Contents

1 Introduction **3**

1.1 The spacelike Cauchy problem in General Relativity 7

1.2 Singularities? 16

1.3 Strong Cosmic Censorship? 19

1.4 (Weak) Cosmic Censorship? 22

1.5 Some answers, in spatially compact space–times with two or more spacelike Killing vector fields. 25

1.6 Spatially open space–times: some small data results. 34

1.7 Beyond the spacelike Cauchy problem: Robinson–Trautman space–times. 38

1.8 Necessary conditions for uniqueness in the large. 43

2 "Highly symmetric" space–times **49**

2.1 From symmetric Cauchy data to symmetric space–times. 49

2.2 $SO(3) \times U(1)$ symmetric space-times, ${}^3\Sigma = L(p,1)$. 59

2.3 $SO(3)$ symmetry, 2-dimensional principal orbits. 61

2.4 Spatially compact Bianchi I space-times. 65

3 $U(1) \times U(1)$ stability of the $(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3})$ Kasner metrics. **68**

3.1 Introduction — notation 68

3.2 Pointwise Estimates 69

3.3 Integral "Decay" Estimates 74

3.4 Pointwise "Decay" Estimates 76

3.5 The Stability Theorem. 81

A On the "hyperboloidal initial data", and Penrose conditions. 87

B On a class of $U(1) \times U(1)$ symmetric metrics found by V.Moncrief. 94

B.1 A harmonic map problem. 94

B.2 Moncrief's space–times. 101

C Maximal developments. 106

C.1 Existence of maximal space–times. 106

C.2 Some maximality criteria. 108

D Some flat metrics with Cauchy horizons. 114

E On a wave equation with a singular source. 117

F Bifurcating geodesics in $C^{1,\alpha}_{loc}$, $0 < \alpha < 1$, metrics. 119