CATALOGUE OF NOTATION

Relations

$x \in A$	set membership	6
$\in_A$	membership of subsets of A	103
$\in \restriction A$	membership restricted to A	307
$g \in f$	membership for subobjects in **Set**	316
$E_{\mathscr{E}}$	membership for set-objects in $\mathscr{E}$	327
$A \subseteq B$	set inclusion	8
$f \subseteq g$	subobject inclusion	76
$r \subseteq s$	inclusion for relations	322
$\leqslant$	'is less than or equal to'	29
$\leqslant\!\!\!\cdot$	ordering of truth-values	137, 139
$a \leqslant b$	restriction-ordering on sheaves	390
$a \between b$	compatibility	390
$s \leqslant t$	*negative* $\leqslant$-ordering of the continuum	429
$r \leqslant c, c \leqslant r$	*positive* $\leqslant$-ordering	431
$r < s$	ordering of ${}^*R$	435
$\sqsubseteq$	partial-ordering	29
$\sqsubset$	antireflexive $\sqsubseteq$-ordering	72
$p \prec q$	'close to'	383
$\neq$	apartness	425
$a \cong b$	isomorphic objects	41
$\mathscr{C} \cong \mathscr{D}$	isomorphic categories	200
$\mathscr{C} \simeq \mathscr{D}$	equivalent categories	200
$f \simeq g$	isomorphic subobjects	77
$r \simeq s$	equality of transitive set-objects	325
$\simeq_{\mathscr{E}}$	equality of set-objects	326
R^{-1}	inverse relation	45
R_f	kernel	66, 111
Δ	diagonal	236
$F \dashv G$	left adjoint	439
$G \vdash F$	right adjoint	439

Objects (elements, sets, structured sets, algebras, spaces)

$\{x: \varphi(x)\}$	set determined by property	7
$A \cap B$	intersection of sets	7
$f \cap g$	intersection of subobjects	147

$r \cap s$	intersection of transitive set-objects	325
$A \cup B$	union of sets	7
$f \cup g$	union of subobjects	147
$r \cup s$	union of tso's	325
$A + B$	disjoint union	54
$a + b$	coproduct	54
$-A$	set complement	7
$-f$	subobject complement	147
$\emptyset$	empty set	7
0	initial object	43
V	class of all sets	9, 11
R	Russell set	9
dom f	domain	19, 24
Im f	image (range) of function	19
cod f	codomain	24
$f^{-1}(C)$	inverse image	65
$\mathscr{C}(a, b)$ $= \hom_{\mathscr{C}}(a, b)$	set of arrows $a \to b$	32, 196
$A \times B$	product set	20, 46
$A \times_C B$	fibred product	65
$A \cdot B$	product-set of Ω-sheaves	399
A^m	m-fold product set	52
$a \times b$	product of objects	47
a^m	m-fold product object	53
P	poset	29
ω	natural numbers	29
M	monoid	31
n	finite ordinal	33
1	terminal object	44
g.l.b.	greatest lower bound	49
$p \sqcap q$	g.l.b.	49
$\bigsqcap A$	g.l.b.	276
l.u.b.	least upper bound	55, 179
$p \sqcup q$	l.u.b.	55
$\bigsqcup A$	l.u.b.	276
$\bigvee B$	join	390
$[a]$	equivalence class	61
B^A	set of functions $A \to B$	70
b^a	exponential	71
$[A \to B]$	exponential for Ω-sheaves	401
$\mathscr{P}(D)$	powerset	76
$\mathscr{P}(a)$	power object	104
Sub (d)	subobjects of d	77

Ω	truth-values object	81, 277
Ω_j	j-sheaf of truth-values	369, 380
Θ	topology	96
Θ_V	open subsets of open set	368
L_M	left ideals	102
$\mathbf{B}$	Boolean algebra (BA)	134
$\mathbf{H}$	Heyting algebra (HA)	183
$\mathbf{P}^+$	HA of hereditary sets	189
$a \Rightarrow b$	relative pseudo-complement (r.p.c.)	182
$f \mid\Rightarrow g$	r.p.c. of subobjects	162
$S \Rightarrow T$	r.p.c. of hereditary sets	190, 213
$\neg a$	pseudo-complement	183
$\neg S$	pseudo-complement of hereditary set	190, 213
Φ_0	sentence letters	130
Φ	propositional sentences	130
Ψ	modal propositional sentences	382
$\mathscr{L}$	first-order language	234
$\mathscr{M}$	**P**-based model	189, 383
$\mathscr{M}(\alpha)$	truth-set in $\mathscr{M}$	189
$\mathfrak{A}$	classical model for $\mathscr{L}$	235, 305
	$\mathscr{E}$-model	246
	Set$^{\mathbf{P}}$-model;	256
	Ω-**Set** model	284
$\mathfrak{A}^*$	completion of Ω-**Set** model	404
$\mathfrak{A}(\mathscr{E})$	model of $\mathscr{E}$-set-objects	327
$\mathfrak{A}_\varphi$	metasubset determined by property	309
$\mathbf{A}_\varphi$	sub-sheaf determined by property	405
S_a	largest a-sieve	205
$[p)$	principal hereditary set	213
S_p	$S \cap [p)$	214
$*$	null entity	268
$\emptyset_{\mathbf{A}}$	null element	398
$\tilde{B}$	set of partial elements	268
$\tilde{a}$	object of partial elements	270
$\mathbf{A}$	Ω-set	276
$\mathbf{A}^*$	completion of Ω-set	393
sup (a)	support	292
$\mu(p)$	monad	383
$\varprojlim$	limit	362
$\varinjlim$	colimit	363
$a \restriction p$	restriction of element (section)	389
Ea	extent	389

F_f	stack of sections	360
p_F	sheaf of germs of sections	362
$\mathbf{C}_X$	Heyting-valued set of continuous partial functions	388
$\mathbf{C}_k$	Heyting-valued set of continuous local sections	389
$F_{\mathbf{A}}$	sheaf corresponding to complete Ω-set	391
$\mathbf{A}_F$	Ω-set corresponding to sheaf	392
F_X	sheaf of continuous partial functions	392
$\dot{\Omega}$	sheaf of truth-values	400
$\bar{\mathbf{X}}$	rigid Ω-set	406
$\mathbf{X}^*$	simple sheaf	406
$\mathbf{A}_a$	sheaf of partial elements	410
N	natural numbers object	302
N^+	positive integers object	414
Z	integers object	414
Q	rationals object	414
R_c	Cauchy reals	414
R_d	Dedekind reals	414ff.
C	object of complex numbers	423
${}^*R$	order-complete reals	423, 431
$\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$	classical number systems	413
$\beta(\Omega)$	soberification	436

Arrows (functions, functors)

$f: A \to B$	function	17
$f: a \to b$	arrow	24
$f: A \rightsquigarrow B$	partial function	268
$f: a \rightsquigarrow b$	partial arrow	268
$1 \to a$	element	78
$1 \rightsquigarrow a$	partial element	268
$A \hookrightarrow B$	inclusion function	19
$h: r \hookrightarrow s$	inclusion of relations	322
id_A	identity function	19
$\mathbf{1}_b$	identity arrow	25
$g \circ f$	composition	20, 24
$\rightarrowtail$	monic	38
$\twoheadrightarrow$	epic	39
f^{-1}	inverse arrow	40
f^{op}	dual arrow	45
$f: a \cong b$	iso arrow	40
0_a	unique arrow $0 \to a$	44

$!_a$	unique arrow $1 \to a$	44
p_A	projection to set	46
pr_a	projection to object	47
pr_j^m	j-th projection $a^m \to a$	53
$\langle f, g \rangle$	product arrow	47
$f \times g$	product of arrows	50
$\dashrightarrow$	uniquely existing arrow	52
i_a	injection	54
$[f, g]$	coproduct arrow	54
$f + g$	coproduct of arrows	55
f_R	natural map	62
ev	evaluation arrow	70–1
ev_a	evaluation on Ω^a	106
$\in_a$	membership relation	104
$\hat{g}$	exponential adjoint	71–2
$\ulcorner f \urcorner$	name of f	78
χ_A	characteristic function	79
χ_f	characteristic arrow (character)	81
true	subobject classifier	80–1
$\top$	*true*	81
$\ulcorner f \urcorner$	unit of **HA** (largest truth-value)	277
$true_a$	$= \top_a = true \circ !_a$	83
$true_j$	classifier for j-sheaves	369, 380
$1 \to \Omega$	truth-value	94
false	character of $0 \to 1$	117
$\bot$	*false*	117
	least truth-value	277
$\neg$	negation truth-arrow	127, 139
$\neg \circ \neg$	double negation	184, 379
$\cap$	conjunction	127, 139
$\cup$	disjunction	128, 139
$\Rightarrow$	implication	128, 139
$\Leftarrow$	ordering on Ω	139
$\mathrm{im}\, f$	image of f	112
$f[g]$	image under f	320
Ω^f	internal images	320
$\{x : \varphi\}$	subobject determined by "property"	107
f^*	epic part of f	112
$F : \mathscr{C} \to \mathscr{D}$	functor	195
$\mathscr{P}, \bar{\mathscr{P}}$	powerset functors	195, 197
$\mathscr{P}(f)$	f-images functor	195
$1_{\mathscr{C}}$	identity functor	195
$- \times a, a \times -$	product functors	196, 200

Sub	subobjects functor	197
$\mathscr{C}(-, a), \mathscr{C}(a, -)$	hom-functors	196, 197
f^{-1}	inverse image functor	447, 454
f^{+}	right adjoint to f^{-1}	447
f^{*}	pulling-back functor	449
Σ_f	'composing-with-f' functor	449
Π_f	right adjoint to f	450, 452
i_a	inclusion into $\mathscr{E} \downarrow a$ functor	455
σ_a	'image in a' functor	456
f^a	internally 'composing-with-f'	337
$\tau : F \dot{\to} G$	natural transformation	199
tw_B	twist map	200
$\forall_A, \exists_A$	quantifier functions on a set	240, 242
$\forall_a, \exists_a$	quantifier arrows for an object	245, 457
$\forall_f, \exists_f$	quantifiers along an arrow	454
$\forall_r, \exists_r$	quantifiers along a relation	457
ρ_t^m	value of term t	243, 246
δ_A, δ_a	Kronecker delta (character of diagonal)	243, 245
Δ_a	diagonal on a	245
T_i^{m+1}	$i/m+1$ substitution	246
$\lvert\varphi\rvert_i^m$		247
$\delta^m[i/t]$	i/ρ_t substitution	250
$\eta_a : a \rightarrowtail \tilde{a}$	partial-arrow classifier	269
$\{\cdot\}_b$	singleton-forming arrow	269
$\{\mathbf{a}\}$	singleton Ω-subset	282, 388
$\emptyset_a$	null partial element	271
$\emptyset_A$	empty section	283
$\{\emptyset_A\}$	null singleton	283
$s \restriction p$	restriction of Ω-subset	283
λ_m	left-multiplication	100
O	zero	302
$\mathscr{s}$	successor	302
$\oplus$	addition	337
$i(f)$	iterate	337
ρ	predecessor	339, 342
$\dot{-}$	subtraction	342
$\circledS$	strict order on N	346
$\otimes$	multiplication	346
$\mathbf{n}$	finite ordinal	350
$\mathscr{S}$	from sheaves to stacks	361
$\mathscr{G}$	from stacks to sheaves	364
j	topology on a topos	378
j_Θ	topology on $\mathbf{St}(I)$	371

J	local operator	371, 379
Cov	pretopology	374
Cov_Θ	pretopology on Θ	375
Cov_Ω	pretopology on Ω	391
j_{Cov}	pretopology on $\mathbf{St}(\mathscr{C})$ determined by Cov	377
$\langle F, G, \theta \rangle$	adjunction	439
η	unit of adjunction	440
ε	counit of adjunction	440

Categories

Set	sets	23
Finset	finite sets	23
Nonset	non-empty sets	23
Mon	monoids	23
Grp	groups	23
Top	topological spaces	23
P	skeletal pre-order (poset)	29, 42
n	n as pre-order	29
Finord	finite ordinals	33
M	one-object category (monoid)	31–2
M-Set	**M**-sets	101
$\mathbf{M}_2$	canonical counter-example	122
$\mathbf{Set}^2$	pairs of sets	34
$\mathscr{C} \times \mathscr{D}$	product category	34
$\mathbf{Set}^{\rightarrow}$	set-functions	34, 219
$\mathscr{C}^{\rightarrow}$	$\mathscr{C}$-arrows	34
$\mathbf{Set}\downarrow X$	X-valued functions	36
$\mathscr{C}\downarrow a$	objects over a	36
$\mathscr{C}\uparrow a$	objects under a	36
$\mathscr{C}^{\mathrm{op}}$	opposite category	45
$\mathscr{E}$	elementary topos	84
$\mathscr{E}_j$	elementary site	378
$\mathbf{Bn}(I)$	bundles	90
$\mathbf{Top}(I)$	sheaves of sets of germs	97
$\mathbf{St}(I)$	stacks over a space	360
$\mathbf{Sh}(I)$	sheaves of sections	362
$\mathbf{Sh}(\mathbf{P})$	sheaves over a poset	366
$(\mathscr{C}, Cov)$	site	375
$\mathbf{St}(\mathscr{C})$	stacks over a category	375
$\mathbf{Sh}(Cov)$	sheaves over a site (Grothendieck topos)	376
$sh_j(\mathscr{E})$	j-sheaves	380

$sh_{\neg\neg}(\mathscr{E})$	double negation sheaves	381
$\mathbf{Sh}(\Omega)$	sheaves over a **CHA**	391
Cat	small categories	198
$\mathscr{D}^{\mathscr{C}}$	functors $\mathscr{C} \to \mathscr{D}$	202
$\mathbf{Set}^{\mathscr{C}}$	set-valued functors	204
$\mathbf{Set}^{\mathbf{P}}$	variable sets	215ff.
$\mathbf{Set}^{\omega}$	sets through time	219
$\mathscr{E}_{\mathscr{L}}$	canonical $\mathscr{L}$-model	265
$\mathscr{E}(\mathfrak{A})$	$\mathfrak{A}$-sets	311
Ω-**Set**	Ω-sets	277
CΩ-**Set**	complete Ω-sets	396
$\mathscr{C}^{\circlearrowright}$	endo's	445

Truth and Validity

$V(\alpha)$	valuation	
	classical	130
	in **B**	135
	in **H**	185
	in $\mathscr{E}$	141, 382
$\vDash \alpha$	tautology	130
$\mathbf{B} \vDash \alpha$	**B**-valid	135
$\mathbf{H} \vDash \alpha$		185
$\mathscr{E} \vDash \alpha$		141
$\mathscr{E}_j \vDash \alpha$	site-valid	382
$\mathscr{M} \vDash_p \alpha$	true at a point of model	189, 383
$\mathscr{M} \vDash \alpha$	true in model	189
$\mathbf{P} \vDash \alpha$	frame-valid	189
$\mathfrak{A} \vDash \varphi[x]$	satisfaction by model-valuation	236
$\mathfrak{A} \vDash \varphi[x_1,\ldots,x_n]$	satisfaction by elements of model	237, 239
$\mathfrak{A} \vDash_p \varphi[x_1,\ldots,x_n]$	satisfaction at a point	257
$\mathfrak{A} \vDash_p \varphi$	true at a point	257
$\mathfrak{A} \vDash \varphi$	true in model	237, 257
$[\![\varphi]\!]^m$	truth-values under interpretation	
	in **Set**	240
	in $\mathscr{E}$	246
$[\![\varphi]\!]_{\mathfrak{A}}$	truth-value in model	
	in **Set**	243
	in $\mathscr{E}$	247
	in Ω-**Set**	284
	in **C**Ω-**Set**	404–6
$\mathfrak{A} \models^{\mathscr{E}} \varphi$	true in topos-model	247

$[\![x \approx y]\!]$	Ω-equality	276
$[\![x \approxeq y]\!]$	equivalence	276
$[\![x \in s]\!]$	Ω-subset membership	280
$[\![Ex]\!] = Ex$	extent (degree of existence)	276, 389
$[\![\mathbf{I}v\varphi]\!]$	definite descriptions	
	in $\mathscr{E}$	287–288
	in sheaves	404
$[\![b \in f(A)]\!]$	membership of subobject	401
$[\![r < s]\!]$, $[\![r \neq s]\!]$, etc.	ordering the continuum	424, 435

Systems, languages, axioms, rules

PL	language of propositional logic	129
CL	classical logic	131
IL	intuitionistic logic	177, 249
LC	logic of linear frames	192, 227–228
$\mathscr{J}$	modal logic of sites	382
$\mathscr{L}$	first-order languages	234
UI, EG	quantifier axioms	238
$(\forall)$, $(\exists)$	quantifier rules	238
$I1$, $I2$	identity axioms	238
PBL	Pullback Lemma	67
Ω-AXIOM	subobject classification	81
EM	Excluded Middle	161
ES	epics split	290
SS	supports split	292
NE	non-initial implies non-empty	292
AC	MacLane's Axiom of Choice	295, 312
NNO	existence of natural numbers object	302
Ext	Extensionality	307
Sep	Separation	308
Inf	Infinity	311
Reg	Regularity	312
Rep	Replacement	312
TA	Transitivity	314
ATR	Transitive Representation	328
APT	Partial Transitivity	330
Z_0	basic classical set theory	309
Z	Z_0 + Reg + TA + ATR	328
ZF	Zermelo–Fraenkel set theory ($= Z_0 + \text{Inf} + \text{Reg} + \text{Rep}$)	11, 312
NBG	Von Neumann–Bernays–Gödel set theory	10

PO-*P*3, *P*3*A*, *P*3*B*	Peano Postulates	348–351
*F*1, *F*2	Freyd Postulates	352
COM	'patching' of compatible elements (sections)	362, 376, 391
$\delta(\mathbf{r})$	Dedekind cut axioms	415, 423
$0(r)$	order-axioms for cuts	427, 430

Logical symbols

$\vee, \wedge, \sim$	'and', 'or', 'not'	126
$\supset$	'implies'	128
$\equiv$	'if and only if'	274
∇	'It is locally the case that'	382
$\approx$	identity/equality	232, 274
$\approx\!\!\approx$	equivalence	274
$\forall$	'for all'	231
$\exists$	'there exists'	231
$\exists!$	'there exists exactly one'	233
$\mathbf{E}(t)$	't exists'	267
$\mathbf{I}v$	'the unique v such that'	287, 404
$\varphi'(v)$	function-value term	288
$\boldsymbol{\varepsilon}$	membership	305
$v\boldsymbol{\varepsilon}\{u\colon \varphi\}$	class abstract	309
$\varphi(v_{i_1},\ldots,v_{i_n})$	open formula	235
$\varphi(v/t)$	substitution	237