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Research Article

Soliton Solutions for the Wick-Type Stochastic KP Equation

Y. F. Guo,[1,2] L. M. Ling,[2] and D. L. Li[1]

[1] *Department of Information and Computation of Science, Guangxi University of Technology, Liuzhou, Guangxi 545006, China*
[2] *Institute of Applied Physics and Computational Mathematics, P.O. Box 8009, Beijing 100088, China*

Correspondence should be addressed to Y. F. Guo, guoyan_feng@yahoo.com.cn

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The Wick-type stochastic KP equation is researched. The stochastic single-soliton solutions and stochastic multisoliton solutions are shown by using the Hermite transform and Darboux transformation.

1. Introduction

In recent decades, there has been an increasing interest in taking random effects into account in modeling, analyzing, simulating, and predicting complex phenomena, which have been widely recognized in geophysical and climate dynamics, materials science, chemistry biology, and other areas, see [1, 2]. If the problem is considered in random environment, the stochastic partial differential equations (SPDEs) are appropriate mathematical models for complex systems under random influences or noise. So far, we know that the random wave is an important subject of stochastic partial differential equations.

In 1970, while studying the stability of the KdV soliton-like solutions with small transverse perturbations, Kadomtsev and Petviashvili [3] arrived at the two-dimensional version of the KdV equation:

$$u_{tx} = (u_{xxx} + 6uu_x)_x + 3\alpha^2 u_{yy}, \tag{1.1}$$

which is known as *Kadomtsev-Petviashvili* (KP) equation. The KP equation appears in physical applications in two different forms with $\alpha = 1$ and $\alpha = i$, usually referred to as the KP-I and the KP-II equations. The number of physical applications for the KP equation is even larger than the number of physical applications for the KdV equation. It is well known that homogeneous balance method [4, 5] has been widely applied to derive the nonlinear transformations and exact solutions (especially the solitary waves) and Darboux transformation [6], as well as the similar reductions of nonlinear PDEs in mathematical physics. These subjects have been researched by many authors.

For SPDEs, in [7], Holden et al. gave white noise functional approach to research stochastic partial differential equations in Wick versions, in which the random effects are taken into account. In this paper, we will use their theory and method to investigate the stochastic soliton solutions of Wick-type stochastic KP equation, which can be obtained in the influence of the random factors.

The Wick-type stochastic KP equation in white noise environment is considered as the following form:

$$U_{tx} = \left(f(t)\Diamond U_{xxx} + 6g(t)\Diamond U\Diamond U_x\right)_x^{\Diamond} + 3\alpha^2 f(t)\Diamond U_{yy} + W(t)\Diamond R^{\Diamond}\left(U, U_x, U_{xx}, U_{xxxx}, U_{yy}\right), \tag{1.2}$$

which is the perturbation of the KP equation with variable coefficients:

$$u_{tx} = \left(f(t)u_{xxx} + 6g(t)uu_x\right)_x + 3\alpha^2 f(t)u_{yy}, \tag{1.3}$$

by random force $W(t)\Diamond R^{\Diamond}(U, U_x, U_{xx}, U_{xxxx}, U_{yy})$, where $\Diamond$ is the Wick product on the Hida distribution space $(S(\mathbb{R}^d))^*$ which is defined in Section 2, $f(t)$ and $g(t)$ are functions of t, $W(t)$ is Gaussian white noise, that is, $W(t) = \dot{B}(t)$ and $B(t)$ is a Brownian motion, $R(u, u_x, u_{xx}, u_{xxxx}, u_{yy}) = \beta u_{xxxx} + 6\gamma u_x^2 + 6\gamma u u_{xx} + 3\alpha^2\beta u_{yy}$ is a function of $u, u_x, u_{xx}, u_{xxxx}, u_{yy}$ for some constants β, γ, and $R^{\Diamond}$ is the Wick version of the function R.

This paper is organized as follows. In Section 2, the work function spaces are given. In Section 3, we present the single-soliton solutions of stochastic KP equation (1.2). Section 4 is devoted to investigate the multisoliton solutions of stochastic KP equation (1.2).

2. SPDEs Driven by White Noise

Let $(S(\mathbb{R}^d))$ and $(S(\mathbb{R}^d))^*$ be the Hida test function and the Hida distribution space on $\mathbb{R}^d$, respectively. The collection $\xi^n = e^{(-x^2/2)h_n(\sqrt{2}x)/(\pi(n-1)!)^{1/2}}$, $n \geq 1$ constitutes an orthogonal basis for $L^2(\mathbb{R})$, where $h_n(x)$ is the d-order Hermite polynomials. The family of tensor products $\xi_\alpha = \xi_{\alpha_1,\ldots,\alpha_d} = \xi_{\alpha_1} \otimes \cdots \otimes \xi_{\alpha_1}$ $(\alpha \in \mathbb{N}^d)$ forms an orthogonal basis for $L^2(\mathbb{R}^d)$, where $\alpha = (\alpha_1, \ldots, \alpha_d)$ is d-dimensional multi-indices with $\alpha_1, \ldots, \alpha_d \in \mathbb{N}$. The multi-indices $\alpha = (\alpha_1, \ldots, \alpha_d)$ are defined as elements of the space $\mathcal{J} = (\mathbb{N}_0^{\mathbb{N}})_c$ of all sequences $\alpha = (\alpha_1, \alpha_2, \ldots)$ with elements $\alpha_i \in \mathbb{N}_0$ and with compact support, that is, with only finite many $\alpha_i \neq 0$. For $\alpha = (\alpha_1, \alpha_2, \ldots)$, we define

$$H_\alpha(\omega) = \prod_{i=1}^{\infty} h_{\alpha_i}(\langle \omega, \eta_i \rangle), \quad \omega \in \left(S\left(\mathbb{R}^d\right)\right)^*. \tag{2.1}$$

If $n \in \mathbb{N}$ is fixed, let $(S)_1^n$ consist of those $x = \sum_\alpha c_\alpha H_\alpha \in \oplus_{k=1}^n L^2(\mu)$ with $c_\alpha \in \mathbb{R}^n$ such that $\|x\|_{1,k}^2 = \sum_\alpha c_\alpha^2 (\alpha!)^2 (2\mathbb{N})^{k\alpha} < \infty$ for all $k \in \mathbb{N}$ with $c_\alpha^2 = |c_\alpha|^2 = \sum_{k=1}^n (c_\alpha^{(k)})^2$ if $c_\alpha = (c_\alpha^{(1)}, \ldots, c_\alpha^{(n)}) \in \mathbb{R}^n$, where μ is the white noise measure on $(S^*(\mathbb{R}), \mathcal{B}(S^*(\mathbb{R})))$, $\alpha! = \prod_{k=1}^\infty \alpha_k!$ and $(2\mathbb{N})^\alpha = \prod_j (2j)^{\alpha_j}$ for $\alpha = (\alpha_1, \alpha_2, \ldots) \in \mathcal{J}$. The space $(S)_{-1}^n$ can be regarded as the dual of

$(S)_1^n$. $(S)_{-1}^n$ consisting of all formal expansion $X = \sum_\alpha b_\alpha H_\alpha$ with $b_\alpha \in \mathbb{R}^n$ such that $\|X\|_{-1,-q} = \sum_\alpha b_\alpha^2 (2\mathbb{N})^{-q\alpha} < \infty$ for some $q \in \mathbb{N}$, by the action $\langle X, x\rangle = \sum_\alpha (b_\alpha, c_\alpha)\alpha!$ and (b_α, c_α) is the usual inner product in $\mathbb{R}^n$.

$X \Diamond Y = \sum_{\alpha,\beta}(a_\alpha, b_\beta) H_{\alpha+\beta}$ is called the Wick product of X and Y, for $X = \sum_\alpha a_\alpha H_\alpha$, $Y = \sum_\alpha b_\alpha H_\alpha \in (S)_{-1}^n$ with $a_\alpha, b_\alpha \in \mathbb{R}^n$. We can prove that the spaces $(S(\mathbb{R}^d))$, $(S(\mathbb{R}^d))^*(S)_1^n$, and $(S)_{-1}^n$ are closed under Wick products.

For $X = \sum_\alpha a_\alpha H_\alpha \in (S)_{-1}^n$ with $a_\alpha \in \mathbb{R}^n$, $\mathscr{H}(X)$ or $\widetilde{X}$ is defined as the Hermite transform of X by $\mathscr{H}(X)(z) = \widetilde{X}(z) = \sum_\alpha a_\alpha z^\alpha \in \mathbb{C}^n$ (when convergent), where $z = (z_1, z_2, \ldots) \in \mathbb{C}^{\mathbb{N}}$ (the set of all sequences of complex numbers) and $z^\alpha = z_1^{\alpha_1} z_2^{\alpha_2} \cdots z_n^{\alpha_n} \cdots$ for $\alpha = (\alpha_1, \alpha_2, \ldots) \in \mathcal{J}$. For $X, Y \in (S)_{-1}^N$, by this definition we have $\widetilde{X \Diamond Y}(z) = \widetilde{X}(z) \cdot \widetilde{Y}(z)$ for all z such that $\widetilde{X}(z)$ and $\widetilde{Y}(z)$ exist. The product on the right-hand side of the above formula is the complex bilinear product between two elements of $\mathbb{C}^N$ defined by $(z_1^1, \ldots, z_n^1) \cdot (z_1^2, \ldots, z_n^2) = \sum_{k=1}^n z_k^1 z_k^2$, where $z_k^i \in \mathbb{C}$. Let $X = \sum_\alpha a_\alpha H_\alpha \in (S)_{-1}^n$. Then the vector $c_0 = \widetilde{X}(0) \in \mathbb{R}^n$ is called the generalized expectation of X denoted by $\mathbb{E}(X)$. Suppose that $f : V \to \mathbb{C}^n$ is an analytic function, where V is a neighborhood of $\mathbb{E}(X)$. Assume that the Taylor series of f around $\mathbb{E}(X)$ has coefficients in $\mathbb{R}^n$. Then the Wick version $f^\Diamond(X) = \mathscr{H}^{-1}(f \circ \widetilde{X}) \in (S)_{-1}^n$.

Suppose that modeling considerations lead us to consider the SPDE expressed formally as $A(t, x, \partial_t, \nabla_x, U, \omega) = 0$, where A is some given function, $U = U(t, x, \omega)$ is the unknown generalized stochastic process, and the operators $\partial_t = \partial/\partial_t$, $\nabla_x = (\partial/\partial_{x_1}, \ldots, \partial/\partial_{x_d})$ when $x = (x_1, \ldots, c_d) \in \mathbb{R}^d$. If we interpret all products as wick products and all functions as their Wick versions, we have

$$A^\Diamond(t, x, \partial_t, \nabla_x, U, \omega) = 0. \tag{2.2}$$

Taking the Hermite transform of (2.2), the Wick product is turned into ordinary products (between complex numbers), and the equation takes the form

$$\widetilde{A}\left(t, x, \partial_t, \nabla_x, \widetilde{U}, z_1, z_2, \ldots\right) = 0, \tag{2.3}$$

where $\widetilde{U} = \mathscr{H}(U)$ is the Hermite transform of U and $z_1, z_2, \ldots$ are complex numbers. Suppose that we can find a solution $u = u(t, x, z)$ of (2.3) for each $z = (z_1, z_2, \ldots) \in \mathbb{K}_q(r)$ for some q, r, where $\mathbb{K}_q(r) = z = (z_1, z_2, \ldots) \in \mathbb{C}^{\mathbb{N}}$ and $\sum_{\alpha \neq 0} |z^\alpha|^2 (2\mathbb{N})^{q\alpha} < r^2$. Then under certain conditions, we can take the inverse Hermite transform $U = \mathscr{H}^{-1} u \in (S)_{-1}$ and thereby obtain a solution U of the original Wick equation (2.2). We have the following theorem, which was proved by Holden et al. in [7].

Theorem 2.1. *Suppose that $u(t, x, z)$ is a solution (in the usual strong, pointwise sense) of* (2.3) *for (t, x) in some bounded open set $G \subset \mathbb{R} \times \mathbb{R}^d$ and $z \in \mathbb{K}_q(r)$ for some q, r. Moreover, suppose that $u(t, x, z)$ and all its partial derivatives, which are involved in* (2.3), *are bounded for $(t, x, z) \in G \times \mathbb{K}_q(r)$, continuous with respect to $(t, x) \in G$ for all $z \in \mathbb{K}_q(r)$, and analytic with respect to $z \in \mathbb{K}_q(r)$ for all $(t, x) \in G$. Then there exists $U(t, x) \in (S)_{-1}$ such that $u(t, x, z) = (\widetilde{U}(t, x))(z)$ for all $(t, x, z) \in G \times \mathbb{K}_q(r)$ and $U(t, x)$ solves (in the strong sense in $(S)_{-1}$)* (2.2) *in $(S)_{-1}$.*

3. Single-Soliton Solution of Stochastic KP Equation

In this section, we investigate the single-soliton solutions of the Wick-type stochastic KP equation (1.2). Using the similar idea of the Darboux transformation about the determinant nonlinear partial differential equations, we can obtain the soliton solutions of (1.2), which can be seen in the following theorem.

Theorem 3.1. *For the Wick-type stochastic KP equation* (1.2) *in white noise environment, one has the single-soliton solution* $U[1] \in (S)_{-1}$ *for KP-I:*

$$U[1] = \frac{\lambda^2}{2k}\left(\operatorname{sech}\left(\frac{\overline{\Phi}}{2}\right)\right)^2, \quad \text{when } \alpha = 1 \tag{3.1}$$

and for KP-II:

$$U[1] = \frac{2a^2}{k}\operatorname{sech}^2\left(\overline{\Phi}_1(t,x,y)\right), \quad \text{when } \alpha = i, \tag{3.2}$$

where $\overline{\Phi}(t,x,y) = \lambda x + \lambda^2 y + 4\lambda^3 \int_0^t f(s)ds + 4\lambda^3 \beta B(t) - 2\lambda^3 \beta t^2$ *and*

$$\overline{\Phi}_1(t,x,y) = ax - 2aby + 4\left(a^3 - 3ab^2\right)\int_0^t f(s)ds + 4\beta\left(a^3 - 3ab^2\right)\left(B(t) - \frac{1}{2}t^2\right). \tag{3.3}$$

Proof. Taking the Hermite transform of (1.2), the equation (1.2) can be changed into

$$\widetilde{U}_{tx} = \left[f(t) + \beta\widetilde{W}(t,z)\right]\widetilde{U}_{xxxx} + 6\left[g(t) + \gamma\widetilde{W}(t,z)\right]\left(\widetilde{U}\widetilde{U}_x\right)_x + 3\alpha^2\left[f(t) + \beta\widetilde{W}(t,z)\right]\widetilde{U}_{yy}, \tag{3.4}$$

where $\widetilde{U}$ is the Hermite transform of U; the Hermite transform of $W(t)$ is defined by $\widetilde{W}(t,z) = \sum_{k=1}^{\infty}\eta_k(t)z_k$ where $z = (z_1, z_2, \ldots) \in (\mathbb{C}^{\mathbb{N}})_c$ is parameter.

Suppose that $g(t)+\gamma\widetilde{W}(t,z) = k[f(t)+\beta\widetilde{W}(t,z)]$. Let $u = k\widetilde{U}$. From (3.4), we can obtain

$$u_{tx} = \left[f(t) + \beta\widetilde{W}(t,z)\right](u_{xxx} + 6uu_x)_x + 3\alpha^2\left[f(t) + \beta\widetilde{W}(t,z)\right]u_{yy}. \tag{3.5}$$

Let $F(t,z) = f(t) + \beta\widetilde{W}(t,z)$; then (3.5) can be changed into

$$u_{tx} = F(t,z)(u_{xxx} + 6uu_x)_x + 3\alpha^2 F(t,z)u_{yy}. \tag{3.6}$$

Now we consider the soliton solutions of (3.6) using Darboux transform. It is more convenient to consider the compatibility condition of the following linear system of partial differential equations, that is, Lax pair of (3.6):

$$\begin{aligned} \phi_y &= \alpha^{-1}\phi_{xx} + \alpha^{-1}u\phi, \\ \phi_t &= 4F(t,z)\phi_{xxx} + 6F(t,z)u\phi_x + 3F(t,z)(\alpha v_y + u_x)\phi. \end{aligned} \tag{3.7}$$

Then we can obtain the Wick-type Lax pair of (1.2):

$$\begin{aligned}
\phi_y &= \alpha^{-1}\phi_{xx} + \alpha^{-1}u\Diamond\phi, \\
\phi_t &= 4\left(f(t)+\beta W(t)\right)\Diamond\phi_{xxx} + 6\left(f(t)+\beta W(t)\right)\Diamond u\Diamond\phi_x \\
&\quad + 3\left(f(t)+\beta W(t)\right)\Diamond\left(\alpha v_y + u_x\right)\Diamond\phi.
\end{aligned} \tag{3.8}$$

Let ϕ_1 be a given solution of (3.8). Using the idea of the Darboux transformation about the determinant nonlinear partial differential equations, by direct computation, it is easy to know that if supposing that $\phi[1] = \phi_x - (\phi_{1x}\Diamond{\phi_1}^{\Diamond(-1)})\Diamond\phi$, where ϕ is an arbitrary solution of (3.8), then $\phi[1]$ satisfies the following equations:

$$\begin{aligned}
\phi_y[1] &= \alpha^{-1}\phi_{xx}[1] + \alpha^{-1}u[1]\Diamond\phi[1], \\
\phi_t[1] &= 4\left(f(t)+\beta W(t)\right)\Diamond\phi_{xxx}[1] + 6\left(f(t)+\beta W(t)\right)\Diamond u[1]\phi_x[1] \\
&\quad + 3\left(f(t)+\beta W(t)\right)\Diamond\left(\alpha v_y[1] + u_x[1]\right)\Diamond\phi[1],
\end{aligned} \tag{3.9}$$

where $u[1] = u + 2(\phi_{1x}\Diamond{\phi_1}^{\Diamond(-1)})_x^{\Diamond}$, $v[1] = v + 2(\phi_{1x}\Diamond{\phi_1}^{\Diamond(-1)})$.

Since (3.6) is nonlinear, it is difficult to solve it in general. In particular, taking $u = 0$ and $v = 0$, then from (3.8), we have

$$\begin{aligned}
\phi_y &= \alpha^{-1}\phi_{xx}, \\
\phi_t &= 4\left(f(t)+\beta W(t)\right)\Diamond\phi_{xxx}.
\end{aligned} \tag{3.10}$$

If $\alpha = 1$, (3.10) have the exponential function solution

$$\phi_1(t,x,y,z) = \exp^{\Diamond}\{\varphi(t,x,y,z)\} + 1, \tag{3.11}$$

where

$$\varphi = \lambda x + \lambda^2 y + 4\lambda^3\left(\int_0^t f(s)ds + \beta B(t)\right), \tag{3.12}$$

and λ is an arbitrary real parameter. Then we can obtain the single-soliton solution of (3.6). By (3.11) and (3.12) there exists a stochastic single-solitary solution of (1.2) as following:

$$U[1] = \frac{2}{k}\left(\phi_{1x}\Diamond{\phi_1}^{\Diamond(-1)}\right)\Diamond\phi = \frac{\lambda^2}{2k}\left(\operatorname{sech}^{\Diamond}\left(\frac{\Phi}{2}\right)\right)^2, \tag{3.13}$$

where

$$\Phi(t,x,y) = \lambda x + \lambda^2 y + 4\lambda^3\int_0^t f(s)ds + 4\lambda^3\beta B(t). \tag{3.14}$$

Since $\exp^{\diamond}\{B(t)\} = \exp\{B(t) - (1/2)t^2\}$ (see Lemma 2.6.16 in [7]), (1.2) has the single-soliton solution

$$U[1] = \frac{\lambda^2}{2k}\left(\operatorname{sech}\left(\frac{\overline{\Phi}}{2}\right)\right)^2, \tag{3.15}$$

where

$$\overline{\Phi}(t,x,y) = \lambda x + \lambda^2 y + 4\lambda^3 \int_0^t f(s)ds + 4\lambda^3 \beta B(t) - 2\lambda^3 \beta t^2. \tag{3.16}$$

In particular, when $f(s) = 1$ we can obtain the solution of (2.2), respectively, as follows:

$$U[1] = \frac{\lambda^2}{2k}\operatorname{sech}^2\left(\frac{1}{2}\left(\lambda x + \lambda^2 y + 4\lambda^3 t + 4\lambda^3 \beta B(t) - 2\lambda^3 \beta t^2\right)\right). \tag{3.17}$$

If $\alpha = i$, (3.10) have the exponential function solution

$$\phi_1(t,x,y,z) = \exp^{\diamond}\{\varphi_1(t,x,y,z)\} + \exp^{\diamond}\{-\overline{\varphi}_1(t,x,y,z)\}, \tag{3.18}$$

where

$$\varphi_1(t,x,y,z) = \lambda x + i\lambda^2 y + 4\lambda^3\left(\int_0^t f(s)ds + \beta B(t)\right), \tag{3.19}$$

$\overline{\varphi}_1$ is the conjugation of $\overline{\varphi}_1$ and λ is an arbitrary complex parameter. Let $\lambda = a+ib$, according to (3.9), from (3.18) and (3.19) there exists a stochastic single-solitary solution of (1.2) as follows:

$$U[1] = \frac{2}{k}\left(\phi_{1x}\diamond\phi_1^{\diamond(-1)}\right)\diamond\phi = \frac{2a^2}{k}\left(\operatorname{sech}^{\diamond}(\Phi_1(t,x,y))\right)^2, \tag{3.20}$$

where

$$\Phi_1(t,x,y) = ax - 2aby + 4\left(a^3 - 3ab^2\right)\int_0^t f(s)ds + 4\left(a^3 - 3ab^2\right)\beta B(t). \tag{3.21}$$

Same as the former case, since $\exp^{\diamond}\{B(t)\} = \exp\{B(t) - (1/2)t^2\}$, (1.2) has the single-soliton solution

$$U[1] = \frac{2a^2}{k}\operatorname{sech}^2\left(\overline{\Phi}_1(t,x,y)\right), \tag{3.22}$$

where

$$\overline{\Phi}_1(t,x,y) = ax - 2aby + 4\left(a^3 - 3ab^2\right)\int_0^t f(s)ds + 4\beta\left(a^3 - 3ab^2\right)\left(B(t) - \frac{1}{2}t^2\right). \tag{3.23}$$

In particular, when $f(s) = 1$ we can obtain the solution of (2.2) as follows:

$$U[1] = \frac{2a^2}{k}\operatorname{sech}^2\left(ax - 2aby + 4\left(a^3 - 3ab^2\right)\left(t - \frac{\beta}{2}t^2 + \beta B(t)\right)\right). \tag{3.24}$$

□

4. Multisoliton Solutions of Stochastic KP Equation

At the same time, the multisoliton solutions of stochastic KP equation can be also considered. It is evident that the Darboux transformation can be applied to (3.9) again. This operation can be repeated arbitrarily. For the second step of this procedure we have

$$\phi[2] = \left(\frac{\partial}{\partial x} - \frac{\phi_{2x}[1]}{\phi_2[1]}\right)\left(\frac{\partial}{\partial x} - \frac{\phi_{1x}}{\phi_1}\right)\phi, \tag{4.1}$$

where $\phi_2[1]$ is the fixed solution of (3.9), which is generated by some fixed solution ϕ_2 of (3.8) and independent of ϕ_1. We know that

$$\phi_2[1] = \phi_{2x} - \frac{\phi_{1x}}{\phi_1}\phi_2, \tag{4.2}$$

$$u[2] = u + 2\frac{\partial^2}{\partial x^2}\ln W(\phi_1, \phi_2). \tag{4.3}$$

By using N-times Darboux transformation, the formula (4.3) can be generalized to obtain the solutions of the initial equations (3.8) without any use of the solutions related to the intermediate iterations of the process.

Let $\phi_1, \phi_2, \ldots, \phi_N$ be different and independent solutions of (3.8). We define the Wronski determinant W of functions $f_1, \ldots, f_m$ as

$$W(f_1, \ldots, f_m) = \det A, \qquad A_{ij} = \frac{d^{i-1} f_j}{dx^{i-1}}, \quad i, j = 1, 2, \ldots, m. \tag{4.4}$$

Theorem 4.1. *For the Wick-type stochastic KP equation* (1.2) *in white noise environment, one has the* N*-soliton solution* $U[N] \in (S)_{-1}$ *satisfying*

$$U[N] = \frac{2}{k}\frac{\partial^2}{\partial x^2}\ln^{\diamond} W^{\diamond}(\phi_1, \ldots, \phi_N). \tag{4.5}$$

Proof. From [6], it is easy to see that the function

$$\phi[N] = \frac{W(\phi_1, \ldots, \phi_N, \phi)}{W(\phi_1, \ldots, \phi_N)} \tag{4.6}$$

satisfies the following equations:

$$\begin{aligned}\phi_y[N] &= \alpha^{-1}\phi_{xx}[N] + \alpha^{-1}u[N]\phi[N],\\ \phi_t[N] &= 4F(t,z)\phi_{xxx}[N] + 6F(t,z)u[N]\phi_x[N]\\ &\quad + 3F(t,z)\left(\alpha v_y[N] + u_x[N]\right)\phi[N],\end{aligned} \tag{4.7}$$

where $u[N] = u + 2(\partial^2/\partial x^2)\ln W(\phi_1,\ldots,\phi_N)$ and $v[N] = v + 2(\partial/\partial x)\ln W(\phi_1,\ldots,\phi_N)$.

Then we have the Wick-type form

$$\phi[N] = \frac{W^{\diamond}(\phi_1,\ldots,\phi_N,\phi)}{W^{\diamond}(\phi_1,\ldots,\phi_N)} \tag{4.8}$$

satisfying the following equations:

$$\begin{aligned}\phi_y[N] &= \alpha^{-1}\phi_{xx}[N] + \alpha^{-1}u[N]\diamond\phi[N],\\ \phi_t[N] &= 4\left(f(t)+W(t)\right)\diamond\phi_{xxx}[N] + 6\left(f(t)+W(t)\right)\diamond u[N]\diamond\phi_x[N]\\ &\quad + 3\left(f(t)+W(t)\right)\diamond\left(\alpha v_y[N] + u_x[N]\right)\diamond\phi[N],\end{aligned} \tag{4.9}$$

where $u[N] = u + 2(\partial^2/\partial x^2)\ln^{\diamond} W^{\diamond}(\phi_1,\ldots,\phi_N)$.

In particular, taking $u = 0$, $v = 0$, we can obtain the N-soliton solution of (1.2):

$$U[N] = \frac{2}{k}\frac{\partial^2}{\partial x^2}\ln^{\diamond} W^{\diamond}(\phi_1,\ldots,\phi_N). \tag{4.10}$$

When $\alpha = 1$ and $\alpha = i$, $\phi_1,\ldots,\phi_N$ are represented by the corresponding forms (3.11) and (3.18), where λ, a, b take the different constants. □

Remark 4.2. However, in generally, in the view of the modeling point, one can consider the situations where the noise has a different nature. It turns out that there is a close mathematical connection between SPDEs driven by Gaussian and Poissonian noise at least for Wick-type equations. It is well known that there is a unitary map to the solution of the corresponding Gaussian SPDE, see [7]. Hence, if the coefficient $f(t)$ is perturbed by Poissonian white noise in (1.2), the stochastic single-soliton solution and stochastic multisoliton solutions also can be obtained by the same discussion.

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