This law may be expressed in the following manner:

If $b = a'$, we have $a = b'$, and conversely, by symmetry.

This proposition makes it possible, in calculations, to transpose the negative from one member of an equality to the other.

The law of double negation makes it possible to conclude the equality of two terms from the equality of their negatives (if $a' = b'$ then $a = b$), and therefore to cancel the negation of both members of an equality.

From the characteristic formulas of negation together with the fundamental properties of 0 and 1, it results that every product which contains two contradictory factors is null, and that every sum which contains two contradictory summands is equal to 1.

In particular, we have the following formulas:

$$a = ab + ab', \quad a = (a + b)\ (a + b'),$$

which may be demonstrated as follows by means of the distributive law:

$$a = a \times 1 = a(b + b') = ab + ab',$$
$$a = a + 0 = a + bb' = (a + b)\ (a + b').$$

These formulas indicate the principle of the method of development which we shall explain in detail later (§§ 21 sqq.)

18. Second Formula for Transforming Inclusions into Equalities:—We can now establish two very important equivalences between inclusions and equalities:

$$(a < b) = (ab' = 0), \quad (a < b) = (a' + b = 1).$$

Demonstration.—1. If we multiply the two members of the inclusion $a < b$ by b' we have

$$(ab' < bb') = (ab' < 0) = (ab' = 0).$$

2. Again, we know that

$$a = ab + ab'.$$

Now if $ab' = 0$,

$$a = ab + 0 = ab.$$

On the other hand: 1. Add a' to each of the two members of the inclusion $a < b$; we have

$$(a' + a < a' + b) = (1 < a' + b) = (a' + b = 1).$$

2. We know that

$$b = (a + b)\ (a' + b).$$

Now, if $a' + b = 1$,

$$b = (a + b) \times 1 = a + b.$$

By the preceding formulas, an inclusion can be transformed at will into an equality whose second member is either 0 or 1. Any equality may also be transformed into an equality of this form by means of the following formulas:

$$(a = b) = (ab' + a'b = 0), \quad (a = b) = [(a + b')\ (a' + b) = 1].$$

Demonstration:

$$(a = b) = (a < b)\ (b < a) = (ab' = 0)\ (a'b = 0) = (ab' + a'b = 0),$$

$$(a = b) = (a < b)\ (b < a) = (a' + b = 1)\ (a + b' = 1) = [(a' + b')\ (a' + b) = 1].$$

Again, we have the two formulas

$$(a = b) = [(a + b)\ (a' + b') = 0], \quad (a = b) = (ab + a'b' = 1),$$

which can be deduced from the preceding formulas by performing the indicated multiplications (or the indicated additions) by means of the distributive law.

19. Law of Contraposition.—We are now able to demonstrate the *law of contraposition*,

$$(a < b) = (b' < a').$$

Demonstration.—By the preceding formulas, we have

$$(a < b) = (ab' = 0) = (b' < a').$$

Again, the law of contraposition may take the form

$$(a < b') = (b < a'),$$

which presupposes the law of double negation. It may be expressed verbally as follows: "Two members of an inclusion may be interchanged on condition that both are denied".