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Research Article

Generalized Fuzzy Soft Expert Set

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In 2011 Alkhazaleh and Salleh defined the concept of soft expert sets where the user can know the opinion of all the experts in one model and give an application of this concept in decision-making problems. Also, they introduced the concept of the fuzzy soft expert set as a combination between the soft expert set and the fuzzy set. In 2010 Majumdar and Samanta presented the concept of a generalized fuzzy soft sets. The purpose of this paper is to combine the work of Alkhazaleh and Salleh (2011) and Majumdar and Samanta (2010), from which we can obtain a new concept: generalized fuzzy soft expert sets (GFSESs). We also introduce its operations, namely, complement, union intersection, "AND" and "OR", and study their properties. The generalized fuzzy soft expert sets are used to analyze a decision-making problem. Also in our model the user can know the opinion of all experts in one model. In this work we also introduce the concept of a generalized fuzzy soft expert sets with multiopinions (four opinions), which will be more effective and useful. Finally, we give an application of this concept in decision-making problem.

1. Introduction

As much of the research completed in economics, engineering, environmental science, sociology, medical science, and other related fields involve uncertain data, it is not always possible to use classical methods when analyzing the information. This can be due to the fact that the information may come in different formats; to solve problems we use data in its many different forms. So we need new mathematical way free from difficulties of dealing with uncertain problems; this method must be efficient tools for dealing with diverse types of uncertainties and imprecision embedded in a system. Molodtsov [1] initiated the concept of soft set theory as a mathematical tool for dealing with uncertainties. After Molodtsov's work, some operations and application of soft sets were studied by Chen et al. [2] and Maji et al. [3, 4]. Also Maji et al. [5] have introduced the concept of fuzzy soft set, a more general concept, which is a combination of fuzzy set and soft set and studied its properties, and also Roy and Maji [6] used this theory to solve some decision-making problems. In 2010 Majumdar and Samanta [7] introduced a concept of generalized fuzzy soft sets and their operations and application of generalised fuzzy soft sets in decision-making problem and medical diagnosis problem. Alkhazaleh and Salleh [8] introduced the concept of a soft expert set and fuzzy soft expert set, where the user can know the opinion of all experts in one model without any operations. Even after any operation, the user can know the opinion of all experts. So in this paper we introduce the concept of a generalised fuzzy soft expert set, which will be more effective and useful which is a combination of a fuzzy soft expert set and generalised fuzzy soft set. We also define its basic operations, namely complement, union, intersection, AND, and OR, and study their properties. Finally, we give an application of this concept in decision-making problem.

2. Preliminaries

In this section, we recall some basic notions related to this work. Molodtsov defined soft set in the following way. Let U be a universe and E a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2.1 (see [1]). A pair (F, A) is called a *soft set* over U, where F is a mapping

$$F : A \longrightarrow P(U). \tag{2.1}$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A).

Definition 2.2 (see [5]). Let U be an initial universal set, and let E be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U. Let $A \subseteq E$. A pair (F, E) is called a *fuzzy soft set* over U where F is a mapping given by

$$F : A \longrightarrow I^U. \tag{2.2}$$

Definition 2.3 (see [7]). Let $U = \{x_1, x_2, \ldots, x_n\}$ be the universal set of elements and $E = \{e_1, e_2, \ldots, e_m\}$ the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to I^U$ and μ a fuzzy subset of E; that is, $\mu : E \to I = [0, 1]$, where I^U is the collection of all fuzzy subsets of U. Let $F_\mu : E \to I^U \times I$ be a function defined as follows:

$$F_\mu(e) = \big(F(e), \mu(e)\big). \tag{2.3}$$

Then F_μ is called a *generalized fuzzy soft set* (GFSS in short) over the soft set (U, E). Here for each parameter e_i, $F_\mu(e_i) = (F(e_i), \mu(e_i))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$. So we can write as follows:

$$F_\mu(e_i) = \left(\left\{\frac{x_1}{F(e_i)(x_1)}, \frac{x_2}{F(e_i)(x_2)}, \ldots, \frac{x_n}{F(e_i)(x_n)}\right\}, \mu(e_i)\right), \tag{2.4}$$

where $F(e_i)(x_1)$, $F(e_i)(x_2)$, $F(e_i)(x_3)$, *and* $F(e_i)(x_n)$ are the degree of belongingness and $\mu(e_i)$ is the degree of possibility of such belongingness.

Let U be a universe, E a set of parameters, X a set of experts (agents), and O a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 2.4 (see [8]). A pair (F, A) is called a *soft expert set* over U, where F is a mapping given by

$$F : A \longrightarrow P(U), \tag{2.5}$$

where $P(U)$ denotes the power set of U.

Let U be a universe, E a set of parameters, X a set of experts (agents), and $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 2.5 (see [9]). A pair (F, A) is called a *fuzzy soft expert set* over U, where F is a mapping given by

$$F : A \longrightarrow I^U, \tag{2.6}$$

where I^U denotes all the fuzzy subsets of U.

Definition 2.6 (see [9]). For two fuzzy soft expert sets (F, A) and (G, B) over U, (F, A) is called a *fuzzy soft expert subset* of (G, B) if

(1) $B \subseteq A$;

(2) for all $\varepsilon \in A, F(\varepsilon)$ is fuzzy subset of $G(\varepsilon)$.

This relationship is denoted by $(F, A)\tilde{\subseteq}(G, B)$. In this case (G, B) is called a fuzzy soft expert superset of (F, A).

Definition 2.7 (see [9]). Two fuzzy soft expert sets (F, A) and (G, B) over U are said to be *equal* if (F, A) is a fuzzy soft expert subset of (G, B) and (G, B) is a fuzzy soft expert subset of (F, A).

Definition 2.8 (see [9]). An *agree-fuzzy soft expert set* $(F, A)_1$ over U is a fuzzy soft expert subset of (F, A) defined as follows:

$$(F, A)_1 = \{F_1(\alpha) : \alpha \in E \times X \times \{1\}\}. \tag{2.7}$$

Definition 2.9 (see [9]). A *disagree-fuzzy soft expert set* $(F, A)_0$ over U is a fuzzy soft expert subset of (F, A) defined as follows:

$$(F, A)_0 = \{F_0(\alpha) : \alpha \in E \times X \times \{0\}\}. \tag{2.8}$$

Definition 2.10 (see [9]). The *complement* of a fuzzy soft expert set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \rceil A)$ where $F^c : \rceil A \rightarrow P(U)$ is a mapping given by

$$F^c(\alpha) = c(F(\rceil\alpha)), \quad \forall \alpha \in \rceil A, \tag{2.9}$$

where c is a fuzzy complement.

Definition 2.11 (see [9]). The *union* of two fuzzy soft expert sets (F, A) and (G, B) over U, denoted by $(F, A)\tilde{\cup}(G, B)$, is the fuzzy soft expert set (H, C) where $C = A \cup B$, and for all $\varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ s(F(\varepsilon), G(\varepsilon)), & \text{if } \varepsilon \in A \cap B, \end{cases} \tag{2.10}$$

where s is an s-norm.

Definition 2.12 (see [9]). The *intersection* of two fuzzy soft expert sets (F, A) and (G, B) over U, denoted by $(F, A)\tilde{\cap}(G, B)$, is the fuzzy soft expert set (H, C) where $C = A \cup B$, and for all $\varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ t(F(\varepsilon), G(\varepsilon)), & \text{if } \varepsilon \in A \cap B, \end{cases} \tag{2.11}$$

where t is a t-norm.

Definition 2.13 (see [9]). If (F, A) and (G, B) are two fuzzy soft expert sets over U, then "(F, A) AND (G, B)" denoted by $(F, A) \wedge (G, B)$ is defined by

$$(F, A) \wedge (G, B) = (H, A \times B), \tag{2.12}$$

such that $H(\alpha, \beta) = t(F(\alpha), G(\beta))$, for all $(\alpha, \beta) \in A \times B$, where t is a t-norm.

Definition 2.14 (see [9]). If (F, A) and (G, B) are two fuzzy soft expert sets over U, then "(F, A) OR (G, B)" denoted by $(F, A) \vee (G, B)$ is defined by

$$(F, A) \vee (G, B) = (H, A \times B), \tag{2.13}$$

such that $H(\alpha, \beta) = s(F(\alpha), G(\beta))$, for all $(\alpha, \beta) \in A \times B$, where s is an s-norm.

3. Generalised Fuzzy Soft Expert Set

In this section we define the concept of the generalized fuzzy soft expert set and study some of its properties. Let U be a universe set, E a set of parameters, X a set of experts (agents), and $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$. μ be a fuzzy set of Z; that is, $\mu : Z \to I = [0, 1]$.

Definition 3.1. A pair (F_μ, A) is called an *generalized fuzzy soft expert set* (GFSES in short) over U, where F_μ is a mapping given by

$$F_\mu : A \longrightarrow I^U \times I, \tag{3.1}$$

where I^U denotes the collection of all fuzzy subsets of U. Here for each parameter e_i, $F_\mu(e_i) = (F(e_i), \mu(e_i))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$.

Example 3.2. Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of universe, and let $E = \{e_1, e_2, e_3\}$ a set of parameters. Let $Z = E \times X \times O$ and $A \subseteq Z$. Let μ be a fuzzy set of Z; that is, $\mu : Z \longrightarrow I = [0,1]$.

Define a function

$$F : A \longrightarrow I^U \times I \tag{3.2}$$

as follows:

$$\begin{aligned}
F_\mu(e_1, m, 1) &= \left(\left\{\frac{u_1}{0.6}, \frac{u_2}{0.3}, \frac{u_3}{0.2}, \frac{u_4}{0.4}\right\}, 0.2\right), & F_\mu(e_1, n, 1) &= \left(\left\{\frac{u_1}{0.7}, \frac{u_2}{0.4}, \frac{u_3}{0.1}, \frac{u_4}{0.3}\right\}, 0.1\right),\\
F_\mu(e_1, r, 1) &= \left(\left\{\frac{u_1}{0.1}, \frac{u_2}{0.3}, \frac{u_3}{0.8}, \frac{u_4}{0.7}\right\}, 0.2\right), & F_\mu(e_2, m, 1) &= \left(\left\{\frac{u_1}{0.5}, \frac{u_2}{0.3}, \frac{u_3}{0.2}, \frac{u_4}{0.7}\right\}, 0.4\right),\\
F_\mu(e_2, n, 1) &= \left(\left\{\frac{u_1}{0.7}, \frac{u_2}{0.8}, \frac{u_3}{0.6}, \frac{u_4}{0.4}\right\}, 0.2\right), & F_\mu(e_2, r, 1) &= \left(\left\{\frac{u_1}{0.7}, \frac{u_2}{0.5}, \frac{u_3}{0.6}, \frac{u_4}{0.7}\right\}, 0.3\right),\\
F_\mu(e_3, m, 1) &= \left(\left\{\frac{u_1}{0.3}, \frac{u_2}{0.6}, \frac{u_3}{0.8}, \frac{u_4}{0.9}\right\}, 0.6\right), & F_\mu(e_3, n, 1) &= \left(\left\{\frac{u_1}{0.2}, \frac{u_2}{0.3}, \frac{u_3}{0.7}, \frac{u_4}{0.6}\right\}, 0.5\right),\\
F_\mu(e_3, r, 1) &= \left(\left\{\frac{u_1}{0.3}, \frac{u_2}{0.4}, \frac{u_3}{0.6}, \frac{u_4}{0.3}\right\}, 0.8\right), & F_\mu(e_1, m, 0) &= \left(\left\{\frac{u_1}{0.3}, \frac{u_2}{0.6}, \frac{u_3}{0.7}, \frac{u_4}{0.6}\right\}, 0.7\right),\\
F_\mu(e_1, n, 0) &= \left(\left\{\frac{u_1}{0.2}, \frac{u_2}{0.7}, \frac{u_3}{0.8}, \frac{u_4}{0.7}\right\}, 0.9\right), & F_\mu(e_1, r, 0) &= \left(\left\{\frac{u_1}{0.6}, \frac{u_2}{0.8}, \frac{u_3}{0.4}, \frac{u_4}{0.5}\right\}, 0.6\right),\\
F_\mu(e_2, m, 0) &= \left(\left\{\frac{u_1}{0.3}, \frac{u_2}{0.6}, \frac{u_3}{0.8}, \frac{u_4}{0.3}\right\}, 0.6\right), & F_\mu(e_2, n, 0) &= \left(\left\{\frac{u_1}{0.2}, \frac{u_2}{0.4}, \frac{u_3}{0.5}, \frac{u_4}{0.7}\right\}, 0.7\right),\\
F_\mu(e_2, r, 0) &= \left(\left\{\frac{u_1}{0.4}, \frac{u_2}{0.6}, \frac{u_3}{0.5}, \frac{u_4}{0.3}\right\}, 0.4\right), & F_\mu(e_3, m, 0) &= \left(\left\{\frac{u_1}{0.8}, \frac{u_2}{0.4}, \frac{u_3}{0.3}, \frac{u_4}{0.1}\right\}, 0.3\right),\\
F_\mu(e_3, n, 0) &= \left(\left\{\frac{u_1}{0.9}, \frac{u_2}{0.6}, \frac{u_3}{0.5}, \frac{u_4}{0.3}\right\}, 0.6\right), & F_\mu(e_3, r, 0) &= \left(\left\{\frac{u_1}{0.8}, \frac{u_2}{0.5}, \frac{u_3}{0.4}, \frac{u_4}{0.6}\right\}, 0.5\right).
\end{aligned} \tag{3.3}$$

Then we can find the a generalized fuzzy soft expert sets (F_μ, Z) as consisting of the following collection of approximations:

$$\begin{aligned}
(F_\mu, Z) = \Bigg\{ &\left((e_1, m, 1), \left(\left\{\frac{u_1}{0.6}, \frac{u_2}{0.3}, \frac{u_3}{0.2}, \frac{u_4}{0.4}\right\}, 0.2\right)\right), \left((e_1, n, 1), \left(\left\{\frac{u_1}{0.7}, \frac{u_2}{0.4}, \frac{u_3}{0.1}, \frac{u_4}{0.3}\right\}, 0.1\right)\right),\\
&\left((e_1, r, 1), \left(\left\{\frac{u_1}{0.1}, \frac{u_2}{0.3}, \frac{u_3}{0.8}, \frac{u_4}{0.7}\right\}, 0.2\right)\right), \left((e_2, m, 1), \left(\left\{\frac{u_1}{0.5}, \frac{u_2}{0.3}, \frac{u_3}{0.2}, \frac{u_4}{0.7}\right\}, 0.4\right)\right),\\
&\left((e_2, n, 1), \left(\left\{\frac{u_1}{0.7}, \frac{u_2}{0.8}, \frac{u_3}{0.6}, \frac{u_4}{0.4}\right\}, 0.2\right)\right), \left((e_2, r, 1), \left(\left\{\frac{u_1}{0.7}, \frac{u_2}{0.5}, \frac{u_3}{0.6}, \frac{u_4}{0.7}\right\}, 0.3\right)\right),\\
&\left((e_3, m, 1), \left(\left\{\frac{u_1}{0.3}, \frac{u_2}{0.6}, \frac{u_3}{0.8}, \frac{u_4}{0.9}\right\}, 0.6\right)\right), \left((e_3, n, 1), \left(\left\{\frac{u_1}{0.2}, \frac{u_2}{0.3}, \frac{u_3}{0.7}, \frac{u_4}{0.6}\right\}, 0.5\right)\right),\\
&\left((e_3, r, 1), \left(\left\{\frac{u_1}{0.3}, \frac{u_2}{0.4}, \frac{u_3}{0.6}, \frac{u_4}{0.3}\right\}, 0.8\right)\right), \left((e_1, m, 0), \left(\left\{\frac{u_1}{0.3}, \frac{u_2}{0.6}, \frac{u_3}{0.7}, \frac{u_4}{0.6}\right\}, 0.7\right)\right),
\end{aligned}$$

$$\left((e_1,n,0),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.7},\frac{u_3}{0.8},\frac{u_4}{0.7}\right\},0.9\right)\right),\left((e_1,r,0),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.8},\frac{u_3}{0.4},\frac{u_4}{0.5}\right\},0.6\right)\right),$$
$$\left((e_2,m,0),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.6},\frac{u_3}{0.8},\frac{u_4}{0.3}\right\},0.6\right)\right),\left((e_2,n,0),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.4},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\},0.7\right)\right),$$
$$\left((e_2,r,0),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\},0.4\right)\right),\left((e_3,m,0),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.1}\right\},0.3\right)\right),$$
$$\left((e_3,n,0),\left(\left\{\frac{u_1}{0.9},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\},0.6\right)\right),\left((e_3,r,0),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.5},\frac{u_3}{0.4},\frac{u_4}{0.6}\right\},0.5\right)\right)\Bigg\}. \tag{3.4}$$

Definition 3.3. For two GFSESs (F_μ, A) and (G_δ, B) over U, (F_μ, A) is called a generalized fuzzy soft expert set subset of (G_δ, B) if

(1) $B \subseteq A$;

(2) for all $\varepsilon \in B$, $G_\delta(\varepsilon)$ is generalized fuzzy subset of $F_\mu(\varepsilon)$.

Example 3.4. Consider Example 3.2. Let where is the rest of the statement and so forth

$$\begin{aligned} A = \{&(e_1,m,1),(e_3,m,1),(e_3,m,0),(e_1,n,1),(e_2,n,1)(e_2,r,0),\\ &(e_3,n,0),(e_2,r,1),(e_3,r,1),(e_3,r,0)\},\\ B = \{&(e_1,m,1),(e_3,m,0),(e_1,n,1),(e_2,n,1),(e_2,r,0),(e_3,r,1),(e_3,r,0)\}. \end{aligned} \tag{3.5}$$

Since B is a fuzzy subset of A, clearly $B \subset A$. Let (G_δ, B) and (F_μ, A) be defined as follows:

$$(F_\mu,A)=\Bigg\{\left((e_1,m,1),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.3},\frac{u_3}{0.2},\frac{u_4}{0.4}\right\},0.2\right)\right),\left((e_1,n,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.4},\frac{u_3}{0.1},\frac{u_4}{0.3}\right\},0.1\right)\right),$$
$$\left((e_2,n,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.6},\frac{u_4}{0.4}\right\},0.2\right)\right),\left((e_2,r,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.7}\right\},0.3\right)\right),$$
$$\left((e_2,r,0),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\},0.0\right)\right),\left((e_3,m,1),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.6},\frac{u_3}{0.8},\frac{u_4}{0.1}\right\},0.6\right)\right),$$
$$\left((e_3,r,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.7}\right\},0.3\right)\right),\left((e_3,m,0),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.4},\frac{u_3}{0.9},\frac{u_4}{0.6}\right\},0.3\right)\right),$$
$$\left((e_3,n,0),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\},0.6\right)\right),\left((e_3,r,0),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.4},\frac{u_4}{0.2}\right\},0.5\right)\right)\Bigg\},$$
$$(G_\delta,B)=\Bigg\{\left((e_1,m,1),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.3},\frac{u_3}{0.1},\frac{u_4}{0.3}\right\},0.1\right)\right),\left((e_1,n,1),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.6},\frac{u_4}{0.5}\right\},0.1\right)\right),$$
$$\left((e_2,n,1),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.2},\frac{u_3}{0.1},\frac{u_4}{0.1}\right\},0.1\right)\right),\left((e_3,r,1),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.4},\frac{u_3}{0.2},\frac{u_4}{0.5}\right\},0.7\right)\right),$$

$$\left((e_2,r,0),\left(\left\{\frac{u_1}{0.1},\frac{u_2}{0.3},\frac{u_3}{0.3},\frac{u_4}{0.1}\right\},0.4\right)\right),\left((e_3,m,0),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.1},\frac{u_3}{0.3},\frac{u_4}{0.6}\right\},0.3\right)\right),$$
$$\left.\left((e_3,r,0),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.4},\frac{u_3}{0.1},\frac{u_4}{0.1}\right\},0.5\right)\right)\right\}. \tag{3.6}$$

Therefore $(G_\delta,B)\subseteq(F_\mu,A)$.

Definition 3.5. Two GFSES (F_μ,A) and (G_δ,B) over U are said to be *equal* if (F_μ,A) is a GFSES subset of (G_δ,A) and (G_δ,A) is a GFSES subset of (F_μ,A).

Definition 3.6. An *agree-GFSES* $(F_\mu,A)_1$ over U is a GFSE subset of (F_μ,A) defined as follows:

$$(F_\mu,A)_1=\{F_1(\alpha):\alpha\in E\times X\times\{1\}\}. \tag{3.7}$$

Definition 3.7. A *disagree-GFSES* $(F_\mu,A)_0$ over U is a GFSE subset of (F_μ,A) defined as follows:

$$(F_\mu,A)_0=\{F_0(\alpha):\alpha\in E\times X\times\{0\}\}. \tag{3.8}$$

Example 3.8. Consider Example 3.2. Then the agree-generalized fuzzy soft expert set $(F_\mu,Z)_1$ over U is

$$(F_\mu,Z)_1=\left\{\left((e_1,m,1),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.3},\frac{u_3}{0.2},\frac{u_4}{0.4}\right\},0.2\right)\right),\left((e_1,n,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.4},\frac{u_3}{0.1},\frac{u_4}{0.3}\right\},0.1\right)\right),\right.$$
$$\left((e_1,r,1),\left(\left\{\frac{u_1}{0.1},\frac{u_2}{0.3},\frac{u_3}{0.8},\frac{u_4}{0.7}\right\},0.2\right)\right),\left((e_2,m,1),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.3},\frac{u_3}{0.2},\frac{u_4}{0.7}\right\},0.4\right)\right),$$
$$\left((e_2,n,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.6},\frac{u_4}{0.4}\right\},0.2\right)\right),\left((e_2,r,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.7}\right\},0.3\right)\right),$$
$$\left((e_3,m,1),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.6},\frac{u_3}{0.8},\frac{u_4}{0.9}\right\},0.6\right)\right),\left((e_3,n,1),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.3},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\},0.5\right)\right),$$
$$\left.\left((e_3,r,1),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.4},\frac{u_3}{0.6},\frac{u_4}{0.3}\right\},0.8\right)\right)\right\}, \tag{3.9}$$

and the disagree-generalized fuzzy soft expert set $(F_\mu,Z)_0$ over U is

$$(F_\mu,Z)_0=\left\{\left((e_1,m,0),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\},0.7\right)\right),\left((e_1,n,0),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.7},\frac{u_3}{0.8},\frac{u_4}{0.7}\right\},0.9\right)\right),\right.$$
$$\left((e_1,r,0),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.8},\frac{u_3}{0.4},\frac{u_4}{0.5}\right\},0.6\right)\right),\left((e_2,m,0),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.6},\frac{u_3}{0.8},\frac{u_4}{0.3}\right\},0.6\right)\right),$$

$$\left((e_2,n,0),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.4},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\},0.7\right)\right),\left((e_2,r,0),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\},0.4\right)\right),$$
$$\left((e_3,m,0),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.1}\right\},0.3\right)\right),\left((e_3,n,0),\left(\left\{\frac{u_1}{0.9},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\},0.6\right)\right),$$
$$\left.\left((e_3,r,0),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.5},\frac{u_3}{0.4},\frac{u_4}{0.6}\right\},0.5\right)\right)\right\}. \tag{3.10}$$

Definition 3.9. The *complement* of a generalized fuzzy soft expert set (F_μ, A) is denoted by $(F_\mu, A)^c$ and is defined by $(F_\mu, A) = (F^c_\mu, \rceil A)$ where $F_\mu{}^c : \rceil A \to I^U$ is a mapping given by

$$F^c_\mu(\alpha) = c\left(F_\mu(\rceil\alpha)\right), \quad \forall \alpha \in \rceil A, \tag{3.11}$$

where c is a generalized fuzzy complement and $\rceil A \subset \{\rceil E \times X \times O\}$.

Example 3.10. Consider Example 3.2. By using the basic fuzzy complement, we have

$$(F_\mu, Z)^c = \left\{\left((e_1,m,1),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.7},\frac{u_3}{0.8},\frac{u_4}{0.6}\right\},0.8\right)\right),\left((e_1,n,1),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.6},\frac{u_3}{0.9},\frac{u_4}{0.7}\right\},0.9\right)\right),\right.$$
$$\left((e_1,r,1),\left(\left\{\frac{u_1}{0.9},\frac{u_2}{0.7},\frac{u_3}{0.2},\frac{u_4}{0.3}\right\},0.8\right)\right),\left((e_2,m,1),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.7},\frac{u_3}{0.8},\frac{u_4}{0.3}\right\},0.6\right)\right),$$
$$\left((e_2,n,1),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.6}\right\},0.8\right)\right),\left((e_2,r,1),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.4},\frac{u_4}{0.3}\right\},0.7\right)\right),$$
$$\left((e_3,m,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.4},\frac{u_3}{0.2},\frac{u_4}{0.1}\right\},0.4\right)\right),\left((e_3,n,1),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.7},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\},0.5\right)\right),$$
$$\left((e_3,r,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.6},\frac{u_3}{0.4},\frac{u_4}{0.7}\right\},0.2\right)\right),\left((e_1,m,0),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\},0.3\right)\right),$$
$$\left((e_1,n,0),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.3},\frac{u_3}{0.2},\frac{u_4}{0.3}\right\},0.1\right)\right),\left((e_1,r,0),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.2},\frac{u_3}{0.6},\frac{u_4}{0.5}\right\},0.4\right)\right),$$
$$\left((e_2,m,0),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.4},\frac{u_3}{0.2},\frac{u_4}{0.7}\right\},0.4\right)\right),\left((e_2,n,0),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\},0.3\right)\right),$$
$$\left((e_2,r,0),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\},0.6\right)\right),\left((e_3,m,0),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.9}\right\},0.7\right)\right),$$
$$\left.\left((e_3,n,0),\left(\left\{\frac{u_1}{0.1},\frac{u_2}{0.4},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\},0.4\right)\right),\left((e_3,r,0),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.4}\right\},0.5\right)\right)\right\}. \tag{3.12}$$

Proposition 3.11. *If (F_μ, A) is a generalized a fuzzy soft expert set over U, then*

(1) $((F_\mu, A)^c)^c = (F_\mu, A)$,

(2) $(F_\mu, A)^c_1 = (F_\mu, A)_0$,

(3) $(F_\mu, A)^c_0 = (F_\mu, A)_1$.

Proof. The proof is straightforward. □

4. Union and Intersection

In this section, we introduce the definitions of union and intersection of a generalized fuzzy soft expert sets, derive their properties, and give some examples.

Definition 4.1. The *union* of two GFSESs (F_μ, A) and (G_δ, B) over U, denoted by $(F_\mu, A)\tilde{\cup}(G_\delta, B)$, is the GFSESs (H_Ω, C) such that $C = A \cup B \subset \{E \times X \times O\}$ and, for all $\varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon)\tilde{\bigcup}G(\varepsilon), & \text{if } \varepsilon \in A \cap B, \end{cases} \tag{4.1}$$

where $\tilde{\bigcup}$ is a generalized fuzzy soft expert sets.

Example 4.2. Consider Example 3.2. Let

$$\begin{aligned} A &= \{(e_1, n, 1), (e_1, r, 1), (e_2, m, 1), (e_2, m, 0), (e_2, r, 1), (e_3, n, 1), (e_3, n, 0), (e_3, r, 0)\}, \\ B &= \{(e_1, n, 1), (e_1, r, 1), (e_2, m, 1), (e_2, r, 0), (e_3, n, 0), (e_3, r, 0)\}. \end{aligned} \tag{4.2}$$

Suppose (F_μ, A) and (G_δ, B) are two GFSESs over U such that

$$\begin{aligned} (F_\mu, A) = \Bigg\{ &\left((e_1, n, 1), \left(\left\{\frac{u_1}{0.7}, \frac{u_2}{0.4}, \frac{u_3}{0.1}, \frac{u_4}{0.3}\right\}, 0.1\right)\right), \left((e_1, r, 1), \left(\left\{\frac{u_1}{0.1}, \frac{u_2}{0.3}, \frac{u_3}{0.8}, \frac{u_4}{0.7}\right\}, 0.2\right)\right), \\ &\left((e_2, m, 1), \left(\left\{\frac{u_1}{0.5}, \frac{u_2}{0.3}, \frac{u_3}{0.2}, \frac{u_4}{0.7}\right\}, 0.4\right)\right), \left((e_2, m, 0), \left(\left\{\frac{u_1}{0.1}, \frac{u_2}{0.2}, \frac{u_3}{0.4}, \frac{u_4}{0.3}\right\}, 0.6\right)\right), \\ &\left((e_2, r, 1), \left(\left\{\frac{u_1}{0.7}, \frac{u_2}{0.5}, \frac{u_3}{0.6}, \frac{u_4}{0.7}\right\}, 0.3\right)\right), \left((e_3, n, 1), \left(\left\{\frac{u_1}{0.2}, \frac{u_2}{0.3}, \frac{u_3}{0.7}, \frac{u_4}{0.6}\right\}, 0.5\right)\right), \\ &\left((e_3, n, 0), \left(\left\{\frac{u_1}{0.4}, \frac{u_2}{0.6}, \frac{u_3}{0.5}, \frac{u_4}{0.3}\right\} 0.6\right)\right), \left((e_3, r, 0), \left(\left\{\frac{u_1}{0.3}, \frac{u_2}{0.5}, \frac{u_3}{0.4}, \frac{u_4}{0.2}\right\} 0.5\right)\right), \\ (G_\delta, B) = \Bigg\{ &\left((e_1, n, 1), \left(\left\{\frac{u_1}{0.5}, \frac{u_2}{0.6}, \frac{u_3}{0.2}, \frac{u_4}{0.1}\right\}, 0.3\right)\right), \left((e_1, r, 1), \left(\left\{\frac{u_1}{0.4}, \frac{u_2}{0.2}, \frac{u_3}{0.8}, \frac{u_4}{0.6}\right\}, 0.1\right)\right), \\ &\left((e_2, m, 1), \left(\left\{\frac{u_1}{0.5}, \frac{u_2}{0.4}, \frac{u_3}{0.0}, \frac{u_4}{0.6}\right\}, 0.4\right)\right), \left((e_2, r, 0), \left(\left\{\frac{u_1}{0.8}, \frac{u_2}{0.6}, \frac{u_3}{0.9}, \frac{u_4}{1.0}\right\}, 0.8\right)\right), \\ &\left((e_3, n, 0), \left(\left\{\frac{u_1}{0.3}, \frac{u_2}{0.4}, \frac{u_3}{0.3}, \frac{u_4}{0.1}\right\}, 0.4\right)\right), \left((e_3, r, 0), \left(\left\{\frac{u_1}{0.5}, \frac{u_2}{0.7}, \frac{u_3}{0.6}, \frac{u_4}{0.4}\right\}, 0.9\right)\right)\Bigg\}. \end{aligned} \tag{4.3}$$

Then $(F,A)\tilde{\cup}(G,B)=(H,C)$ where

$$\begin{aligned}(H_\Omega,C)=\Bigg\{&\left((e_1,n,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.6},\frac{u_3}{0.2},\frac{u_4}{0.3}\right\},0.3\right)\right),\left((e_1,r,1),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.3},\frac{u_3}{0.8},\frac{u_4}{0.7}\right\},0.2\right)\right),\\&\left((e_2,m,1),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.4},\frac{u_3}{0.2},\frac{u_4}{0.7}\right\},0.4\right)\right),\left((e_2,m,0),\left(\left\{\frac{u_1}{0.1},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.3}\right\},0.6\right)\right),\\&\left((e_2,r,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.7}\right\},0.3\right)\right),\left((e_2,r,0),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.6},\frac{u_3}{0.9},\frac{u_4}{1.0}\right\},0.8\right)\right),\\&\left((e_3,n,1),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.3},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\},0.5\right)\right),\left((e_3,n,0),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\},0.6\right)\right),\\&\left((e_3,r,0),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.7},\frac{u_3}{0.6},\frac{u_4}{0.4}\right\},0.9\right)\right)\Bigg\}.\end{aligned}\tag{4.4}$$

Proposition 4.3. *If (F_μ,A), (G_δ,B) and (H_Ω,C) are three GFSESs over U, then*

(1) $(F_\mu,A)\tilde{\cup}\,((G_\delta,B)\ \tilde{\cup}\ (H_\Omega,C))=((F_\mu,A)\ \tilde{\cup}\ (G_\delta,B))\ \tilde{\cup}\ (H_\Omega,C)$,

(2) $(F_\mu,A)\tilde{\cup}\ (F_\mu,A)=(F_\mu,A)$.

Proof. The proof is straightforward. □

Definition 4.4. The *intersection* of two GFSESs (F_μ,A) and (G_δ,B) over U, denoted by $(F_\mu,A)\tilde{\cup}(G_\delta,B)$, is the GFSES (H_Ω,C) such that $C=A\cup B\subset\{E\times X\times O\}$ and, for all $\varepsilon\in C$,

$$H(\varepsilon)=\begin{cases}F(\varepsilon), & \text{if } \varepsilon\in A-B\\ G(\varepsilon), & \text{if } \varepsilon\in B-A\\ F(\varepsilon)\tilde{\bigcap}G(\varepsilon), & \text{if } \varepsilon\in A\cap B,\end{cases}\tag{4.5}$$

where $\tilde{\bigcap}$ is an interval-valued fuzzy intersection.

Example 4.5. Consider Example 4.2; we have $(F,A)\tilde{\cap}(G,B)=(H,C)$ where

$$\begin{aligned}(H_\Omega,C)=\Bigg\{&\left((e_1,n,1),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.4},\frac{u_3}{0.1},\frac{u_4}{0.1}\right\},0.1\right)\right),\left((e_1,r,1),\left(\left\{\frac{u_1}{0.1},\frac{u_2}{0.2},\frac{u_3}{0.8},\frac{u_4}{0.6}\right\},0.1\right)\right),\\&\left((e_2,m,1),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.3},\frac{u_3}{0.0},\frac{u_4}{0.6}\right\},0.4\right)\right),\left((e_2,m,0),\left(\left\{\frac{u_1}{0.1},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.3}\right\},0.6\right)\right),\\&\left((e_2,r,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.7}\right\},0.3\right)\right),\left((e_2,r,0),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.6},\frac{u_3}{0.9},\frac{u_4}{1.0}\right\},0.8\right)\right),\\&\left((e_3,n,1),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.3},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\},0.5\right)\right),\left((e_3,n,0),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.1}\right\},0.4\right)\right),\\&\left((e_3,r,0),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.4},\frac{u_4}{0.2}\right\},0.5\right)\right)\Bigg\}.\end{aligned}\tag{4.6}$$

Proposition 4.6. *If* (F_μ, A), (G_δ, B), *and* (H_Ω, C) *are three GFSESs over* U, *then*

(1) $(F_\mu, A)\tilde{\cap}\, ((G_\delta, B) \,\tilde{\cap}\, (H_\Omega, C)) = ((F_\mu, A) \,\tilde{\cap}\, (G_\delta, B)) \,\tilde{\cap}\, (H_\Omega, C)$,

(2) $(F_\mu, A)\tilde{\cap}\, (F_\mu, A) = (F_\mu, A)$.

Proof. The proof is straightforward. □

Proposition 4.7. *If* (F_μ, A), (G_δ, B), *and* (H_Ω, C) *are three GFSESs over* U, *then*

(1) $(F_\mu, A)\tilde{\cup}((G_\delta, B)\tilde{\cap}(H_\Omega, C)) = ((F_\mu, A)\tilde{\cup}(G_\delta, B))\tilde{\cap}((F_\mu, A)\tilde{\cup}(H_\Omega, C))$,

(2) $(F_\mu, A)\tilde{\cap}((G_\delta, B)\tilde{\cup}(H_\Omega, C)) = ((F_\mu, A)\tilde{\cap}(G_\delta, B))\tilde{\cup}((F_\mu, A)\tilde{\cap}(H_\Omega, C))$.

Proof. The proof is straightforward. □

5. AND and OR Operations

In this section, we introduce the definitions of AND and OR operations for GFSES, derive their properties, and give some examples.

Definition 5.1. If (F_μ, A) and (G_δ, B) are two GFSES over U, then "(F_μ, A) AND (G_δ, B)" denoted by $(F_\mu, A) \wedge (G_\delta, B)$ is defined by

$$(F_\mu, A) \wedge (G_\delta, B) = (H_\Omega, A \times B) \tag{5.1}$$

such that $H(\alpha, \beta) = F(\alpha)\tilde{\bigcap}G(\beta)$, for all $(\alpha, \beta) \in A \times B$, where $\tilde{\bigcap}$ is GFSES.

Example 5.2. Consider Example 3.2. Let

$$\begin{aligned} A &= \{(e_2, m, 1), (e_2, n, 0), (e_3, r, 1), (e_3, r, 0)\}, \\ B &= \{(e_2, m, 1), (e_2, r, 1), (e_3, n, 0)\}. \end{aligned} \tag{5.2}$$

Suppose (F_μ, A) and (G_δ, B) are two fuzzy soft expert sets over U such that

$$\begin{aligned} (F_\mu, A) = \Bigg\{&\left((e_2, m, 1), \left(\left\{\frac{u_1}{0.5}, \frac{u_2}{0.2}, \frac{u_3}{0.4}, \frac{u_4}{0.8}\right\}, 0.2\right)\right), \left((e_2, n, 0), \left(\left\{\frac{u_1}{0.3}, \frac{u_2}{0.6}, \frac{u_3}{0.5}, \frac{u_4}{0.7}\right\}, 0.3\right)\right), \\ &\left((e_3, r, 1), \left(\left\{\frac{u_1}{0.4}, \frac{u_2}{0.8}, \frac{u_3}{0.5}, \frac{u_4}{0.9}\right\}, 0.7\right)\right), \left((e_3, r, 0), \left(\left\{\frac{u_1}{0.1}, \frac{u_2}{0.4}, \frac{u_3}{0.7}, \frac{u_4}{0.8}\right\}, 0.5\right)\right)\Bigg\}, \\ (G_\delta, B) = \Bigg\{&\left((e_2, m, 1), \left(\left\{\frac{u_1}{0.3}, \frac{u_2}{0.4}, \frac{u_3}{0.3}, \frac{u_4}{0.6}\right\}, 0.4\right)\right), \left((e_2, r, 1), \left(\left\{\frac{u_1}{0.6}, \frac{u_2}{0.7}, \frac{u_3}{0.4}, \frac{u_4}{0.7}\right\}, 0.5\right)\right), \\ &\left((e_3, n, 0), \left(\left\{\frac{u_1}{0.2}, \frac{u_2}{0.7}, \frac{u_3}{0.8}, \frac{u_4}{0.6}\right\}, 0.8\right)\right)\Bigg\}. \end{aligned} \tag{5.3}$$

Then

$$
\begin{aligned}
(F_\mu, A) \wedge (G_\delta, B) &= (H_\Omega, A \times B) \\
&= \left\{ \left(((e_2, m, 1), (e_2, m, 1)), \left(\left\{ \frac{u_1}{0.3}, \frac{u_2}{0.2}, \frac{u_3}{0.3}, \frac{u_4}{0.6} \right\}, 0.2 \right) \right), \right. \\
&\quad \left(((e_2, m, 1), (e_2, r, 1)), \left(\left\{ \frac{u_1}{0.5}, \frac{u_2}{0.2}, \frac{u_3}{0.4}, \frac{u_4}{0.7} \right\}, 0.2 \right) \right), \\
&\quad \left(((e_2, m, 1), (e_3, n, 0)), \left(\left\{ \frac{u_1}{0.2}, \frac{u_2}{0.2}, \frac{u_3}{0.4}, \frac{u_4}{0.6} \right\}, 0.2 \right) \right), \\
&\quad \left(((e_2, n, 0), (e_2, m, 1)), \left(\left\{ \frac{u_1}{0.3}, \frac{u_2}{0.4}, \frac{u_3}{0.3}, \frac{u_4}{0.6} \right\}, 0.3 \right) \right), \\
&\quad \left(((e_2, n, 0), (e_2, r, 1)), \left(\left\{ \frac{u_1}{0.3}, \frac{u_2}{0.6}, \frac{u_3}{0.4}, \frac{u_4}{0.7} \right\}, 0.3 \right) \right), \\
&\quad \left(((e_2, n, 0), (e_3, n, 0)), \left(\left\{ \frac{u_1}{0.2}, \frac{u_2}{0.6}, \frac{u_3}{0.5}, \frac{u_4}{0.6} \right\}, 0.3 \right) \right), \\
&\quad \left(((e_3, r, 1), (e_2, m, 1)), \left(\left\{ \frac{u_1}{0.3}, \frac{u_2}{0.4}, \frac{u_3}{0.3}, \frac{u_4}{0.6} \right\}, 0.4 \right) \right), \\
&\quad \left(((e_3, r, 1), (e_2, r, 1)), \left(\left\{ \frac{u_1}{0.4}, \frac{u_2}{0.7}, \frac{u_3}{0.4}, \frac{u_4}{0.7} \right\}, 0.5 \right) \right), \\
&\quad \left(((e_3, r, 1), (e_3, n, 0)), \left(\left\{ \frac{u_1}{0.2}, \frac{u_2}{0.7}, \frac{u_3}{0.5}, \frac{u_4}{0.6} \right\}, 0.7 \right) \right), \\
&\quad \left(((e_3, r, 0), (e_2, m, 1)), \left(\left\{ \frac{u_1}{0.1}, \frac{u_2}{0.4}, \frac{u_3}{0.3}, \frac{u_4}{0.6} \right\}, 0.4 \right) \right), \\
&\quad \left(((e_3, r, 0), (e_2, r, 1)), \left(\left\{ \frac{u_1}{0.1}, \frac{u_2}{0.4}, \frac{u_3}{0.4}, \frac{u_4}{0.7} \right\}, 0.5 \right) \right), \\
&\quad \left. \left(((e_3, r, 0), (e_3, n, 0)), \left(\left\{ \frac{u_1}{0.1}, \frac{u_2}{0.4}, \frac{u_3}{0.7}, \frac{u_4}{0.6} \right\}, 0.5 \right) \right) \right\}.
\end{aligned}
\tag{5.4}
$$

Definition 5.3. If (F_μ, A) and (G_δ, B) are two GFSES over U, then "(F_μ, A) OR (G_δ, B)" denoted by $(F_\mu, A) \bigvee (G_\delta, B)$ is defined by

$$
(F_\mu, A) \bigvee (G_\delta, B) = (H_\delta, A \times B) \tag{5.5}
$$

such that $H(\alpha, \beta) = F(\alpha) \widetilde{\bigcup} G(\beta)$, for all $(\alpha, \beta) \in A \times B$, where $\widetilde{\bigcup}$ is an generalized fuzzy union.

Example 5.4. Consider Example 5.2 we have

$$
\begin{aligned}
(F_\mu, A) \vee (G_\delta, B) &= (H_\Omega, A \times B) \\
&= \left\{ \left(((e_2, m, 1), (e_2, m, 1)), \left(\left\{ \frac{u_1}{0.5}, \frac{u_2}{0.4}, \frac{u_3}{0.4}, \frac{u_4}{0.8} \right\}, 0.4 \right) \right), \right. \\
&\quad \left(((e_2, m, 1), (e_2, r, 1)), \left(\left\{ \frac{u_1}{0.6}, \frac{u_2}{0.7}, \frac{u_3}{0.4}, \frac{u_4}{0.8} \right\}, 0.5 \right) \right),
\end{aligned}
$$

$$\left(((e_2,m,1),(e_3,n,0)),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.7},\frac{u_3}{0.8},\frac{u_4}{0.8}\right\},0.8\right)\right),$$
$$\left(((e_2,n,0),(e_2,m,1)),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\},0.4\right)\right),$$
$$\left(((e_2,n,0),(e_2,r,1)),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.7},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\},0.5\right)\right),$$
$$\left(((e_2,n,0),(e_3,n,0)),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.7},\frac{u_3}{0.8},\frac{u_4}{0.7}\right\},0.8\right)\right),$$
$$\left(((e_3,r,1),(e_2,m,1)),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.8},\frac{u_3}{0.5},\frac{u_4}{0.9}\right\},0.7\right)\right),$$
$$\left(((e_3,r,1),(e_2,r,1)),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.8},\frac{u_3}{0.5},\frac{u_4}{0.9}\right\},0.7\right)\right),$$
$$\left(((e_3,r,1),(e_3,n,0)),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.8},\frac{u_3}{0.8},\frac{u_4}{0.9}\right\},0.8\right)\right),$$
$$\left(((e_3,r,0),(e_2,m,1)),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.4},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\},0.5\right)\right),$$
$$\left(((e_3,r,0),(e_2,r,1)),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.7},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\},0.5\right)\right),$$
$$\left.\left(((e_3,r,0),(e_3,n,0)),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.7},\frac{u_3}{0.8},\frac{u_4}{0.8}\right\},0.8\right)\right)\right\}. \tag{5.6}$$

6. An Application of Generalized Fuzzy Soft Expert Set in Decision Making

In this section, we present an application of the generalized fuzzy soft expert set theory in a decision making problem. Suppose that one of the broadcasting channels wants to invite experts to evaluate their show through the discussion of a controversial issue and obtain their opinion of the situation. The producers of the show used the following criteria to determine how to evaluate their findings. Their four alternatives are as follows: $U = \{u_1, u_2, u_3, u_4\}$, suppose there are five parameters $E = \{e_1, e_2, e_3, e_4, e_5,\}$; choose the experts for the programs. For $i = 1, 2, 3, 4, 5$ the parameters e_i $(i = 1, 2, 3, 4, 5)$ stand for "this criteria to discriminate," "this criteria is independent of the other criteria," "this criteria measures one thing," "the universal criteria", and "the criteria that is important to some of the stakeholders." Let $X = \{m, n, r\}$ be a set of committee members. From those findings we can find the most suitable choice for the decision. After a serious discussion, the committee constructs the following generalised fuzzy soft expert set:

$$(F_\mu, Z) = \left\{\left((e_1,m,1),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.1},\frac{u_3}{0.0},\frac{u_4}{0.2}\right\},0.3\right)\right),\left((e_1,n,1),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.2},\frac{u_3}{0.1},\frac{u_4}{0.1}\right\},0.2\right)\right),\right.$$
$$\left((e_1,r,1),\left(\left\{\frac{u_1}{0.1},\frac{u_2}{0.0},\frac{u_3}{0.6},\frac{u_4}{0.5}\right\},0.5\right)\right),\left((e_2,m,1),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.1},\frac{u_3}{0.4},\frac{u_4}{0.7}\right\},0.4\right)\right),$$

$$\left((e_2,n,1),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.7},\frac{u_3}{0.8},\frac{u_4}{0.4}\right\},0.3\right)\right),\left((e_2,r,1),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.9}\right\},0.6\right)\right),$$
$$\left((e_3,m,1),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.6},\frac{u_3}{0.8},\frac{u_4}{0.9}\right\},0.1\right)\right),\left((e_3,n,1),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.5},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\},0.7\right)\right),$$
$$\left((e_3,r,1),\left(\left\{\frac{u_1}{0.1},\frac{u_2}{0.2},\frac{u_3}{0.5},\frac{u_4}{0.4}\right\},0.3\right)\right),\left((e_4,m,1),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.5}\right\},0.8\right)\right),$$
$$\left((e_4,n,1),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.7},\frac{u_3}{0.9},\frac{u_4}{0.7}\right\},0.6\right)\right),\left((e_4,r,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.5},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\},0.5\right)\right),$$
$$\left((e_5,m,1),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.6},\frac{u_3}{0.6},\frac{u_4}{0.8}\right\},0.3\right)\right),\left((e_5,n,1),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.7},\frac{u_3}{0.6},\frac{u_4}{0.9}\right\},0.7\right)\right),$$
$$\left((e_5,r,1),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.3},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\},0.4\right)\right),\left((e_1,m,0),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.8},\frac{u_3}{0.9},\frac{u_4}{0.7}\right\},0.9\right)\right),$$
$$\left((e_1,n,0),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.8},\frac{u_3}{0.6},\frac{u_4}{0.9}\right\},0.5\right)\right),\left((e_1,r,0),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.8},\frac{u_3}{0.3},\frac{u_4}{0.7}\right\},0.4\right)\right),$$
$$\left((e_2,m,0),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.4}\right\},0.3\right)\right),\left((e_2,n,0),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.1},\frac{u_3}{0.3},\frac{u_4}{0.7}\right\},0.2\right)\right),$$
$$\left((e_2,r,0),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.2},\frac{u_3}{0.2},\frac{u_4}{0.4}\right\},0.5\right)\right),\left((e_3,m,0),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.2},\frac{u_3}{0.4},\frac{u_4}{0.3}\right\},0.8\right)\right),$$
$$\left((e_3,n,0),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\},0.9\right)\right),\left((e_3,r,0),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\},0.6\right)\right),$$
$$\left((e_4,m,0),\left(\left\{\frac{u_1}{0.9},\frac{u_2}{0.4},\frac{u_3}{0.2},\frac{u_4}{0.6}\right\},0.7\right)\right),\left((e_4,n,0),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.2}\right\},0.6\right)\right),$$
$$\left((e_4,r,0),\left(\left\{\frac{u_1}{0.1},\frac{u_2}{0.3},\frac{u_3}{0.6},\frac{u_4}{0.8}\right\},0.5\right)\right),\left((e_5,m,0),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.3},\frac{u_4}{0.4}\right\},0.2\right)\right),$$
$$\left((e_5,n,0),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.5}\right\},0.1\right)\right),\left((e_5,r,0),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\},0.5\right)\right)\bigg\}. \tag{6.1}$$

In Tables 1 and 2 we present the agree-generalized fuzzy soft expert set and disagree-generalized fuzzy soft expert set. Now to determine the best choices, we first mark the highest numerical grade (underline) in each row in agree-generalized fuzzy soft expert set and disagree-generalized fuzzy soft expert set excluding the last column which is the grade of such belongingness of a expert against of parameters. Then we calculate the score of each of such expert in agree-generalized fuzzy soft expert set and disagree-generalized fuzzy soft expert set by taking the sum of the products of these numerical grades with the corresponding values of λ. Then we calculate the final score by subtracting the scores of expert in the agree-generalized fuzzy soft expert set from the scores of expert in disagree-generalized fuzzy soft expert set. The expert with the highest score is the desired expert.

Table 1: Agree-generalized fuzzy soft expert set.

U	u_1	u_2	u_3	u_4	λ
(e_1, m)	0.4	0.1	0.0	0.2	0.3
(e_2, m)	0.3	0.1	0.4	0.7	0.4
(e_3, m)	0.5	0.6	0.8	0.9	0.1
(e_4, m)	0.3	0.6	0.7	0.5	0.8
(e_5, m)	0.7	0.6	0.6	0.8	0.4
(e_1, n)	0.5	0.2	0.1	0.1	0.2
(e_2, n)	0.6	0.7	0.8	0.4	0.3
(e_3, n)	0.4	0.5	0.7	0.6	0.7
(e_4, n)	0.4	0.7	0.9	0.7	0.6
(e_5, n)	0.4	0.7	0.6	0.9	0.7
(e_1, r)	0.1	0.0	0.6	0.5	0.5
(e_2, r)	0.5	0.5	0.6	0.9	0.6
(e_3, r)	0.1	0.2	0.5	0.4	0.3
(e_4, r)	0.7	0.5	0.3	0.4	0.5
(e_5, r)	0.6	0.3	0.5	0.7	0.4

Table 2: Disagree-generalized fuzzy soft expert set.

U	u_1	u_2	u_3	u_4	λ
(e_1, m)	0.4	0.8	0.9	0.7	0.9
(e_2, m)	0.5	0.6	0.7	0.4	0.3
(e_3, m)	0.3	0.2	0.4	0.3	0.8
(e_4, m)	0.9	0.4	0.2	0.6	0.7
(e_5, m)	0.3	0.5	0.3	0.4	0.2
(e_1, n)	0.3	0.8	0.6	0.9	0.5
(e_2, n)	0.2	0.1	0.3	0.7	0.2
(e_3, n)	0.7	0.8	0.3	0.4	0.9
(e_4, n)	0.7	0.4	0.3	0.2	0.6
(e_5, n)	0.8	0.6	0.7	0.5	0.1
(e_1, r)	0.5	0.8	0.3	0.7	0.4
(e_2, r)	0.3	0.2	0.2	0.4	0.5
(e_3, r)	0.8	0.6	0.5	0.7	0.6
(e_4, r)	0.1	0.3	0.6	0.8	0.5
(e_5, r)	0.2	0.6	0.5	0.3	0.5

Now, we compute the score of u_i by using the data in Table 3:

$$\text{Score } (u_1) = (0.4 * 0.3) + (0.5 * 0.2) + (0.7 * 0.5) = 0.57,$$

$$\text{Score } (u_2) = 0,$$

$$\begin{aligned}\text{Score } (u_3) &= (0.7 * 0.8) + (0.8 * 0.3) + (0.7 * 0.7) + (0.9 * 0.6) \\ &\quad + (0.6 * 0.5) + (0.5 * 0.3) = 2.28,\end{aligned}$$

$$\begin{aligned}\text{Score } (u_4) &= (0.7 * 0.4) + (0.9 * 0.1) + (0.8 * 0.4) + (0.9 * 0.7) \\ &\quad + (0.9 * 0.6) + (0.7 * 0.4) = 2.5.\end{aligned}$$

Table 3: Grade table of agree-generalized fuzzy soft expert set.

R	x_i	Highest numerical grade	λ
(e_1, m)	u_1	0.4	0.3
(e_2, m)	u_4	0.7	0.4
(e_3, m)	u_4	0.9	0.1
(e_4, m)	u_3	0.7	0.8
(e_5, m)	u_4	0.8	0.4
(e_1, n)	u_1	0.5	0.2
(e_2, n)	u_3	0.8	0.3
(e_3, n)	u_3	0.7	0.7
(e_4, n)	u_3	0.9	0.6
(e_5, n)	u_4	0.9	0.7
(e_1, r)	u_3	0.6	0.5
(e_2, r)	u_4	0.9	0.6
(e_3, r)	u_3	0.5	0.3
(e_4, r)	u_1	0.7	0.5
(e_5, r)	u_4	0.7	0.4

Table 4: Grade table of disagree-generalized fuzzy soft expert set.

R	x_i	Highest numerical grade	λ
(e_1, m)	u_3	0.9	0.9
(e_2, m)	u_3	0.7	0.3
(e_3, m)	u_3	0.4	0.8
(e_4, m)	u_1	0.9	0.7
(e_5, m)	u_2	0.5	0.2
(e_1, n)	u_4	0.9	0.5
(e_2, n)	u_4	0.7	0.2
(e_3, n)	u_2	0.8	0.9
(e_4, n)	u_1	0.7	0.6
(e_5, n)	u_1	0.8	0.1
(e_1, r)	u_4	0.7	0.4
(e_2, r)	u_4	0.4	0.5
(e_3, r)	u_1	0.8	0.6
(e_4, r)	u_4	0.8	0.5
(e_5, r)	u_2	0.6	0.5

Now, we compute the score of u_i by using the data in Table 4:

$$\text{Score } (u_1) = (0.9 * 0.7) + (0.7 * 0.6) + (0.8 * 0.1) + (0.8 * 0.6) = 1.61,$$
$$\text{Score } (u_2) = (0.5 * 0.2) + (0.8 * 0.9) + (0.6 * 0.5) = 1.12,$$
$$\text{Score } (u_3) = (0.9 * 0.9) + (0.7 * 0.3) + (0.4 * 0.8) = 1.34,$$
$$\text{Score } (u_4) = (0.9 * 0.5) + (0.7 * 0.2) + (0.7 * 0.4) + (0.4 * 0.5) + (0.8 * 0.5) = 1.47.$$

The final score of u_i is as follows:

$$\text{Score } (u_1) = 0.57 - 1.61 = -1.04,$$
$$\text{Score } (u_2) = 0 - 1.12 = -1.12,$$

Table 5: Accepted opinions of generalized fuzzy soft expert set.

U	u_1	u_2	u_3	u_4	λ
(e_1, m, a)	0.2	0.4	0.3	0.6	0.2
(e_1, n, a)	0.0	0.5	0.1	0.4	0.4
(e_1, r, a)	0.4	0.3	0.3	0.5	0.5
(e_1, m, b)	0.4	0.5	0.4	0.7	0.2
(e_1, n, b)	0.1	0.6	0.3	0.5	0.4
(e_1, r, b)	0.5	0.4	0.5	0.6	0.5
(e_2, m, a)	0.5	0.7	0.7	0.8	0.5
(e_2, n, a)	0.5	0.6	0.5	0.6	0.3
(e_2, r, a)	0.3	0.5	0.5	0.7	0.4
(e_2, m, b)	0.6	0.7	0.8	0.8	0.5
(e_2, n, b)	0.7	0.8	0.6	0.9	0.3
(e_2, r, b)	0.5	0.7	0.6	0.8	0.4
(e_3, m, a)	0.8	0.9	0.6	0.7	0.7
(e_3, n, a)	0.9	0.7	0.7	0.6	0.3
(e_3, r, a)	0.6	0.6	0.8	0.7	0.2
(e_3, m, b)	0.9	1.0	0.8	0.9	0.7
(e_3, n, b)	0.9	0.8	0.7	0.8	0.3
(e_3, r, b)	0.7	0.8	0.8	0.9	0.2
(e_4, m, a)	0.3	0.6	0.8	0.5	0.2
(e_4, n, a)	0.1	0.5	0.6	0.4	0.8
(e_4, r, a)	0.2	0.4	0.6	0.4	0.6
(e_4, m, b)	0.5	0.7	0.8	0.6	0.2
(e_4, n, b)	0.3	0.7	0.8	0.5	0.8
(e_4, r, b)	0.4	0.6	0.7	0.7	0.6
(e_5, m, a)	0.7	0.5	0.9	0.4	0.9
(e_5, n, a)	0.8	0.6	0.7	0.3	0.5
(e_5, r, a)	0.6	0.4	0.7	0.5	0.6
(e_5, m, b)	0.8	0.6	0.9	0.6	0.9
(e_5, n, b)	0.9	0.8	0.9	0.5	0.5
(e_5, r, b)	0.7	0.6	0.8	0.6	0.6

$$\text{Score } (u_3) = 2.28 - 1.34 = 0.94,$$

$$\text{Score } (u_4) = 2.5 - 1.47 = 1.03.$$

Then the decision is u_4.

7. An Application of Generalized Fuzzy Soft Expert Set with Multiopinions in Decision Making

In this section, we present an application of the generalized fuzzy soft expert set theory in a decision making problem. In this section, we present an application of generalized fuzzy soft expert set with multiopinions in decision making problem. Let U be a universe set, E a set of parameters, X a set of experts (agents), and $O = \{a = \text{strongly agree}, b = \text{agree}, c = \text{disagree}, d = \text{strongly disagree}\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Consider the previous application 6. Suppose that the management of broadcasting channels takes the opinion of the committee once again with multiopinions. Let where is the rest of the statement and so forth:

$$
\begin{aligned}
(F,Z)= \Bigg\{ & \left((e_1,m,a),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.6}\right\},0.2\right)\right),\left((e_1,n,a),\left(\left\{\frac{u_1}{0.0},\frac{u_2}{0.5},\frac{u_3}{0.1},\frac{u_4}{0.4}\right\},0.4\right)\right),\\
& \left((e_1,r,a),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.3},\frac{u_3}{0.3},\frac{u_4}{0.5}\right\},0.5\right)\right),\left((e_2,m,a),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.7},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\},0.5\right)\right),\\
& \left((e_2,n,a),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\},0.3\right)\right),\left((e_2,r,a),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.5},\frac{u_3}{0.5},\frac{u_4}{0.7}\right\},0.4\right)\right),\\
& \left((e_3,m,a),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.9},\frac{u_3}{0.6},\frac{u_4}{0.7}\right\},0.7\right)\right),\left((e_3,n,a),\left(\left\{\frac{u_1}{0.9},\frac{u_2}{0.7},\frac{u_3}{0.7},\frac{u_4}{0.6}\right\},0.3\right)\right),\\
& \left((e_3,r,a),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.6},\frac{u_3}{0.8},\frac{u_4}{0.7}\right\},0.2\right)\right),\left((e_4,m,a),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.6},\frac{u_3}{0.8},\frac{u_4}{0.5}\right\},0.2\right)\right),\\
& \left((e_4,n,a),\left(\left\{\frac{u_1}{0.1},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.4}\right\},0.8\right)\right),\left((e_4,r,a),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.4},\frac{u_3}{0.6},\frac{u_4}{0.4}\right\},0.6\right)\right),\\
& \left((e_5,m,a),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.5},\frac{u_3}{0.9},\frac{u_4}{0.4}\right\},0.9\right)\right),\left((e_5,n,a),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.3}\right\},0.5\right)\right),\\
& \left((e_5,r,a),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\},0.6\right)\right),\left((e_1,m,b),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.5},\frac{u_3}{0.4},\frac{u_4}{0.7}\right\},0.2\right)\right),\\
& \left((e_1,n,b),\left(\left\{\frac{u_1}{0.1},\frac{u_2}{0.6},\frac{u_3}{0.3},\frac{u_4}{0.5}\right\},0.4\right)\right),\left((e_1,r,b),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.4},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\},0.5\right)\right),\\
& \left((e_2,m,b),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.7},\frac{u_3}{0.8},\frac{u_4}{0.8}\right\},0.5\right)\right),\left((e_2,n,b),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.6},\frac{u_4}{0.9}\right\},0.3\right)\right),\\
& \left((e_2,r,b),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.7},\frac{u_3}{0.6},\frac{u_4}{0.8}\right\},0.4\right)\right),\left((e_3,m,b),\left(\left\{\frac{u_1}{0.9},\frac{u_2}{1.0},\frac{u_3}{0.8},\frac{u_4}{0.9}\right\},0.7\right)\right),\\
& \left((e_3,n,b),\left(\left\{\frac{u_1}{0.9},\frac{u_2}{0.8},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\},0.3\right)\right),\left((e_3,r,b),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.8},\frac{u_4}{0.9}\right\},0.2\right)\right),\\
& \left((e_4,m,b),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.7},\frac{u_3}{0.8},\frac{u_4}{0.6}\right\},0.2\right)\right),\left((e_4,n,b),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.7},\frac{u_3}{0.8},\frac{u_4}{0.5}\right\},0.8\right)\right),\\
& \left((e_4,r,b),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.6},\frac{u_3}{0.7},\frac{u_4}{0.7}\right\},0.6\right)\right),\left((e_5,m,b),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.6},\frac{u_3}{0.9},\frac{u_4}{0.6}\right\},0.9\right)\right),\\
& \left((e_5,n,b),\left(\left\{\frac{u_1}{0.9},\frac{u_2}{0.8},\frac{u_3}{0.9},\frac{u_4}{0.5}\right\},0.5\right)\right),\left((e_5,r,b),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.6},\frac{u_3}{0.8},\frac{u_4}{0.6}\right\},0.6\right)\right),\\
& \left((e_1,m,c),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.3}\right\},0.2\right)\right),\left((e_1,n,c),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.7},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\},0.4\right)\right),\\
& \left((e_1,r,c),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.7},\frac{u_3}{0.6},\frac{u_4}{0.7}\right\},0.5\right)\right),\left((e_2,m,c),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.2},\frac{u_3}{0.1},\frac{u_4}{0.1}\right\},0.5\right)\right),\\
& \left((e_2,n,c),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.1}\right\},0.3\right)\right),\left((e_2,r,c),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.3},\frac{u_3}{0.4},\frac{u_4}{0.1}\right\},0.4\right)\right),
\end{aligned}
$$

$$\left((e_3,m,c),\left(\left\{\frac{u_1}{0.1},\frac{u_2}{0.1},\frac{u_3}{0.2},\frac{u_4}{0.2}\right\},0.7\right)\right),\left((e_3,n,c),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.3},\frac{u_3}{0.1},\frac{u_4}{0.3}\right\},0.3\right)\right),$$
$$\left((e_3,r,c),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.2},\frac{u_3}{0.1},\frac{u_4}{0.2}\right\},0.2\right)\right),\left((e_4,m,c),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.2},\frac{u_3}{0.1},\frac{u_4}{0.4}\right\},0.2\right)\right),$$
$$\left((e_4,n,c),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.3},\frac{u_3}{0.4},\frac{u_4}{0.5}\right\},0.8\right)\right),\left((e_4,r,c),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.5}\right\},0.6\right)\right),$$
$$\left((e_5,m,c),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.3},\frac{u_3}{0.0},\frac{u_4}{0.4}\right\},0.9\right)\right),\left((e_5,n,c),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.4},\frac{u_3}{0.3},\frac{u_4}{0.5}\right\},0.5\right)\right),$$
$$\left((e_5,r,c),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.5},\frac{u_3}{0.2},\frac{u_4}{0.4}\right\},0.6\right)\right),\left((e_1,m,d),\left(\left\{\frac{u_1}{0.8},\frac{u_2}{0.7},\frac{u_3}{0.8},\frac{u_4}{0.5}\right\},0.2\right)\right),$$
$$\left((e_1,n,d),\left(\left\{\frac{u_1}{0.9},\frac{u_2}{0.8},\frac{u_3}{0.7},\frac{u_4}{0.7}\right\},0.4\right)\right),\left((e_1,r,d),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.8},\frac{u_3}{0.7},\frac{u_4}{0.8}\right\},0.5\right)\right),$$
$$\left((e_2,m,d),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.3},\frac{u_3}{0.2},\frac{u_4}{0.1}\right\},0.5\right)\right),\left((e_2,n,d),\left(\left\{\frac{u_1}{0.6},\frac{u_2}{0.5},\frac{u_3}{0.6},\frac{u_4}{0.2}\right\},0.3\right)\right),$$
$$\left((e_2,r,d),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.3}\right\},0.4\right)\right),\left((e_3,m,d),\left(\left\{\frac{u_1}{0.2},\frac{u_2}{0.0},\frac{u_3}{0.3},\frac{u_4}{0.2}\right\},0.7\right)\right),$$
$$\left((e_3,n,d),\left(\left\{\frac{u_1}{0.3},\frac{u_2}{0.4},\frac{u_3}{0.2},\frac{u_4}{0.4}\right\},0.3\right)\right),\left((e_3,r,d),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.3},\frac{u_3}{0.2},\frac{u_4}{0.4}\right\},0.2\right)\right),$$
$$\left((e_4,m,d),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.3},\frac{u_3}{0.3},\frac{u_4}{0.5}\right\},0.2\right)\right),\left((e_4,n,d),\left(\left\{\frac{u_1}{0.9},\frac{u_2}{0.4},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\},0.8\right)\right),$$
$$\left((e_4,r,d),\left(\left\{\frac{u_1}{0.7},\frac{u_2}{0.6},\frac{u_3}{0.5},\frac{u_4}{0.6}\right\},0.6\right)\right),\left((e_5,m,d),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.5},\frac{u_3}{0.1},\frac{u_4}{0.5}\right\},0.9\right)\right),$$
$$\left((e_5,n,d),\left(\left\{\frac{u_1}{0.4},\frac{u_2}{0.5},\frac{u_3}{0.4},\frac{u_4}{0.7}\right\},0.5\right)\right),\left((e_5,r,d),\left(\left\{\frac{u_1}{0.5},\frac{u_2}{0.5},\frac{u_3}{0.3},\frac{u_4}{0.6}\right\},0.6\right)\right)\Bigg\}. \tag{7.1}$$

7.1. Algorithm

Our goal is to change the generalised Fuzzy soft expert set with multi opinions to generalised fuzzy soft set to find the optimal decision. Note: here we say that the term accept means agree and strongly agree and the term reject means disagree and strongly disagree.

Now we find the simplified for Table 5 where each entries $a_{ij} = \sum_{x\in\{X\times\{a,b\}\}} F(e_i)$, for all $j = 1,2,\dots,\|U\|$ and the last entry $a_{i(\|U\|+1)} = \sum_{x\in\{X\times\{a,b\}\}} \mu(e_i)$, for all $i = 1,2,\dots,\|E\|$.

Now we find the simplified for Table 6 where each entries $a_{ij} = \sum_{x\in\{X\times\{c,d\}\}} F(e_i)$, for all $j = 1,2,\dots,\|U\|$ and the last entry $a_{i(\|U\|+1)} = \sum_{x\in\{X\times\{c,d\}\}} \mu(e_i)$, for all $i = 1,2,\dots,\|E\|$.

Now to determine the best choice, we first mark the highest numerical grade (indicated in underline) in each row excluding the last column which is the grade of such belongingness of the expert against each pair of parameters. Now the score of each of the expert is calculated by taking the sum of the products of these numerical grades with the corresponding values of λ. The expert with the highest score is the desired expert.

Table 6: 1-rejected opinions of generalized fuzzy soft expert set.

U	u_1	u_2	u_3	u_4	λ
(e_1, m, c)	0.4	0.5	0.4	0.7	0.2
(e_1, n, c)	0.2	0.3	0.5	0.4	0.4
(e_1, r, c)	0.4	0.3	0.4	0.3	0.5
(e_1, m, d)	0.2	0.3	0.2	0.5	0.2
(e_1, n, d)	0.1	0.2	0.3	0.3	0.4
(e_1, r, d)	0.3	0.2	0.3	0.2	0.5
(e_2, m, c)	0.7	0.8	0.9	0.9	0.5
(e_2, n, c)	0.5	0.6	0.7	0.9	0.3
(e_2, r, c)	0.4	0.7	0.6	0.9	0.4
(e_2, m, d)	0.6	0.7	0.8	0.9	0.5
(e_2, n, d)	0.4	0.5	0.4	0.8	0.3
(e_2, r, d)	0.3	0.4	0.5	0.7	0.4
(e_3, m, c)	0.9	0.9	0.8	0.8	0.7
(e_3, n, c)	0.8	0.7	0.9	0.7	0.3
(e_3, r, c)	0.7	0.8	0.9	0.8	0.2
(e_3, m, d)	0.8	1.0	0.7	0.8	0.7
(e_3, n, d)	0.7	0.6	0.8	0.6	0.3
(e_3, r, d)	0.6	0.7	0.8	0.6	0.2
(e_4, m, c)	0.4	0.8	0.9	0.6	0.2
(e_4, n, c)	0.3	0.7	0.6	0.5	0.8
(e_4, r, c)	0.4	0.6	0.7	0.5	0.6
(e_4, m, d)	0.3	0.7	0.7	0.5	0.2
(e_4, n, d)	0.1	0.6	0.5	0.4	0.8
(e_4, r, d)	0.3	0.4	0.5	0.4	0.6
(e_5, m, c)	0.8	0.7	1.0	0.6	0.9
(e_5, n, c)	0.8	0.6	0.7	0.5	0.5
(e_5, r, c)	0.6	0.5	0.8	0.6	0.6
(e_5, m, d)	0.6	0.5	0.9	0.5	0.9
(e_5, n, d)	0.6	0.5	0.6	0.3	0.5
(e_5, r, d)	0.5	0.5	0.7	0.4	0.6

Table 7: Simplified accepted opinions of GFSES.

U	u_1	u_2	u_3	u_4	λ
$(e_1, \mathrm{X}, +)$	1.6	2.7	1.9	3.3	2.2
$(e_2, \mathrm{X}, +)$	3.1	4.0	3.7	4.6	2.4
$(e_3, \mathrm{X}, +)$	4.8	4.8	4.4	4.6	2.4
$(e_4, \mathrm{X}, +)$	1.8	3.5	4.3	3.1	3.2
$(e_5, \mathrm{X}, +)$	4.5	3.5	4.9	2.9	4.0

The committee can use the following algorithm.

(1) Input the generalised fuzzy soft expert set with multiopinions (F, Z).

(2) Find the accept opinions of generalised fuzzy soft expert set (Table 5).

(3) Find the 1-reject opinion of generalised fuzzy soft expert set (Table 6).

Table 8: Simplified 1-rejected opinions of GFSES.

U	u_1	u_2	u_3	u_4	λ
$(e_1, X, -)$	1.6	1.8	2.1	2.4	2.2
$(e_2, X, -)$	2.9	3.7	3.9	5.1	2.4
$(e_3, X, -)$	4.5	4.7	4.9	4.3	2.4
$(e_4, X, -)$	1.8	3.8	3.9	2.9	3.2
$(e_5, X, -)$	3.9	3.3	4.7	2.9	4.0

Table 9: Translation from GFSES—with multiopinion to GFSS.

U	u_1	u_2	u_3	u_4	λ
$G(e_1)$	0.266	0.375	0.333	$\underline{0.475}$	0.366
$G(e_2)$	0.500	0.641	0.633	$\underline{0.808}$	0.400
$G(e_3)$	0.775	$\underline{0.791}$	0.775	0.741	0.400
$G(e_4)$	0.300	0.608	$\underline{0.683}$	0.500	0.533
$G(e_5)$	0.700	0.566	$\underline{0.800}$	0.483	0.666

Table 10: Grade table of GFSS.

R	x_i	Highest numerical grade	λ
(e_1)	u_4	0.475	0.366
(e_2)	u_4	0.808	0.400
(e_3)	u_2	0.791	0.400
(e_4)	u_3	0.683	0.533
(e_5)	u_3	0.800	0.666

(4) Find the simplified accept opinion of GFSES and the simplified 1-reject opinion of GFSES (Tables 7 and 8).

(5) Find (G_δ, E) where $G_\delta(e)$ defined as follows: for all $u \in U$ and for all $e \in E$,

$$G_\delta(e) = \left(\left\{ \frac{u}{(1/\|R\|) \sum_{p\in R}\left(\left(\mu^{+}(p)\right) + \left(1 - \mu^{-}(p)\right)\right)} \right\}, \frac{\delta}{(1/\|R\|) \sum_{(p)\in R}\left(\left(\lambda^{+}((p))\right) + \left(1 - \lambda^{-}(p)\right)\right)} \right), \tag{7.2}$$

where $R = O \times X$, Table 9.

(6) Apply Majumdar and Samanta algorithm on GFSS.

(i) Construct the grade table of GFSS, Table 10.

(ii) Compute the score of u_i.

(iii) The decision will select with highest score.

Now, we compute the score of u_i by using the data in Table 10:

Score $(u_1) = 0$,

Score $(u_2) = (0.791 * 0.400) = 0.361$,

$$\text{Score}\ (u_3) = (0.683 * 0.533) + (0.800 * 0.666) = 0.896,$$
$$\text{Score}\ (u_4) = (0.475 * 0.366) + (0.808 * 0.400) = 0.496.$$

Then the committee will select the expert with highest score. Hence they will select u_3.

8. Conclusions

In this work we have introduced the concept of generalised fuzzy soft expert set and studied some of its properties. The complement, union, and intersection operations have been defined on the generalised fuzzy soft expert set. An application of this theory is given in solving a decision making problem.

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