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An asymptotic formula for the primitive of Hardy's function

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Abstract. Let $Z(t)$ be the classical Hardy function in the theory of Riemann's zeta-function. An asymptotic formula with an error term $O(T^{1/6}\log T)$ is given for the integral of $Z(t)$ over the interval $[0,T]$, with special attention paid to the critical cases when the fractional part of $\sqrt{T/2\pi}$ is close to $\frac{1}{4}$ or $\frac{3}{4}$.

1. Introduction

Hardy's function

$$Z(t)=\chi\left(\tfrac{1}{2}+it\right)^{-1/2}\zeta\left(\tfrac{1}{2}+it\right)$$

with $\chi(s)=2^s\pi^{s-1}\sin\left(\frac{1}{2}\pi s\right)\Gamma(1-s)$ as in the functional equation $\zeta(s)=\chi(s)\zeta(1-s)$ for Riemann's zeta-function $\zeta(s)$ is a useful tool for the study of the zeros of the zeta-function on the critical line $\operatorname{Re} s=\frac{1}{2}$. Namely, since $Z(t)$ is real for real t, its sign changes detect such "critical" zeros. The integral

$$F(T)=\int_0^T Z(t)\,dt$$

is a book-keeper for the balance between the positive and negative values of $Z(t)$. At first sight, the positivity and negativity aspects of $Z(t)$ seem to be perfectly comparable. However, recently it has turned out that occasionally one or the other sign is *favored*, and this happens if the fractional part $\left\{\sqrt{t/2\pi}\right\}$ is close to $\frac{1}{4}$ or $\frac{3}{4}$. This phenomenon was first observed by M. A. Korolev ([3] and [4]) as a corollary of an asymptotic formula for $F(T)$, for fixed $\vartheta=\left\{\sqrt{T/2\pi}\right\}$. We proved recently a version of Korolev's theorem which is uniform in ϑ (see [2], Theorem 2). The starting point of our argument was a formula of Atkinson type (Lemma 1 below) for $F(T)$. A simple main term for $F(T)$—a step function in terms of ϑ—can be exhibited except when the distance from ϑ to $\frac{1}{4}$ or $\frac{3}{4}$ is positive and small, namely of the order $\ll T^{-1/6}$. In that case, just an estimate for $F(T)$ was obtained. On the other hand, the cases $\vartheta=\frac{1}{4}$ or $\frac{3}{4}$ were settled satisfactorily in [3] and [4], and in [2] as well. Our goal in this paper is to work out an explicit main term for $F(T)$ in the whole range for ϑ. In the above mentioned critical ranges, the main term involves *Airy functions*, that is certain special Bessel functions. As a corollary of the Atkinson formula, or of the present new formula for $F(T)$, we may deduce a *quasiperiodicity* property for the function $G(\tau)=F(2\pi\tau^2)$, namely

$$G(\tau+2)=G(\tau)+O(T^{1/6}\log T) \tag{1.1}$$

with $T=2\pi\tau^2$. Alternative proofs of this will be discussed in the end of Section 3.1. For comparison, if n is a positive integer, then it follows from [2], or also from [3] and [4], that

$$\left|G\left(n+\tfrac{1}{2}\right)-G\left(n-\tfrac{1}{2}\right)\right|=4\pi\left(\frac{T}{2\pi}\right)^{1/4}+O(T^{1/6}\log T)$$

with $T=2\pi n^2$.

We introduce the *Airy functions* in the form

$$\beta(u)=\frac{1}{\pi}\int_0^\infty \cos(Ay^3-2\pi uy)\,dy,$$

where $A>0$ is a parameter which will depend on T. We applied such functions in [1] in connection with the original Atkinson formula, and we are just repeating the same argument in the context of its new variant. The connection between the Airy and Bessel functions is as follows (see [5], p. 190):

$$\beta(u)=\frac{\sqrt{|2\pi u|}}{3\pi\sqrt{A}}K_{1/3}\left(\frac{2|2\pi u|^{3/2}}{3\sqrt{3A}}\right)\quad\text{for } u<0 \tag{1.2}$$

and

$$\beta(u)=\frac{\sqrt{2\pi u}}{3\sqrt{3A}}\left(J_{1/3}\left(\frac{2(2\pi u)^{3/2}}{3\sqrt{3A}}\right)+J_{-1/3}\left(\frac{2(2\pi u)^{3/2}}{3\sqrt{3A}}\right)\right)\quad\text{for } u>0. \tag{1.3}$$

Turning to the formulation of our main result, we introduce two auxiliary functions. First, let $K(x)$ be an odd periodic step function with period 2, which on the interval $[0,1]$ is defined as follows:

$$K(x)=\begin{cases}0 & \text{for } 0\le x<\frac{1}{4} \text{ and } \frac{3}{4}<x\le 1,\\ \pi & \text{for } x=\frac{1}{4} \text{ and } x=\frac{3}{4},\\ 2\pi & \text{for } \frac{1}{4}<x<\frac{3}{4}.\end{cases}$$

Secondly, let $w(u)$ be a smooth weight function such that

$$w(u)=\begin{cases}1 & \text{for } |u|\le\frac{1}{4},\\ 0 & \text{for } |u|\ge\frac{1}{2},\end{cases}$$

and $w^{(\nu)}(u)\ll_\nu 1$ for sufficiently many derivatives.

The main novelty of the following theorem, compared with Theorem 2 of [2], is the emergence of a "correction term" in the form of an integral in (1.5). Since the function $K(x)$ is piecewise constant, we will have integrals of $w(u)\beta(u)$ over subintervals of $\left[-\frac{1}{2},\frac{1}{2}\right]$ to deal with. In particular, the integrals over $\left[-\frac{1}{2},0\right]$ and $\left[0,\frac{1}{2}\right]$ will be given in (2.11) and (2.12), and these values are helpful in an analysis of (1.5) in different cases as to the location of ϑ.

Theorem. *Let T be a large positive integer and write $\sqrt{T/2\pi}=L+\vartheta$ with $L\in\mathbb{N}$ and $0\le\vartheta<1$. Let $\beta(u)$ be defined by (1.2)–(1.3) with $A=\frac{1}{12}\sqrt{2\pi^3}T^{-1/2}$. Then*

$$F(T)=\left(\frac{T}{2\pi}\right)^{1/4}(-1)^L\widetilde{K}(\vartheta)+O(T^{1/6}\log T), \tag{1.4}$$

where

$$\widetilde{K}(\vartheta)=K(\vartheta)+2\pi\int_{-1/2}^{1/2}w(u)\beta(u)(K(\vartheta+u)-K(\vartheta))\,du. \tag{1.5}$$

Further, define $\vartheta_0=\min\left(\left|\vartheta-\frac{1}{4}\right|,\left|\vartheta-\frac{3}{4}\right|\right)$. Then for $\vartheta_0\ne0$ we have

$$K(\vartheta)-\widetilde{K}(\vartheta)\ll\min\left(1,T^{-1/8}\vartheta_0^{-3/4}\right). \tag{1.6}$$

Also,

$$\widetilde{K}\left(\tfrac{1}{4}\right)=\tfrac{4}{3}\pi+O(T^{-1/8}), \tag{1.7}$$

$$\widetilde{K}\left(\tfrac{3}{4}\right)=\tfrac{2}{3}\pi+O(T^{-1/8}). \tag{1.8}$$

The estimate (1.6) shows that (1.4) is indeed a refinement of Theorem 2 of [2]. It also says that in a neighborhood of length about $T^{1/3}$ around any point $T=2\pi(n+j/4)^2$ with a natural number n and $j=1$ or $j=3$, the function $F(t)$ jumps an amount $\asymp T^{1/4}$ upwards or downwards and the theorem moreover indicates *how* these jumps take place, at least approximately. Here the notation $A\asymp B$ means that $A\ll B\ll A$. The existence of the jumps can be illustrated as follows: the center of gravity of the curve $Z(t)$ over any of the critical intervals mentioned above lies at a distance $\asymp T^{-1/12}$ above or below the axis.

2. Lemmas

The proof of our theorem will be based on the following formula of Atkinson type for $F(T)$ (see [2], Theorem 1).

Lemma 1. *Let T be a large positive number, $N \asymp T$, and*

$$N' = \frac{T}{2\pi} + \frac{N}{2} - \sqrt{\frac{N^2}{4} + \frac{NT}{2\pi}}.$$

Define

$$e(T,n) = \left(1+\frac{\pi n}{2T}\right)^{-1/4} \left\{ \sqrt{\frac{2T}{\pi n}} \operatorname{arsinh} \sqrt{\frac{\pi n}{2T}} \right\}^{-1}, \tag{2.1}$$

$$f(T,n) = 2T \operatorname{arsinh}\left(\sqrt{\frac{\pi n}{2T}}\right) + (2\pi n T + \pi^2 n^2)^{1/2} - \frac{\pi}{4}, \tag{2.2}$$

and

$$g(T,n) = T \log\left(\frac{T}{2\pi n}\right) - T + \frac{\pi}{4}.$$

Then

$$F(T) = S_1(T) + S_2(T) + O((\log T)^{5/4}), \tag{2.3}$$

where

$$S_1(T) = 2\sqrt{2}\left(\frac{T}{2\pi}\right)^{1/4} \sum_{0 \le n \le \sqrt{N}} (-1)^{n(n+1)/2} e\left(T, \left(n+\tfrac{1}{2}\right)^2\right)\left(n+\tfrac{1}{2}\right)^{-1}$$
$$\times \cos\left(\tfrac{1}{2} f\left(T, \left(n+\tfrac{1}{2}\right)^2\right) - \frac{3\pi}{8}\right) \tag{2.4}$$

and

$$S_2(T) = -4 \sum_{1 \le n \le \sqrt{N'}} n^{-1/2} \left(\log \frac{T}{2\pi n^2}\right)^{-1} \cos\left(\tfrac{1}{2} g(T, n^2) + \frac{\pi}{4}\right).$$

For the purpose of simplifying the trigonometric factors in (2.4), we are going to need the following lemma from [1] (with a bit different scaling); it is here that the

Airy function $\beta(u)$ appears in our argument. Note that it involves the parameter A as in (1.2)–(1.3) though this dependence is not indicated in the notation.

Lemma 2. *Let $A>0$, B, and C be real numbers. Then, for all real y, we have*

$$\sin(Ay^3+By+C)=2\pi\int_{-\infty}^{\infty}\beta(u)\sin((B+2\pi u)y+C)\,du. \tag{2.5}$$

The proof of this formula is based on the calculation and inversion of the Fourier transform of the function e^{iAy^3}. Since this function does not lie in L_1 or L_2, classical theorems on the Fourier inversion do not apply here, but one may verify the inversion directly by following the standard argument in textbooks on Fourier analysis without appealing to any general theorem. Namely, due to the oscillatory nature of our function, one may use known estimates (the first and second derivative tests) for exponential integrals.

Some properties of the function $\beta(u)$, with A specified as in the theorem, will be needed in the next section, and to verify these, we recall a couple of familiar results from the theory of Bessel functions.

Let ν be a fixed real number which is not an integer. By the definitions of Bessel functions, we have

$$J_\nu(x)\ll x^\nu \text{ and } K_\nu(x)\ll x^{-|\nu|} \quad \text{for } 0<x\ll 1.$$

For $x\gg 1$, the following asymptotic formulae hold (see [5], pp. 199 and 202):

$$K_\nu(x)\sim\sqrt{\frac{\pi}{2x}}e^{-x} \tag{2.6}$$

and

$$J_\nu(x)=\sqrt{\frac{2}{\pi x}}\cos\Big(x-\frac{\pi\nu}{2}-\frac{\pi}{4}\Big)+O(x^{-3/2}). \tag{2.7}$$

Lemma 3. *Let the functions $\beta(u)$ and $w(u)$ be as in the theorem. Then*

$$\beta(u)\ll\min(T^{1/6},T^{1/8}|u|^{-1/4}). \tag{2.8}$$

Further, for $0<U<\frac{1}{2}$,

$$\int_U^{1/2}w(u)\beta(u)\,du\ll\min(1,T^{-1/8}U^{-3/4}), \tag{2.9}$$

and likewise for the integral over $\big[-\frac{1}{2},-U\big]$. If Δ is an interval of length U, then

$$\int_\Delta w(u)\beta(u)\,du\ll\min(1,UT^{1/6}). \tag{2.10}$$

Finally,

$$2\pi\int_0^{1/2} w(u)\beta(u)\,du = \frac{2}{3} + O(T^{-1/8}), \tag{2.11}$$

$$2\pi\int_{-1/2}^{0} w(u)\beta(u)\,du = \frac{1}{3} + O(T^{-1/8}). \tag{2.12}$$

Proof. The bound (2.8) is an immediate corollary of the estimates for Bessel functions mentioned above. As to the integrals (2.9) and (2.10), note that the function $\beta(u)$ is non-oscillatory for $u<0$, and also for $0<u\ll T^{-1/6}$, whereas for $u\gg T^{-1/6}$ it is oscillatory by (2.7). Therefore, to estimate these integrals, we may apply integration by parts or the first derivative test for exponential integrals when the integrand is oscillatory, and otherwise the integrals are just estimated directly. In any case, the estimates (2.9) and (2.10) are easy to verify.

Finally, let us consider the proofs of (2.11) and (2.12). Those integrals, where the weight function is omitted, can be calculated precisely. In fact, we have

$$2\pi\int_0^{\infty} \beta(u)\,du = \frac{2}{3}, \tag{2.13}$$

$$2\pi\int_{-\infty}^{0} \beta(u)\,du = \frac{1}{3}. \tag{2.14}$$

The latter equation follows, after a change of the variable, from the integral (see [5], p. 388, (8))

$$\int_0^{\infty} K_{1/3}(x)\,dx = \frac{\pi}{\sqrt{3}}.$$

The former equation is then immediate since

$$2\pi\int_{-\infty}^{\infty} \beta(u)\,du = 1$$

by Lemma 2 (choose $y=0$ and $C=\pi/2$). Finally, to prove the formulae (2.11)–(2.12), we have to insert the weight function $w(u)$ into the integrands in (2.13)–(2.14), and the effect of this modification can be estimated as above using properties of Bessel functions. □

3. Proof of the theorem

3.1. Proof of (1.4)

We start from the Atkinson formula (2.3) for $F(T)$, noting first that the estimation of $S_2(T)$ goes back to that of the zeta-function. Therefore $S_2(T)=O(T^{1/6}\log T)$ unconditionally, and $S_2(T)=O(T^{\varepsilon})$ for any $\varepsilon>0$ on the assumption of Lindelöf's hypothesis. In any case, $S_2(T)$ can be viewed as an error term.

Turning to $S_1(T)$, we first truncate and simplify it. In [2], Section 4.3, it is shown that the sum in $S_1(T)$ can be truncated to any length $\gg T^{2/9}$ with an error $\ll T^{1/6}\log T$. Actually we truncate this sum to a length $\asymp T^{1/4}$, and it is clear that the truncated sum can be equipped with the weights

$$a_n=\begin{cases}1 & \text{for } 1\le 2n+1<M/2,\\ 2-(4n+2)/M & \text{for } M/2\le 2n+1\le M,\end{cases}$$

where $M\asymp T^{1/4}$ is an even integer. Further, since $e\big(T,\big(n+\frac12\big)^2\big)=1+O\big(\big(n+\frac12\big)^2T^{-1}\big)$ by (2.1), we may omit the e-factors with an error $\ll T^{-1/4}$.

Next we simplify the trigonometric factors involving the function $f\big(T,\big(n+\frac12\big)^2\big)$. Developing the right-hand side of (2.2) into a power series we see, as in [2], that

$$\cos\left(\tfrac12 f\big(T,\big(n+\tfrac12\big)^2\big)-\frac{3\pi}{8}\right)=(-1)^L\sin\big(2\pi\big(n+\tfrac12\big)\vartheta+A\big(n+\tfrac12\big)^3+O\big(\big(n+\tfrac12\big)^5T^{-3/2}\big)\big).$$

Here the error term can be omitted, at the cost of a negligible error term $\ll 1$ for $S_1(T)$. Then, writing

$$b_n=2\sqrt{2}(-1)^{n(n+1)/2}\big(n+\tfrac12\big)^{-1}$$

for short, we have

$$S_1(T)=(-1)^L\left(\frac{T}{2\pi}\right)^{1/4}K_0(\vartheta)+O(T^{1/6}\log T), \tag{3.1}$$

where

$$K_0(\vartheta)=\sum_{1\le 2n+1<M}a_nb_n\sin\big(2\pi\big(n+\tfrac12\big)\vartheta+A\big(n+\tfrac12\big)^3\big). \tag{3.2}$$

We reformulate the sum $K_0(x)$ using the Fourier series

$$K(x)=\sum_{n=0}^{\infty}b_n\sin\big(2\pi\big(n+\tfrac12\big)x\big),$$

Cesàro summation and Lemma 2. The function $K(y/\pi)$ has period 2π in the variable y and the Cesàro partial sum

$$\sigma_{M-1}(y)=\sum_{1\le 2n+1<M}\left(1-\frac{2n+1}{M}\right)b_n\sin((2n+1)y)$$

for its Fourier series can be written, by Fejér's formula, as

$$\sigma_{M-1}(y)=\frac{1}{2\pi}\int_{-\pi}^{\pi}K\left(\frac{y+t}{\pi}\right)F_{M-1}(t)\,dt$$

in terms of the Fejér kernel

$$F_{M-1}(t)=\frac{\sin^2(Mt/2)}{M\sin^2(t/2)}.$$

Now the sum

$$\sigma(x)=\sum_{1\le 2n+1<M}a_nb_n\sin\bigl(2\pi\bigl(n+\tfrac12\bigr)x\bigr)$$

equals $2\sigma_{M-1}(\pi x)-\sigma_{M/2-1}(\pi x)$, and hence, with

$$\widetilde{F}_M(v)=2F_{M-1}(\pi v)-F_{M/2-1}(\pi v),$$

it follows that

$$\sigma(x)=\frac12\int_{-1}^{1}K(x+v)\widetilde{F}_M(v)\,dv. \tag{3.3}$$

By well-known properties of the Fejér kernels, we have that

$$\int_{-1}^{1}\widetilde{F}_M(v)\,dv=2 \tag{3.4}$$

and

$$\widetilde{F}_M(v)\ll\min\bigl(M,M^{-1}v^{-2}\bigr)\quad\text{for } 0<|v|\le 1. \tag{3.5}$$

On the other hand, the sum $K_0(\vartheta)$ can be written, by Lemma 2, in the form

$$K_0(\vartheta)=2\pi\int_{-\infty}^{\infty}\beta(u)\sigma(\vartheta+u)\,du. \tag{3.6}$$

It is convenient to truncate this integral to a finite range by means of the weight function $w(u)$. Integration by parts or direct estimations show that

$$\int_{-\infty}^{\infty}(1-w(u))\beta(u)\sin\bigl(2\pi\bigl(n+\tfrac12\bigr)(\vartheta+u)\bigr)\,du\ll T^{-1/8}$$

if $1 \le 2n+1 < M$ and $M/T^{1/4}$ is sufficiently small. Indeed, by (2.6), the function $\beta(u)$ is very small for $u \le -\frac{1}{4}$, while the main term in the asymptotic formula (2.7) for the J-Bessel functions gives an exponential integral over $u \ge \frac{1}{4}$ without a saddle point and it can be estimated by the "first derivative test"; the contribution of the error term in (2.7) can be estimated directly. Therefore we may insert the weight function into the integrand in (3.6), and then

$$K_0(\vartheta) = 2\pi \int_{-1/2}^{1/2} w(u)\beta(u)\sigma(\vartheta+u)\, du + O(T^{-1/8}\log T).$$

Combining this with (3.3), we obtain

$$\begin{aligned} K_0(\vartheta) &= \pi \int_{-1/2}^{1/2} w(u)\beta(u)\left(\int_{-1}^{1} K(\vartheta+u+v)\widetilde{F}_M(v)\, dv\right) du + O(T^{-1/8}\log T) \\ &= K(\vartheta) + 2\pi \int_{-1/2}^{1/2} w(u)\beta(u)(K(\vartheta+u) - K(\vartheta))\, du \\ &\quad + \pi \int_{-1/2}^{1/2} w(u)\beta(u)\left(\int_{-1}^{1} (K(\vartheta+u+v) - K(\vartheta+u))\widetilde{F}_M(v)\, dv\right) du \\ &\quad + O(T^{-1/8}\log T) \\ &= \widetilde{K}(\vartheta) + K_1(\vartheta) + O(T^{-1/8}\log T), \end{aligned}$$

say. Here we used (2.11), (2.12), and (3.4). Together with (3.1), this yields (1.4) if we can show that $K_1(\vartheta)$ can be viewed as an error term, that is $K_1(\vartheta) \ll T^{-1/12}\log T$.

Turning to the estimation of $K_1(\vartheta)$, we first invert the order of the integrations:

$$K_1(\vartheta) = \pi \int_{-1}^{1} \widetilde{F}_M(v)\left(\int_{-1/2}^{1/2} w(u)\beta(u)(K(\vartheta+u+v) - K(\vartheta+u))\, du\right) dv.$$

For given v, the function $K(\vartheta+u+v) - K(\vartheta+u)$ is non-zero and constant in a finite number of intervals of length at most $|v|$. Therefore, by (3.5) and (2.10), we have that

$$K_1(\vartheta) \ll \int_0^1 \min(T^{1/4}, T^{-1/4}v^{-2}) \min(1, vT^{1/6})\, dv \ll T^{-1/12}\log T,$$

and the proof of (1.4) is complete.

Remark. Consider now the quasiperiodicity property (1.1). Let T take two values of the form $2\pi\tau^2$ and $2\pi(\tau+2)^2$, so that ϑ and the parity of L remain unchanged. The values of the parameter A for these choices of T differ by an amount $\asymp T^{-1}$, and if we let M be the same in both cases, then the corresponding sums $K_0(\vartheta)$ in (3.2) differ by an amount $\asymp T^{-1/8}$. Therefore the difference of the values of $S_1(T)$ in (3.1) is $\ll T^{1/6}\log T$, and this proves (1.1). Another way to prove the same result is to start from the main theorem, and using properties of Bessel functions one may verify that the difference of the values of $\beta(u)$ in (1.5) for our two choices of T is $\ll T^{-1/8}$. Then the change of the leading term in (1.4) is $\ll T^{-1/8}$, and (1.1) follows again, admittedly in a less direct way.

3.2. Proofs of (1.6)–(1.8)

Suppose first that $\vartheta_0\neq 0$, and consider the difference

$$|K(\vartheta)-\widetilde{K}(\vartheta)|=2\pi\left|\int_{-1/2}^{1/2} w(u)\beta(u)(K(\vartheta+u)-K(\vartheta))\,du\right|.$$

Here the integrand vanishes unless $|u|\geq\vartheta_0$, and then the difference $K(\vartheta+u)-K(\vartheta)$ is piecewise constant. Therefore, by (2.9), we have

$$K(\vartheta)-\widetilde{K}(\vartheta)\ll\min(1,T^{-1/8}\vartheta_0^{-3/4}),$$

which proves (1.6).

Finally, let $\vartheta_0=0$, that is $\vartheta=\frac14$ or $\vartheta=\frac34$, say $\vartheta=\frac14$. Then $K\bigl(\frac14\bigr)=\pi$ and

$$\begin{aligned}2\pi\int_{-1/2}^{1/2} w(u)\beta(u)\bigl(K\bigl(\tfrac14+u\bigr)-K\bigl(\tfrac14\bigr)\bigr)\,du &= (2\pi)^2\int_0^{1/2} w(u)\beta(u)\,du\\ &\quad -2\pi^2\int_{-1/2}^{1/2} w(u)\beta(u)\,du\\ &=\frac{\pi}{3}+O(T^{-1/8}),\end{aligned}$$

where we used (2.11) and (2.12). Hence (1.7) follows immediately, and (1.8) can be verified similarly.

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