Correction to

On the Diophantine equation $1^k + 2^k + \ldots + x^k + R(x) = y^2$

by

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(Acta Mathematica, 143 (1979), 1–8)

In the above article the authors claim that a polynomial $P$ with rational integer coefficients which is congruent (mod 4) to

$3x^8 + 2x^4 + x^4 + 2x^2$

has at least three simple roots. Their argumentation is incorrect. In this corrigendum, they wish to repair this defect by proving claim (i) in case B of Lemma 4 in a correct way.

Suppose $P$ can be written as

$P(x) = Q(x) T^2(x), \ (*)$

with $\deg Q \leq 2$.

If $\deg Q = 0$, then clearly $Q$ is an odd constant, so $T^2(x) \equiv x^8 + x^4 \pmod{2}$, hence $T(x) \equiv x^4 + x^2 \pmod{2}$ and $T^2(x) \equiv x^8 + 2x^4 + x^4 \pmod{4}$, which is clearly not the case. If $\deg Q = 1$, then either $Q(x) \equiv x$ or $Q(x) \equiv x + 1 \pmod{2}$. In both cases, the quotient of $P$ and $Q$ cannot be written as a square (mod 2). If $\deg Q = 2$, then either

$Q(x) \equiv x^2$ or $Q(x) \equiv x^2 + x$ or $Q(x) \equiv x^2 + 1 \pmod{2}$,

since $x^2 + x + 1$ does not divide $P$ (mod 2). In the first case $T(x) \equiv x^3 + x \pmod{2}$, hence $T^2(x) \equiv x^6 + 2x^4 + x^2 \pmod{4}$ which does not divide $P$ (mod 4). In the second case, the
quotient of $P$ and $Q$ is not even a square (mod 2). In the third case $T(x) \equiv x^3 + x^2$ (mod 2), hence $T^2(x) \equiv x^6 + 2x^3 + x^4$ (mod 4) which does not divide $P$ (mod 4). We conclude that $P$ cannot be written in the form (*) with deg $Q<3$, proving our claim.

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