PREFACE

As part of the special year devoted to the *Application and Numerical Solutions to Partial Differential Equations*, the Centre for Mathematical Analysis at the Australian National University, Canberra, hosted a Mini–conference on *Free and Moving Boundary and Diffusion Problems*, on June 14–16, 1990. The primary aim was to stimulate strong interaction between practitioners (scientists, industrialists and engineers) with specific free and moving boundary and diffusion problems and mathematicians (pure, applied and computational) working on the mathematical theory as well as on the exact and approximate solution of such problems. In addition, the Mini-conference aimed to foster the interest of younger colleagues in research connected with these problems. A number of Australian and overseas speakers were invited to participate. They were:

Prof. Jim Hill University of Wollongong	Similarity Solutions for Nonlinear Diffusion and Related Phenomena
Dr. Francis Rose DSTO, Melbourne	Diffusion with Trapping and Fast Diffusion Paths
Prof. Giles Auchmuty University of Houston	Algorithms for Computing Equilibria of Rotating Self-Gravitating Fluids
Dr. Jeff Dewynne University of Southampton	Phase Change and Loose Change: Unstable Stefan Problems and the Stock Market
Dr. Rodney Weber ADFA, Canberra	Fire Spread as a Moving Boundary
Prof. Michael Barber School of Mathematical Sciences, ANU	Velocity Selection in Dentritic Growth
Prof. John Mason Royal Military College, Schrivenham	A Method of Particular Solutions for Free Boundary Problems

Prof. Ernie Tuck University of Adelaide	Water Non-Waves
Dr. K. Gopalsamy Flinders University	Oscillations in Parabolic Neutral Systems
Dr. Amiya Pani IIT, Bombay	Finite Element Method for a Free Boundary Problems Arising in Polymer Diffusion
Dr. John Knight CSIRO, Canberra	Improving the Boussinesque Approximation for Transient Ground Water Free Surface Movement
Prof. Phil Broadbridge University of Wollongong	Moving Boundary Problems involving Linearisable Models in Hydrology
Dr. Graham Williams University of Wollongong	Regularity for Obstacle Problems with Applications to the Free Boundary in the Constrained Least Gradient Problems
Prof. Alex McNabb Massey University, NZ	The Freezing Time for Pseudo–Ellipsoids
Dr. Kerry Landman University of Melbourne	Moving Boundary Value Problems in Solid Liquid Separation
Dr. Noel Barton CSIRO, Sydney	Moving Boundary Problems in Industry
Dr. Noel Smyth University of Wollongong	Higher Order Nonlinear Diffusion
Dr. Robin Wooding CSIRO, Canberra	Deposition from a Curving Shallow flow,Treated as a Moving Boundary of Constant Form

This volume contains the proceeding of the Mini-conference. The papers are arranged in their order of presentation at the conference. Alternatively, the papers could have been organised in terms of the focus they gave to deliberations about the application and numerical solution of free and moving boundary and diffusion problems. In particular, such a reorganization would fall naturally under the following headings: specific and novel applications, analytic and semianalytic methods, numerical techniques, and theoretical studies.

Specific and Novel Applications. Free and moving boundary and diffusion problems arise naturally in a great variety of concrete phenomena such as solidification of metals, dentritic growth, crystal pulling, combustion theory, and fluid flow through porous media. In this conference, a number of talks focussed on applications.

Dr. Rose discussed two diffusion problems arising in aircraft fabrication. The first one was connected with hydrogen embrittlement of highstrength steels, while the second one involved the transient redistribution of lithium during heat treatment of alluminium-lithium alloys.

Most of the models in fire spread consider only heat transfer, and hence, can be classified as Stefan-type models, which are mathematically quite similar to ablation problems. But based on 'combustion theory', another alternate model can be formulated which considers chemical kinetic effects alongwith heat transfer. Dr. Weber compared these two types of models using examples and experimental observations.

A problem of major industrial importance is the separation of fine solids from liquids. Dr. Landman and Professor White outlined the dynamics of consolidation for suspended solids under the influence of a body force, such as gravity in batch settling, or of an applied pressure in pressure filtration.

Most mathematical models for solidification are classical Stefan or Stefan-type problems and are inadequate for describing supercooling of pure liquids or liquids in the presence of dilute impurities. A number of modifications to these problems have been proposed in the past in order to include the Gibbs-Thompson effect (i.e. the effect of surface tension and the local mean curvature of the interface which provides the mechanism for dendrite growth) and /or a kinetic undercooling condition (i.e. the velocity dependent term) at the free boundary. Dr. Dewynne examined Stefan models for supercooling including the effect of such modifications. Professor Barber discussed recent developments connected with the velocity and shape selection in dendrite growth.

Two papers examined free boundary problems connected with fluid flow.

Professor Tuck discussed free surface flows associated with the movement of ships through water. Dr. Wooding examined the growth of limestone dams and terraces through deposition from supersaturated and supercooled flows.

For decision making, all that is usually needed is some appropriate rules-of-thumb. Drs. McNabb and Anderssen examined the utility and applicability for the freezing of food stuffs of rules-of-thumb derived from a study of the freezing of ellipsoidal bodies.

Analytic and Semianalytic Methods. Analytic and semianalytic solutions play a vital role in the understanding mathematical models. Professor Hill reviewed recent results relating to similarity solution for nonlinear diffusion equations with powerlaw diffusivities. Professor Broadbridge presented exact solutions, derived using Lie-Bäcklund transformation techniques, for two nonlinear moving boundary problems . In his paper, Dr. Smyth considered asymptotic solutions for a fourth order diffusion equation which models a number of physical processes such as the flow of a surface-tension-dominated thin liquid film, and the diffusion of dopant in semiconductors.

Numerical Techniques. Three papers are included in this category. For both fixed and free boundary value problems, Drs. Mason and Weber discussed three aspects of the particular-solution strategy including the choice of particular-solutions, approximation procedures for satisfying the boundary conditions, and the choice of iterative techniques. Approximations examined include bivariate piecewise polynomials, harmonic functions, variable separable expansions, and combinations of them.

Dr. Landman and Professor White, in the later part of their paper, considered economically viable computational method for the moving boundary problems in solid-liquid separation. Their strategy was to apply the method of lines and then solve the resulting equations using Runge-Kutta and shooting methods.

In order to provide confidence for numerical methods as well as insight into their performance, numerical analysts like to make definite statements regarding convergence rates in terms of the discretization. This is the subject of the contribution by Drs. Pani and Anderssen. They presented an error analysis for a finite element method applied to a free boundary problem in polymer technology.

Theoretical Studies. Two out of the three contributions in this category are connected with constrained minimization problems. Prof. Auchmuty analysed a constrained variational principle for the axisymmetric equilibria of rotating self–gravitating polytopes. They represent simple hydrodynamical models of stars and planets with their shapes defined by the free boundaries. In the later part of his paper, a relaxed Lagrangean formulation was discussed as well as globally convergent iterative algorithms for its solution.

The problem of finding a bar of lightest weight with constant cross-section, which supports a given load, can be modelled as a constrained least square gradient problem. The free boundary appears in regions where the magnitude of the gradient is either zero or one. The position and nature of these free boundaries can be related to results connected with obstacle problems for minimal surfaces. Dr. Williams discussed regularity results for such problems.

Finally, Dr. Gopalsamy examined sufficient conditions which guarantee the oscillation of all solutions of linear neutral parabolic system with homogeneous Neumann and Dirichlet boundary conditions.

In conclusion, the editors would like to thank the participants in the mini–conference for their contributions, and in particular, the speakers, and authors of the papers in this Proceeding. The editors also wish to acknowledge the support and assistance of the Centre for Mathematical Analysis. Finally, a special thanks to Professor Neil Trudinger (the Director of the Centre), Marilyn Gray and Jillian Smith, for their contributions to ensuring the success of the mini–conference.

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