int_{\mathbb{R}^3}\left|\Lambda^3u\right|^2+\int_{\mathbb{R}^3}\left|\Lambda^3\theta\right|^2+\int_{\mathbb{R}^3}\left|\Lambda^3\phi\right|^2\right)\\
&\quad\leqslant C(\|\nabla u\|_{L^\infty}+1)(e+A(t)).
\end{aligned}\tag{3.19}$$

By Lemma 2.3 and (3.19), we infer that

$$\begin{aligned}
\frac{d}{dt}\ln(e+A(t))&\leqslant C(\|\nabla u\|_{L^\infty}+1)\\
&\leqslant C\left(1+\|\nabla\times u\|_{\mathrm{BMO}}\sqrt{\ln(e+\|u\|_{H^3})}\right)\\
&\leqslant C\left(1+\frac{\|\nabla\times u\|_{\mathrm{BMO}}}{\sqrt{\ln(e+\|\nabla\times u\|_{\mathrm{BMO}})}}\right)\\
&\quad\times\left[1+\ln\left(e+\left\|\Lambda^3u\right\|_{L^2}^2+\left\|\Lambda^3\theta\right\|_{L^2}^2+\left\|\Lambda^3\phi\right\|_{L^2}^2\right)\right]\\
&\leqslant C\left(1+\frac{\|\nabla\times u\|_{\mathrm{BMO}}}{\sqrt{\ln(e+\|\nabla\times u\|_{\mathrm{BMO}})}}\right)[1+\ln(e+A(t))].
\end{aligned}\tag{3.20}$$

Using Gronwall's inequality, we have

$$\begin{aligned}
&\ln\left(e+\left\|\Lambda^3u\right\|_{L^2}^2+\left\|\Lambda^3\theta\right\|_{L^2}^2+\left\|\Lambda^3\phi\right\|_{L^2}^2\right)\\
&\quad\leqslant\ln(\|u(T^*)\|_{H^3}+\|\theta(T^*)\|_{H^3}+\left\|\phi(T^*)\right\|_{H^3})\exp^{C(T+\int_{T^*}^{T}(\|\nabla\times u\|_{\mathrm{BMO}}/\sqrt{\ln(e+\|\nabla\times u\|_{\mathrm{BMO}})})dt)}.
\end{aligned}\tag{3.21}$$

Therefore, $\|u\|_{H^3} + \|\theta\|_{H^3} + \|\phi\|_{H^3}$ can be bounded provided that

$$\int_{T^*}^{T} \frac{\|\nabla \times u\|_{\mathrm{BMO}}}{\sqrt{\ln(e + \|\nabla \times u\|_{\mathrm{BMO}})}} dt < \infty. \tag{3.22}$$

This completes the proof of Theorem 1.1. □

Appendix

In this section, we give the following local well-posedness of the system (1.1).

Theorem A.1 (local well-posedness). *Let $s > 3/2+1$, and $(u_0, \theta_0, \phi_0) \in H^s(\mathbb{R}^3)$. Then there exist $T > 0$ and a unique solution (u, θ, ϕ) of the system* (1.1) *such that*

$$u \in C\Big([0,T]; H^s\Big(\mathbb{R}^3\Big)\Big), \quad (\theta, \phi) \in C\Big([0,T]; H^s\Big(\mathbb{R}^3\Big)\Big) \cap L^2\Big(0,T; H^{s+1}\Big(\mathbb{R}^3\Big)\Big). \tag{A.1}$$

Proof. The proof is similar to Theorem 3.1 in [4, 13], so we omit it here. □

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