45. The Law of Causes.—The method of finding the consequences of a given equality suggests directly the method of finding its *causes*, namely, the propositions of which it is the consequence. Since we pass from the cause to the consequence by eliminating known terms, *i. e.*, by suppressing constituents, we will pass conversely from the consequence to the cause by adjoining known terms, *i. e.*, by adding constituents to the given equality. Now, the number of constituents that may be added to it, *i. e.*, that do not already appear in it, is $2^n - m$. We will obtain all the possible causes (in the universe of the n terms under consideration) by forming all the additive combinations of these constituents, and adding them to the first member of the equality in virtue of the general formula

$$(A + B = 0) < (A = 0),$$

which means that the equality $(A = 0)$ has as its cause the equality $(A + B = 0)$, in which B is any term. The number of causes thus obtained will be equal to the number of the aforesaid combinations, or $2^{2^n - m}$.

This method may be applied to the investigation of the causes of the premises of the syllogism

$$(a < b)\ (b < c),$$

which, as we have seen, is equivalent to the developed equality

$$abc' + ab'c + ab'c' + a'bc' = 0.$$

This equality contains four of the eight (2^3) constituents of the universe of three terms, the four others being

$$abc,\ a'bc,\ a'b'c,\ a'b'c'.$$

The number of their combinations is 16 (2^4), this is also the number of the causes sought, which are:

1. $(abc + abc' + ab'c + ab'c' + a'bc' = 0)$
$= (a + bc' = 0) = (a = 0)\ (b < c);$

2. $(abc' + ab'c + ab'c' + a'bc + a'bc' = 0)$
$= (abc' + ab' + a'b = 0) = (ab < c)\ (a = b);$

3. $(abc' + ab'c + ab'c' + a'bc' + a'b'c = 0)$
$= (bc' + b'c + ab'c' = 0) = (b = c)\ (a < b + c);$

4. $(abc' + ab'c + ab'c' + a'bc' + a'b'c' = 0)$
$= (c' + ab' = 0) = (c = 1)\ (a < b);$

5. $(abc + abc' + ab'c + ab'c' + a'bc + a'bc' = 0)$
$= (a + b = 0) = (a = 0)\ (b = 0);$

6. $(abc + abc' + ab'c + ab'c' + a'bc' + a'b'c = 0)$
$= (a + bc' + b'c = 0) = (a = 0)\ (b = c);$

7. $(abc + abc' + ab'c + ab'c' + a'bc' + a'b'c' = 0)$
$= (a + c' = 0) = (a = 0)\ (c = 1)$[1];

8. $(abc' + ab'c + ab'c' + a'bc + a'bc' + a'b'c = 0)$
$= (ac' + a'c + ab'c + a'bc' = 0)$
$= (a = c)\ (ac < b < a + c) = (a = b = c);$

9. $(abc' + ab'c + ab'c' + a'bc + a'bc' + a'b'c' = 0)$
$= (c' + ab' + a'b = 0) = (c = 1)\ (a = b);$

10. $(abc' + ab'c + ab'c' + a'bc' + a'b'c + a'b'c' = 0)$
$= (b' + c' = 0) = (b = c = 1).$

Before going any further, it may be observed that when the sum of certain constituents is equal to 0, the sum of the rest is equal to 1. Consequently, instead of examining the sum of seven constituents obtained by ignoring one of the four missing constituents, we can examine the equalities obtained by equating each of these constituents to 1:

11. $(a'b'c' = 1) = (a + b + c = 0) = (a = b = c = 0);$

12. $(a'b'c = 1) = (a + b + c' = 0) = (a = b = 0)\ (c = 1);$

13. $(a'bc = 1) = (a + b' + c' = 0) = (a = 0)\ (b = c = 1);$

14. $(abc = 1) \qquad\qquad = (a = b = c = 1).$

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[1] It will be observed that this cause is the only one which is independent of b; and indeed, in this case, whatever b is, it will always contain a and will always be contained in c. Compare Cause 5, which is independent of c, and Cause 10, which is independent of a.

Note that the last four causes are based on the inclusion

$$0 < 1.$$

The last two causes (15. and 16.) are obtained either by adding *all* the missing constituents or by not adding any. In the first case, the sum of all the constituents being equal to 1, we find

15. $$1 = 0,$$

that is, absurdity, and this confirms the paradoxical proposition that the false (the absurd) implies any proposition (is its cause). In the second case, we obtain simply the given equality, which thus appears as one of its own causes (by the principle of identity):

16. $$ab' + bc' = 0.$$

If we disregard these two extreme causes, the number of causes properly so called will be

$$2^{2^n - m} - 2.$$

46. Forms of Consequences and Causes.—We can apply the law of forms to the consequences and causes of a given equality so as to obtain all the forms possible to each of them. Since any equality is equivalent to one of the two forms

$$N = O, \qquad N' = 1,$$

each of its consequences has the form[1]

$$NX = 0, \qquad \text{or } N' + X' = 1,$$

and each of its causes has the form

$$N + X = 0, \qquad \text{or } N'X' = 1.$$

[1] In § 44 we said that a consequence is obtained by taking a part of the constituents of the first member N, and not by multiplying it by a term X; but it is easily seen that this amounts to the same thing. For, suppose that X (like N) be developed with respect to the n terms of discourse. It will be composed of a certain number of constituents. To perform the multiplication of N by X, it is sufficient to multiply all their constituents each by each. Now, the product of two identical constituents is equal to each of them, and the product of two different constituents is 0. Hence the product of N by X becomes reduced to the sum of the constituents common to N and X, which is, of course, contained in N. So, to multiply N by an arbitrary term is tantamount to taking a part of its constituents (or all, or none).