Perspectives in Mathematical Logic

Manuel Lerman

Degrees of Unsolvability

Local and Global Theory

Springer-Verlag
Berlin Heidelberg New York Tokyo

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With 56 Figures

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To Maxine,
Elliot and Sharon

Preface to the Series
Perspectives in Mathematical Logic

(Edited by the Ω-group for "Mathematische Logik" of the Heidelberger Akademie der Wissenschaften)

On Perspectives. *Mathematical logic arose from a concern with the nature and the limits of rational or mathematical thought, and from a desire to systematise the modes of its expression. The pioneering investigations were diverse and largely autonomous. As time passed, and more particularly in the last two decades, interconnections between different lines of research and links with other branches of mathematics proliferated. The subject is now both rich and varied. It is the aim of the series to provide, as it were, maps or guides to this complex terrain. We shall not aim at encyclopaedic coverage; nor do we wish to prescribe, like Euclid, a definitive version of the elements of the subject. We are not committed to any particular philosophical programme. Nevertheless we have tried by critical discussion to ensure that each book represents a coherent line of thought; and that, by developing certain* themes, *it will be of greater interest than a mere assemblage of results and techniques.*

The books in the series differ in level: some are introductory, some highly specialised. They also differ in scope: some offer a wide view of an area, others present a single line of thought. Each book is, at its own level, reasonably self-contained. Although no book depends on another as prerequisite, we have encouraged authors to fit their book in with other planned volumes, sometimes deliberately seeking coverage of the same material from different points of view. We have tried to attain a reasonable degree of uniformity of notation and arrangement. However, the books in the series are written by individual authors, not by the group. Plans for books are discussed and argued about at length. Later, encouragement is given and revisions suggested. But it is the authors who do the work; if, as we hope, the series proves of value, the credit will be theirs.

History of the Ω-Group. *During 1968 the idea of an integrated series of monographs on mathematical logic was first mooted. Various discussions led to a meeting at Oberwolfach in the spring of 1969. Here the founding members of the group (R. O. Gandy, A. Levy, G. H. Müller, G. E. Sacks, D. S. Scott) discussed the project in earnest and decided to go ahead with it. Professor F. K. Schmidt and Professor Hans Hermes gave us encouragement and support. Later Hans Hermes joined the group. To begin with all was fluid. How ambitious should we be? Should we write the books ourselves? How long would it take? Plans for authorless books were promoted, savaged and scrapped. Gradually there emerged a form and a method. At the end of an infinite discussion we found our name, and that of the series. We established our centre in Heidelberg. We agreed to meet twice a year together with authors, consultants and* *assistants, generally in Oberwolfach. We soon found the value of collaboration: on the one hand the permanence of the founding group gave coherence to the over-all plans; on the other hand the stimulus of new contributors kept the project alive and flexible. Above all, we found how intensive discussion could modify the authors' ideas and our own. Often the battle ended with a detailed plan for a better book which the author was keen to write and which would indeed contribute a perspective.*

Oberwolfach, September 1975

Acknowledgements. *In starting our enterprise we essentially were relying on the personal confidence and understanding of Professor Martin Barner of the Mathematisches Forschungsinstitut Oberwolfach, Dr. Klaus Peters of Springer-Verlag and Dipl.-Ing. Penschuck of the Stiftung Volkswagenwerk. Through the Stiftung Volkswagenwerk we received a generous grant (1970 – 1973) as an initial help which made our existence as a working group possible.*

Since 1974 the Heidelberger Akademie der Wissenschaften (Mathematisch-Naturwissenschaftliche Klasse) has incorporated our enterprise into its general scientific program. The initiative for this step was taken by the late Professor F. K. Schmidt, and the former President of the Academy, Professor W. Doerr.

Through all the years, the Academy has supported our research project, especially our meetings and the continuous work on the Logic Bibliography, in an outstandingly generous way. We could always rely on their readiness to provide help wherever it was needed.

Assistance in many various respects was provided by Drs. U. Felgner and K. Gloede (till 1975) and Drs. D. Schmidt and H. Zeitler (till 1979). Last but not least, our indefatigable secretary Elfriede Ihrig was and is essential in running our enterprise.

We thank all those concerned.

Heidelberg, September 1982

R. O. Gandy *H. Hermes*
A. Levy *G. H. Müller*
G. E. Sacks *D. S. Scott*

Author's Preface

I first seriously contemplated writing a book on degree theory in 1976 while I was visiting the University of Illinois at Chicago Circle. There was, at that time, some interest in an Ω-series book about degree theory, and through the encouragement of Bob Soare, I decided to make a proposal to write such a book. Degree theory had, at that time, matured to the point where the local structure results which had been the mainstay of the earlier papers in the area were finding a steadily increasing number of applications to global degree theory. Michael Yates was the first to realize that the time had come for a systematic study of the interaction between local and global degree theory, and his papers had a considerable influence on the content of this book.

During the time that the book was being written and rewritten, there was an explosion in the number of global theorems about the degrees which were proved as applications of local theorems. The global results, in turn, pointed the way to new local theorems which were needed in order to make further progress. I have tried to update the book continuously, in order to be able to present some of the more recent results. It is my hope to introduce the reader to some of the fascinating combinatorial methods of Recursion Theory while simultaneously showing how to use these methods to prove some beautiful global theorems about the degrees.

This book has gone through several drafts. An earlier version was used for a one semester course at the University of Connecticut during the Fall Semester of 1979, at which time a special year in Logic was taking place. Many helpful comments were received from visitors to UConn and UConn faculty at that time. Klaus Ambos, David Miller and James Schmerl are to be thanked for their helpful comments. Steven Brackin and Peter Fejer carefully read sizable portions of that version and supplied me with many corrections and helpful suggestions on presentation. Richard Shore, Stephen Simpson and Robert Soare gave helpful advice about content and presentation of material. Other people whose comments, corrections and suggestions were of great help are Richard Epstein, Harold Hodes, Carl Jockusch, Jr. Azriel Levy and George Odifreddi. I am especially grateful to David Odell who carefully read the manuscript which I expected to be the final one, and to Richard Shore who used that same manuscript for a course at Cornell University during the Fall Semester of 1981. They supplied me with many corrections and helpful suggestions on presentation of material which have been incorporated into the book and which, I hope, have greatly enhanced the readability of the book.

Also, the meetings of the Ω-group provided me with many suggestions which influenced the continuously evolving formulation of the book.

I owe a debt of gratitude to my teachers, Anil Nerode and Thomas McLaughlin, who introduced me to Recursion Theory, and to Gerald Sacks who continued my education and provided me with much needed encouragement and dubious advice. Finally, I thank my colleagues who have shown an interest in my work and have stimulated me with theirs.

Storrs, February 5, 1983 Manuel Lerman

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