$$abcd + ab'c'uv + a'bd'uv' + a'cd'u'v + b'c'du'v' = 0$$

or since, by hypothesis, the resultant $abcd = 0$ is verified,

$$ab'c'uv + a'bd'uv' + a'cdu'v + b'c'du'v' = 0.$$

This is an "equation of condition" which the indeterminates u and v must verify; it can always be verified, since its resultant is identically true,

$$ab'c'.a'bd'.a'cd'.b'c'd = aa'.bb'.cc'.dd' = 0,$$

but it is not verified by any pair of values attributed to u and v.

Some general symmetrical solutions, *i. e.*, symmetrical solutions in which the unknowns are expressed in terms of several independent indeterminates, can however be found. This problem has been treated by SCHRÖDER[1], by WHITEHEAD[2] and by JOHNSON.[3]

This investigation has only a purely technical interest; for, from the practical point of view, we either wish to eliminate one or more unknown quantities (or even all), or else we seek to solve the equation with respect to one particular unknown. In the first case, we develop the first member with respect to the unknowns to be eliminated and equate the product of its coefficients to 0. In the second case we develop with respect to the unknown that is to be extricated and apply the formula for the solution of the equation of one unknown quantity. If it is desired to have the solution in terms of some unknown quantities or in terms of the known only, the other unknowns (or all the unknowns) must first be eliminated before performing the solution.

41. The Problem of Boole.—According to BOOLE the most general problem of the algebra of logic is the following[4]:

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1 *Algebra der Logik*, Vol. I, § 24.

2 *Universal Algebra*, Vol. I, §§ 35—37.

3 "Sur la théorie des égalités logiques", *Bibl. du Cong. intern. de Phil.*, Vol. III, p. 185 (Paris, 1901).

4 *Laws of Thought*, Chap. IX, § 8.

Given any equation (which is assumed to be possible)

$$f(x, y, z, \ldots) = 0,$$

and, on the other hand, the expression of a term t in terms of the variables contained in the preceding equation

$$t = \varphi\,(x, y, z, \ldots),$$

to determine the expression of t in terms of the constants contained in f and in φ.

Suppose f and φ developed with respect to the variables $x, y, z \ldots$ and let $p_1, p_2, p_3, \ldots$ be their constituents:

$$f(x, y, z, \ldots) = \mathrm{A}p_1 + \mathrm{B}p_2 + \mathrm{C}p_3 + \ldots,$$
$$\varphi\,(x, y, z, \ldots) = ap_1 + bp_2 + cp_3 + \ldots.$$

Then reduce the equation which expresses t so that its second member will be 0:

$$(t\varphi' + t'\varphi = 0) = [(a'p_1 + b'p_2 + c'p_3 + \ldots)\,t$$
$$+ (ap_1 + bp_2 + cp_3 + \ldots)\,t' = 0].$$

Combining the two equations into a single equation and developing it with respect to t:

$$[(A + a')p_1 + (B + b')p_2 + (C + c')p_3 + \ldots]\,t$$
$$+ [(A + a)p_1 + (B + b)p_2 + (C + c)p_3 + \ldots]\,t' = 0.$$

This is the equation which gives the desired expression of t. Eliminating t, we obtain the resultant

$$Ap_1 + Bp_2 + Cp_3 + \ldots = 0,$$

as we might expect. If, on the other hand, we wish to eliminate $x, y, z, \ldots$ (*i. e.*, the constituents $p_1, p_2, p_3 \ldots$), we put the equation in the form

$$(A + a't + at')p_1 + (B + b't + bt')p_2 + (C + c't + ct')p_3 + \ldots = 0,$$

and the resultant will be

$$(A + a't + at')\,(B + b't + bt')\,(C + c't + ct')\ldots = 0,$$

an equation that contains only the unknown quantity t and the constants of the problem (the coefficients of f and of φ). From this may be derived the expression of t in terms of these constants. Developing the first member of this equation

$$(A + a')(B + b')(C + c')\ldots \times t + (A + a)\,(B + b)\,(C + c)\ldots \times t' = 0.$$

The solution is

$$t = (A + a)\ (B + b)\ (C + c) \ldots + u\,(A'a + B'b + C'c + \ldots).$$

The resultant is verified by hypothesis since it is

$$ABC \ldots = 0,$$

which is the resultant of the given equation

$$f(x, y, z, \ldots) = 0.$$

We can see how this equation contributes to restrict the variability of t. Since t was defined only by the function φ, it was determined by the double inclusion

$$abc \ldots < t < a + b + c + \ldots.$$

Now that we take into account the condition $f = 0$, t is determined by the double inclusion

$$(A + a)\ (B + b)\ (C + c) \ldots < t < (A'a + B'b + C'c + \ldots).$$ [1]

The inferior limit can only have increased and the superior limit diminished, for

$$abc \ldots < (A + a)\ (B + b)\ (C + c) \ldots$$

and

$$A'a + B'b + C'c \ldots < a + b + c \ldots.$$

The limits do not change if $A = B = C = \ldots = 0$, that is, if the equation $f = 0$ is reduced to an identity, and this was evident *a priori*.

42. The Method of Poretsky.—The method of BOOLE and SCHRÖDER which we have heretofore discussed is clearly inspired by the example of ordinary algebra, and it is summed up in two processes analogous to those of algebra, namely the solution of equations with reference to unknown quantities and elimination of the unknowns. Of these processes the second is much the more important from a logical point of view, and BOOLE was even on the point of considering deduction as essentially consisting in the *elimination of middle*

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[1] WHITEHEAD, *Universal Algebra*, p. 63.