# Preface

This volume presents contributions focused on recent developments of CR geometry and overdetermined systems. Some of the papers are based on the lectures delivered at a conference of the same title held at Osaka, Japan, from December 19 to 21, 1994, on the occasion of Kuranishi's 70th birthday.

The notion of CR manifold is an abstraction of a smooth boundary of a complex manifold or a complex space equipped with the tangential Cauchy-Riemann operator $\overline{\partial}_b$, where a formal integrability condition is imposed as in the case of an almost complex structure to be a complex structure. When the strict pseudoconvexity is assumed, one can employ differential geometric formalism and the method of harmonic integrals for the $\overline{\partial}_b$ complex analogous to the $\overline{\partial}$-Neumann problem. How does the CR structure on the boundary determine the complex structure inside? This is a central theme of CR geometry. A fundamental question is the embeddability of a CR manifold $M$ as a real hypersurface of a complex space. An affirmative answer for the local embeddability was given by Kuranishi under the assumption that $\dim_{\mathbb{R}} M$ is not too small, indeed, $\dim_{\mathbb{R}} M \geq 7$ is sufficient. ($\dim_{\mathbb{R}} M$ must be odd.) The case $\dim_{\mathbb{R}} M = 3$ is exceptional, and most of the CR structures on $M$ are not embeddable even when $M$ is compact.

In the early 1970s, Kuranishi proposed to develop a deformation theory of isolated singularities via that of embeddable CR structures. His idea was presented in 1975 by series of lectures at the AMS summer institute, Williamstown, and at RIMS, Kyoto University. Since then, great progress was made of the theory and applications of CR geometry, synchronously with attempts to realize Kuranishi's idea. This volume reports on such progress and related topics as follows.

Methods of studying isolated singularities are developed since 1975, and the article by Ohsawa overviews such development. To investigate the deformations of a three dimensional CR manifold, one must consider the spaces of embeddable and non-embeddable CR structures, and this is done in the papers by Bland, Epstein and Lempert. The local embedding problem for 5-dimensional CR manifolds is still open, and Webster investigates this through a model problem. The paper by Luk and Stephen Yau discusses problems related to the minimal embedding dimension of a compact CR manifold in the Euclidean space. Since the work of E. Cartan, the method of prolongation has been successfully used in CR geometry. The paper by Han and Yoo determines the freedom of the CR mappings via the prolongation, while Veloso discusses the Cartan connection of CR structure through the general theory of prolongation. The weakly pseudoconvex case is difficult due to the lack of differential geometric formalism, and the paper by Stanton deals with this case by studying infinitesimal CR automorphisms. The higher codimensional CR geometry is also difficult, and the paper by Garrity and Mizner discuss invariants of the Levi form in this case.

There is also a variety of interesting contributions on related topics. The article by DeTurk and Goldschmidt considers problems of seeking a Riemannian metric with the Ricci curvature prescribed. The paper by Gasqui and Goldschmidt deals with a theorem on the spectral rigidity. The paper by Glazebrook and Sundararaman is concerned with the deformations of self-dual bundles. The article by Phong and Silvotti should be useful to mathematicians interested in the conformal field theory. The paper by Sasaki, Yamaguchi and Yoshida is concerned with a conjecture on hypergeometric functions of several variables. Umemura gives a new insight to the infinite dimensional differential Galois theory.

Three survey lecture notes are included for the reader's convenience. The surveys by Akahori and by Miyajima are concerned with recent development on the deformations of isolated singularities of higher dimension via those of CR structures. The survey by Hirachi and Komatsu are on the invariant theory of the Bergman kernel initiated by Fefferman. These are attempts to understand the relation between the CR structure on the boundary and the local biholomorphic geometry inside.

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M. Namba and K. Yamaguchi
On behalf of the editors

*All papers in this volume have been refereed and are in final form. No version of any of them will be submitted for publication elsewhere.*