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Research Article

Analytic Solution for MHD Falkner-Skan Flow over a Porous Surface

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This paper discusses the MHD Falkner-Skan flow over a porous surface. The solution to nonlinear problem is first constructed and analyzed for the emerging parameters

1. Introduction

Motivated by significant applications in packed bed reactor, geothermal system, extractions of crude oil, water or nuclear pollution, and so forth, the wedge flow over shaped bodies has attracted the attention of various researchers as the early formulation given by Falkner and Skan [1]. Later, Asaithambi [2] analyzed the Falkner-Skan equation by using finite difference scheme. Magnetohydrodynamics effects on the Falkner Skan wedge flow are studied by Abbasbandy and Hayat [3, 4]. They used Hankel-Pade and homotopy analysis methods for the derivation of the solutions. Rajagopal et al. [5] discussed the Falkner-Skan flow of a non-Newtonian fluid. Massoudi and Ramezan [6] extended the idea of Rajagopal et al. [5] for suction and blowing cases. Forced convection boundary layer flow over a wedge with uniform suction or injection is analyzed by Yih [7]. Kuo [8] discussed the heat transfer for the Falkner-Skan wedge flow by using differential transformation method. Numerical treatment for the Falkner-Skan wedge flow of a power law fluid saturating the porous space has been discussed by Kim [9]. More recently, Hayat et al. [10] extended the idea of Kim [9] to study the mixed convection flow. In this paper, Falkner-Skan flow over a porous surface is considered. Case of uniform suction/blowing is taken into account. Stream function formulation and suitable transformations reduce the arising problem to ordinary differential equation which has been solved by a homotopy analysis method HAM [11–23]. Finally the solutions are sketched and analyzed.

2. Problem Statement

We consider the two-dimensional Falkner-Skan flow in an incompressible magnetohydrodynamic (MHD) viscous fluid. Under the usual boundary layer approximations, the equations which govern the Falkner-Skan flow are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$$
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu\frac{\partial^2 v}{\partial y^2}-\frac{\sigma B^2 u}{\rho}(u-U), \tag{2.1}$$

where u and v are the velocity components in the x and y directions, respectively, ρ is the fluid density, ν is the kinematic viscosity of the fluid, σ is the electrical conductivity, U is the characteristic velocity, and a uniform magnetic field B is applied. The induced magnetic field is neglected in view of small magnetic Reynolds number. The boundary conditions are

$$u=U_w,\qquad v=V_w,\quad \text{at } y=0,$$
$$u\longrightarrow U(x),\quad \text{as } y\longrightarrow\infty, \tag{2.2}$$

where

$$U(x)=ax^m,$$
$$B(x)=B_0x^{(m-1)/2}, \tag{2.3}$$

where V_w is the porous velocity and U_w is the surface velocity.

Denoting by Ψ the stream function and defining

$$\eta=\sqrt{\frac{m+1}{2}}\sqrt{\frac{U}{\nu x}}y,\qquad \psi=\sqrt{\frac{m+1}{2}}\sqrt{\nu xU}f(\eta),\qquad u=Uf'(\eta),$$
$$v=-\sqrt{\frac{m+1}{2}}\sqrt{\frac{\nu U}{x}}\left[f(\eta)+\frac{m-1}{m+1}\eta f'(\eta)\right], \tag{2.4}$$
$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x},$$

the continuity equations are automatically satisfied and the other equations yield

$$\frac{d^3f}{d\eta^3}+f\frac{d^2f}{d\eta^2}+\beta\left[1-\left(\frac{df}{d\eta}\right)^2\right]-M^2\left(\frac{df}{d\eta}-1\right)=0,$$
$$f(0)=\gamma,\qquad f'(0)=\lambda, \tag{2.5}$$
$$f'(\infty)=1.$$

Table 1: [10, 10] homotopy-Pade approximation at $m = 2$, $\lambda = 0$, $\gamma = 0$.

M	Present results	Numerical results by Abbasbandy and Hayat [4]
1	1.719465	1.71946540
2	2.439498	2.43949833
5	5.190959	5.19095945
10	10.096775	10.09677545
50	50.019440	50.01944071
100	100.009721	100.00972170

Table 2: [10, 10] homotopy-Pade approximation at $m = -3/5$, $\lambda = 0$, $\gamma = 0$.

M	Present results	Numerical results by Abbasbandy and Hayat [4]
3	2.273388	2.27338836
4	3.488148	3.48814857
5	4.600754	4.60075494
10	9.806464	9.80646420
15	14.871674	14.87167484
20	19.903937	19.90393701

Here primes denote the differentiation with respect to η and β, M, Pr, and β are the dimensionless numbers. These are defined as

$$M^2 = \frac{2\sigma B_0^2}{\rho a(1+m)}, \qquad \beta = \frac{2m}{m+1}, \qquad \lambda = \frac{U_w}{U}, \qquad \gamma = -V_w\left((m+1)\frac{vU}{2x}\right)^{-1/2}. \tag{2.6}$$

3. Homotopy Analysis Solutions

The velocity can be expressed as the set of base function

$$\left\{\eta^k \exp(-n\eta), k, n\eta \geq 0\right\} \tag{3.1}$$

by selecting the initial guess

$$f_0(\eta) = \gamma + \eta - \frac{1-\lambda}{\delta}(1 - \delta \exp(-\eta)) \tag{3.2}$$

and the auxiliary linear operator $\mathcal{L}_f$

$$\mathcal{L}_f(f) = \frac{d^3 f}{d\eta^3} + \delta \frac{d^2 f}{d\eta^2}, \tag{3.3}$$

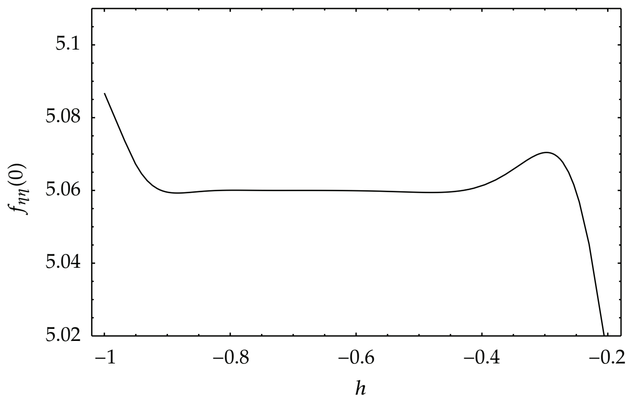

Figure 1: The h curve of $f''(0)$ at 20th order of approximation for $M = 5$ and $\delta = 6$.

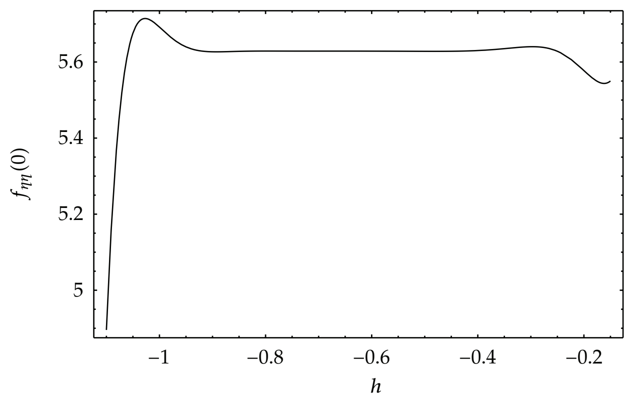

Figure 2: The h curve of $f''(0)$ at 20th order of approximation for $m = -3/5$, $M = 3$ and $\delta = 3$.

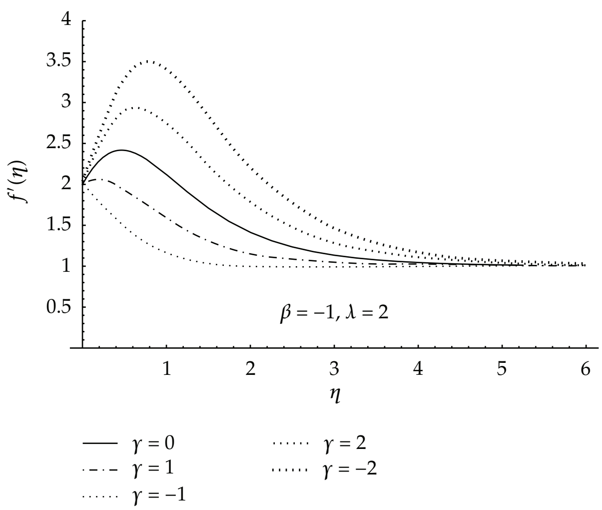

Figure 3: Velocity profiles for suction/injection parameter $\gamma = 1, 0, -1$ for $\lambda = 2$.

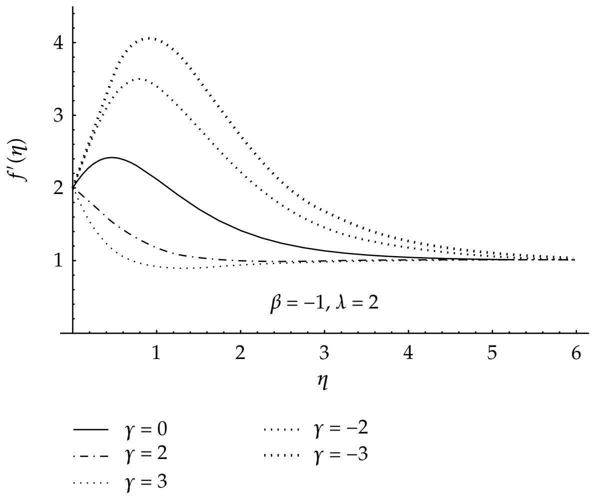

Figure 4: Velocity profiles for suction/injection parameter $\gamma = 2, 0, -2$ for $\lambda = 2$.

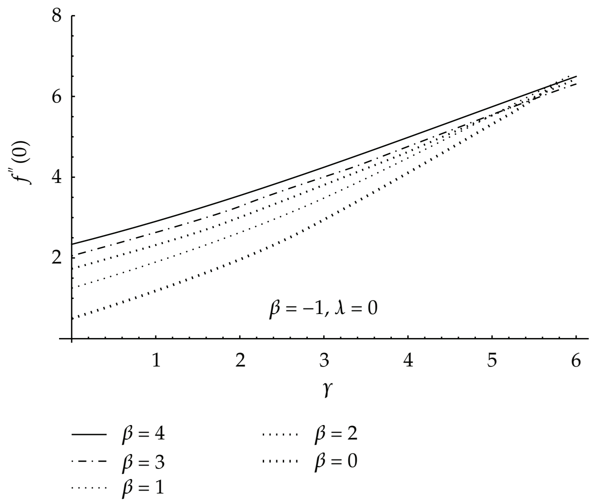

Figure 5: Skin friction coefficient for different values of stretching parameter $\beta = 4, 3, 2, 1, 0$.

with

$$\mathcal{L}_f\left[C_1 + C_2 \exp(2\eta) + C_3 \exp(-\eta)\right] = 0, \tag{3.4}$$

in which C_i, $(i = 1\text{–}3)$ are the arbitrary constants. If $p \in [0, 1]$ is the embedding parameter and $\hbar_f$, is the nonzero auxiliary parameter, then the zeroth-order deformation problem is given as follows:

$$\begin{gathered}(1-p)\mathcal{L}_f\left[\widehat{f}(\eta;p) - f_0(\eta)\right] = p\hbar_f N_f\left[\widehat{f}(\eta;p)\right], \\ \widehat{f}(0;p) = \gamma, \qquad \widehat{f}'(0;p) = \lambda, \qquad \widehat{f}'(\infty;p) = 1,\end{gathered} \tag{3.5}$$

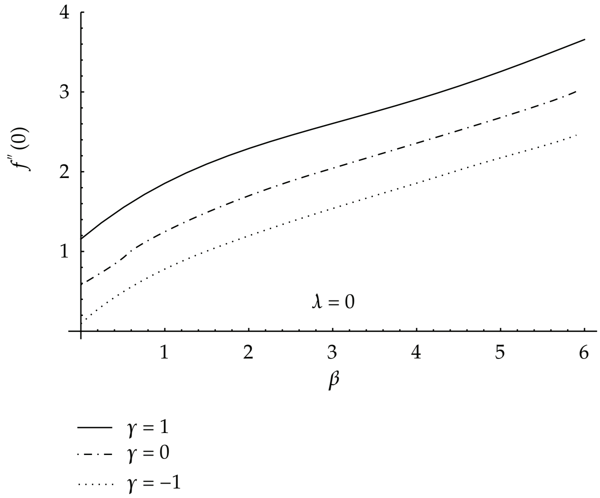

Figure 6: Shear stress $f''(0)$ versus the stretching parameter β for various values of the wall suction/injection parameter $\gamma = 1, 0, -1$.

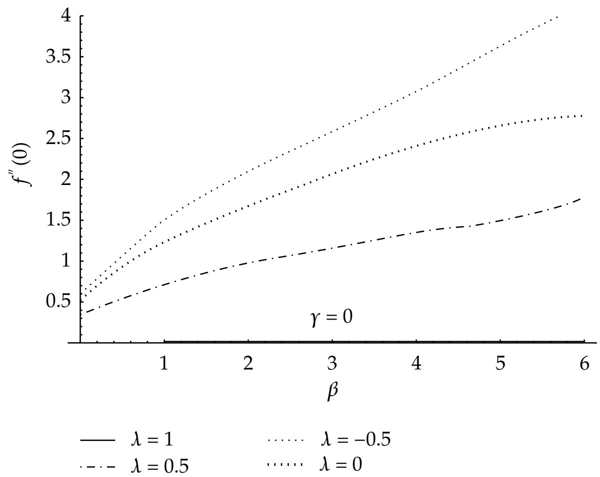

Figure 7: Shear stress $f''(0)$ versus the stretching parameter β for various values $\lambda = 1, 0.5, 0$ and -0.5.

in which the nonlinear operator N_f is of the following form:

$$N_f\left[\hat{f}(\eta;p)\right] = \frac{\partial^3 \hat{f}(\eta;p)}{\partial \eta^3} + \hat{f}(\eta;p)\frac{\partial^2 \hat{f}(\eta;p)}{\partial \eta^2} + \beta\left[1 - \left(\frac{\partial \hat{f}(\eta;p)}{\partial \eta}\right)^2\right] - M^2\left(\frac{\partial \hat{f}(\eta;p)}{\partial \eta} - 1\right). \tag{3.6}$$

For $p = 0$ and $p = 1$, the aforementioned zeroth-order deformation equation has the solutions:

$$\hat{f}(\eta;0) = f_0(\eta), \qquad \hat{f}(\eta;1) = f(\eta). \tag{3.7}$$

when p increases from 0 to 1, $\hat{f}(\eta;p)$ vary from $f_0(\eta)$ to the exact solutions $f(\eta)$. In view of Taylor's theorem and (3.7), we can write

$$\hat{f}(\eta;p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)p^m, \tag{3.8}$$

where

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m \hat{f}(\eta;p)}{\partial p^m}\bigg|_{p=0}. \tag{3.9}$$

The auxiliary parameter is so properly chosen that the series (3.8) converge at $p = 1$. Hence

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta). \tag{3.10}$$

The mth-order deformation problem is

$$\begin{aligned}
&\mathcal{L}_f\left[f_m(\eta) - \chi_m f_{m-1}(\eta)\right] = \hbar_f \mathcal{R}_m^f(\eta),\\
\mathcal{R}_m^f\left[\hat{f}(\eta;p)\right] &= \frac{\partial^3 \hat{f}_{m-1}(\eta;p)}{\partial \eta^3} + \hat{f}(\eta;p)\frac{\partial^2 \hat{f}_{m-1}(\eta;p)}{\partial \eta^2} + \beta\left[1 - \left(\frac{\partial \hat{f}_{m-1}(\eta,p)}{\partial \eta}\right)^2\right]\\
&\quad - M^2\left(\frac{\partial \hat{f}_{m-1}(\eta,p)}{\partial \eta} - 1\right),
\end{aligned} \tag{3.11}$$

$$f_m(0) = f_m'(0) = f_m'(\infty) = 0,$$

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases}$$

4. Analysis and Discussion

Here convergence is checked at 20th order of approximations. Figures 1 and 2 show the $\hbar$ curves at various values of the emerging parameters. The range of $\hbar$ curves is $-0.8 \le \hbar \le -0.4$ for Figure 1 and $-0.8 \le \hbar \le -0.4$ for Figure 2, respectively. It is obvious from these figures that auxiliary parameter h is necessary for the convergence purposes. Tables 1 and 2 are made to give the comparison with numerical results at $[10, 10]$ order of approximation for various values of magnetic parameter M and Falkner-Skan flow parameter m. The results are similar to those of Abbasbandy and Hayat [4]. Figure 3 is plotted for various values of suction/injection parameter γ. For positive values of γ, the velocity decreases whereas for negative values the velocity increases. In Figure 4 with the fixed value of slip condition $\lambda = 2$, the peak value of velocity is high when compared with Figure 3. The skin friction versus γ is sketched in

Figure 5 for various values of β with slip velocity equal to zero. It is obvious that with an increase in β the skin friction increases. Figure 6 is prepared for different values of γ. It is shown that $f''(0)$ decreases for injection and increases for suction. The skin friction versus stretching parameter β is displayed in Figure 7. It is noticed that for negative values of slip velocity, the skin friction increases.

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