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Spontaneous partial breaking of $\mathcal{N} = 2$ supersymmetry and the $U(N)$ gauge model

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Abstract.

We briefly review properties of the $\mathcal{N} = 2$ $U(N)$ gauge model composed of $\mathcal{N} = 1$ superfields. This model can be regarded as a low-energy effective action of $\mathcal{N} = 2$ Yang–Mills theory equipped with electric and magnetic Fayet–Iliopoulos terms. In this model, the $\mathcal{N} = 2$ supersymmetry is spontaneously broken to $\mathcal{N} = 1$, and the Nambu–Goldstone fermion comes from the overall $U(1)$ part of $U(N)$ gauge group. We also give $\mathcal{N} = 1$ supermultiplets appearing in the vacua. In addition, we give a manifestly $\mathcal{N} = 2$ symmetric formulation of the model by employing the unconstrained $\mathcal{N} = 2$ superfields in harmonic superspace. Finally, we study a decoupling limit of the Nambu–Goldstone fermion and identify the origin of the fermionic shift symmetry with the second, spontaneously broken supersymmetry.

§1. Introduction

Supersymmetry has been one of the most attractive ideas in theoretical physics. As string theory does not contain adjustable coupling constants and generically leads to four-dimensional theories with extended supersymmetries, it is important to derive $\mathcal{N} = 1$ supersymmetry from spontaneous partial breaking of extended supersymmetries. However, there is a no-go theorem for partial breaking of extended supersymmetries:

The supercharge algebra:

$$\left\{\bar{Q}^A{}_{\alpha},\ Q_{B\dot{\alpha}}\right\} = 2\delta_{\alpha\dot{\alpha}}\delta^A{}_B H \qquad (A = 1,\ 2,\ \ldots,\ \mathcal{N})$$

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is positive definite, so that
• if $Q_A|0\rangle = 0$ for some A, then $\langle 0|H|0\rangle = 0$. This implies $Q_A|0\rangle = 0$ for all A and supersymmetries are unbroken.
• if $Q_A|0\rangle \neq 0$ for some A, then $\langle 0|H|0\rangle \neq 0$. This implies $Q_A|0\rangle \neq 0$ for all A and supersymmetries are all broken.

This theorem prohibits partial spontaneous breaking of extended rigid supersymmetries.[1] But, there is a loophole to this theorem. We consider the local version of the supercharge algebra, so called, supercurrent algebra :

$$(1) \qquad \int d^3y \left\{ \bar{J}^A_{\dot{\alpha}}(y), J^m_{\alpha B}(x) \right\} = 2(\sigma^n)_{\alpha\dot{\alpha}} \delta^A{}_B T^m_n(x) + (\sigma^m)_{\alpha\dot{\alpha}} C^A{}_B$$

where J and T are the supercurrents and the energy momentum tensor respectively. Note that the supercurrent algebra admits a constant matrix C, which does not break the Jacobi identity.

In [1], Antoniadis, Partouche and Taylor (APT) constructed a $U(1)$ gauge model which realizes the supercurrent algebra (1). This model can be regarded as a low-energy effective action of $U(1)$ Yang–Mills theory equipped with electric and magnetic Fayet–Iliopoulos (FI) terms. In [9, 10], we gave the $U(N)$ generalization of the APT model in terms of $\mathcal{N} = 1$ superfields. The key for it is the special Kähler geometry and the discrete $SU(2)_R$ symmetry. Analyzing the vacua of this model, we find the following properties; the $\mathcal{N} = 2$ supersymmetry and the $U(N)$ gauge symmetry are spontaneously broken to $\mathcal{N} = 1$ and $\prod_{i=1}^n U(N_i)$ respectively; the Nambu–Goldstone fermion comes from the overall $U(1)$ part of $U(N)$ gauge group; the supercurrent algebra develops a space-time independent constant "central charge" in (1). The $\mathcal{N} = 1$ supermultiplets appearing in the vacua are also given. In [11], a manifestly $\mathcal{N} = 2$ symmetric formulation of the model was given by employing the unconstrained $\mathcal{N} = 2$ superfields in harmonic superspace [12], and it is generalized to one coupling with hypermultiplets in adjoint representation.

In [8], some features of "Gauge-Matrix duality" with the use of the $U(N)$ gauge model [9, 11] are discussed. It was conjectured in [5] that non-perturbative quantities in a low energy effective gauge theory can be computed by a bosonic matrix model. This conjecture was confirmed

[1]This theorem is not valid for local supersymmetries because it relies on a positive definiteness of the Hilbert space. Partial breaking of local $\mathcal{N} = 2$ supersymmetry was discussed in a number of papers [7].

in [2] for the case of an $\mathcal{N}=1$ $U(N)$ gauge theory with a chiral superfield Φ in the adjoint representation of $U(N)$. These, the $\mathcal{N}=1$ action is obtained by "soft" breaking of $\mathcal{N}=2$ supersymmetry by adding the tree-level superpotential. The group $SU(N)$ is confined and there is a symmetry of shifting the $U(1)$ gaugino by an anticommuting c-number. It is called "fermionic shift symmetry". Thanks to this symmetry, effective superpotential is written as $W_{\text{eff}} = \int d^2\chi \mathcal{F}$, for some function $\mathcal{F}$. "Gauge-Matrix duality" implies that this function $\mathcal{F}$ is given by the free energy $F_{\text{m.m.}}$ of a bosonic matrix model. The fermionic shift symmetry is due to a free fermion and should be related to a second, spontaneously broken supersymmetry [3, 4]. In [9], it is discussed that a scaling limit generates the fermionic shift symmetry. In fact, it is shown more precisely in [8] (See also [13]) that the fermionic shift symmetry arises from the decoupling limit of the Nambu–Goldstone fermion with partial breaking of $\mathcal{N}=2$ supersymmetry.

This paper is organized as follows. After introducing the $\mathcal{N}=2$ $U(N)$ gauge model in sections 2, we analyze the vacua of the model in section 3. A manifestly $\mathcal{N}=2$ supersymmetric formulation is given in section 4. In the last section we discuss the decoupling limit of the Nambu–Goldstone fermion.

§2. The $\mathcal{N}=2$ $U(N)$ gauge model

The $\mathcal{N}=2$ $U(N)$ gauge model constructed in [9] is composed of $\mathcal{N}=1$ chiral multiplets $\Phi=\Phi^a t_a$ and $\mathcal{N}=1$ vector multiplets $V=V^a t_a$, where $N\times N$ hermitian matrices t_a $(a=0,\dots N^2-1)$ generate $u(N)$, $[t_a,t_b]=if_{ab}^c t_c$.[2] The index 0 refers to the overall $U(1)$ generator. These superfields, Φ^a and V^a, contain component fields (A^a,ψ^a,F^a) and (v_m^a,λ^a,D^a), respectively.[3] This model is described by an analytic function (prepotential) $\mathcal{F}(\Phi)$. The kinetic term of Φ is given by the Kähler potential[4] $K(\Phi^a,\Phi^{*a})=\frac{i}{2}(\Phi^a\mathcal{F}_a^*-\Phi^{*a}\mathcal{F}_a)$. The Kähler metric $g_{ab}\equiv\partial_a\partial_{b^*}K(A^a,A^{*a})=\text{Im}\mathcal{F}_{ab}$ admits isometry $U(N)$ generated by the Killing vector $k_a=k_a{}^b\partial_b=-ig^{bc}\partial_{c^*}\mathfrak{D}_a\partial_b$ where $\mathfrak{D}_a=-ig_{ab}f_{cd}^b A^{*c}A^d$ is the Killing potential. The gauged action is given

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[2] $u(N)$ Cartan generators t_i are normalized as $\text{tr}(t_it_j)=\frac{1}{2}\delta_{ij}$, so that the overall $u(1)$ generator is $t_0=\frac{1}{\sqrt{2N}}\mathbf{1}_{N\times N}$.

[3] We follow the notation of [14].

[4] We denote $\mathcal{F}_a\equiv\frac{\partial\mathcal{F}}{\partial\Phi^a}$, $\mathcal{F}_{ab}\equiv\frac{\partial^2\mathcal{F}}{\partial\Phi^a\partial\Phi^b}$ and so on.

by introducing the counterterm Γ for $U(N)$ gauging as

$$\mathcal{L}_{K+\Gamma} = \int d^2\theta d^2\bar{\theta}(K+\Gamma)\ ,\ \Gamma = \left[\int_0^1 d\alpha e^{\frac{i}{2}\alpha v^a(k_a - k_a^*)} v^c \mathfrak{D}_c\right]_{v^a \to V^a}\ ,$$

which is simply rewritten as $\frac{1}{4}\mathrm{Im}\int d^4\theta(\bar{\Phi}e^{\mathrm{ad}V})^a\mathcal{F}_a$. The gauge kinetic term is

$$\mathcal{L}_{\mathcal{W}^2} = -\frac{i}{4}\int d^2\theta^2 \mathcal{F}_{ab}(\Phi)\mathcal{W}^a\mathcal{W}^b + c.c\ , \tag{2}$$

where $\mathcal{W}^a$ is the gauge field strength of V^a. We also introduce the gauge invariant superpotential term

$$\mathcal{L}_W = \int d\theta^2 W + c.c.\ , \quad W = e\Phi^0 + m\mathcal{F}_0\ , \tag{3}$$

with real constant e and m, and the FI D-term [6]

$$\mathcal{L}_D = \sqrt{2}\xi D^0\ , \tag{4}$$

which does not affect the $\mathcal{N}=2$ supersymmetry. In [9], it is shown that the total action $S = \int d^4x(\mathcal{L}_{K+\Gamma} + \mathcal{L}_{\mathcal{W}^2} + \mathcal{L}_W + \mathcal{L}_D)$ is invariant under the discrete $\mathfrak{R}$ transformation composed of a discrete element of the $SU(2)$ R-symmetry and a sign flip of the FI parameter

$$R : \lambda_I^a \to \epsilon^{IJ}\lambda_J^a \qquad \& \qquad R_\xi : \xi \to -\xi\ , \tag{5}$$

where $\lambda_I^a = \binom{\lambda^a}{\psi^a}$, so that $S^{(+\xi)} \xrightarrow{R} S^{(-\xi)} \xrightarrow{R_\xi} S^{(+\xi)}$. We have made the sign of the FI parameter manifest. This ensures the $\mathcal{N}=2$ supersymmetry of our action. In fact, acting $\mathfrak{R}$ on the first supersymmetry transformation $\delta_{\eta_1}S^{(+\xi)} = 0$, we have, $\delta_{\eta_1}S^{(+\xi)} = 0 \xrightarrow{R} R(\delta_{\eta_1})S^{(-\xi)} = 0 \xrightarrow{R_\xi} \mathfrak{R}(\delta_{\eta_1})S^{(+\xi)} = 0$, which implies that the resulting $\mathfrak{R}$-invariant action is invariant under the second supersymmetry $\delta_{\eta_2} \equiv \mathfrak{R}(\delta_{\eta_1})$ as well. By applying the $\mathfrak{R}$-action on the first supersymmetry transformation, we obtain the $\mathcal{N}=2$ supersymmetry transformation of the fermion as

$$\delta\lambda_J^a = i(\tau\cdot\widetilde{\boldsymbol{D}}^a)_J{}^K\eta_K + \cdots\ ,\ \widetilde{\boldsymbol{D}}^a = -\sqrt{2}g^{ab^*}\partial_{b^*}\left(\mathcal{E}A^{*0} + \mathcal{M}\mathcal{F}_0^*\right) \tag{6}$$

where τ are Pauli matrices. The rigid $SU(2)$ has been fixed by making $\mathcal{E}$ and $\mathcal{M}$ point to specific directions, $\mathcal{E} = (0,\ -e,\ \xi)$ and $\mathcal{M} = (0,\ -m,\ 0)$.

Gathering these all together and eliminating the auxiliary fields by using their equations of motion, the total action of the $\mathcal{N}=2$ $U(N)$ model is given as

$$\mathcal{L}_{\mathcal{N}=2} = \mathcal{L}_{\mathrm{kin}} + \mathcal{L}_{\mathrm{pot}} + \mathcal{L}_{\mathrm{Pauli}} + \mathcal{L}_{\mathrm{Yukawa}} + \mathcal{L}_{\mathrm{fermi}}\ , \tag{7}$$

with

$$\mathcal{L}_{\rm kin} = -g_{ab}\mathcal{D}_m A^a \mathcal{D}^m A^{*b} - \frac{1}{4}g_{ab}v^a_{mn}v^{bmn} - \frac{1}{8}\mathrm{Re}(\mathcal{F}_{ab})\epsilon^{mnpq}v^a_{mn}v^b_{pq}$$
$$+\Big(-\frac{1}{2}\mathcal{F}_{ab}\lambda^a\sigma^m\mathcal{D}_m\bar\lambda^b - \frac{1}{2}\mathcal{F}_{ab}\psi^a\sigma^m\mathcal{D}_m\bar\psi^b + c.c.\Big)\ ,$$
$$\mathcal{L}_{\rm pot} = -\frac{1}{2}g^{ab}\left(\frac{1}{2}\mathfrak{D}_a + \sqrt{2}\xi\delta^0_a\right)\left(\frac{1}{2}\mathfrak{D}_b + \sqrt{2}\xi\delta^0_b\right) - g^{ab}\partial_a W\partial_{b^*}W^*,$$
$$\mathcal{L}_{\rm Pauli} = i\frac{\sqrt{2}}{8}\mathcal{F}_{abc}\psi^c\sigma^m\bar\sigma^n\lambda^a v^b_{mn} + c.c.\ ,$$
$$\mathcal{L}_{\rm Yukawa} = \left(-\frac{i}{4}\mathcal{F}_{abc}g^{cd}\partial_d W - \frac{1}{2}\partial_a\partial_b W\right)\psi^a\psi^b - \frac{i}{4}\mathcal{F}_{abc}g^{cd}\partial_{d^*}W^*\lambda^a\lambda^b$$
$$+\left\{-\frac{1}{4\sqrt{2}}\mathcal{F}_{abc}g^{cd}\left(\mathfrak{D}_d + 2\sqrt{2}\xi\delta^0_d\right) + \frac{1}{\sqrt{2}}g_{ac}k^{*c}_b\right\}\psi^a\lambda^b + c.c.\ ,$$
$$\mathcal{L}_{\rm fermi} = g_{ab}\hat F^a\hat F^{*b} + \frac{1}{2}g_{ab}\hat D^a\hat D^b + \Big(-\frac{i}{8}\mathcal{F}_{abcd}\psi^c\psi^d\lambda^a\lambda^b$$
$$+\frac{i}{4}\mathcal{F}_{abc}\hat F^{*c}\psi^a\psi^b + \frac{i}{4}\mathcal{F}_{abc}\hat F^c\lambda^a\lambda^b + \frac{1}{2\sqrt{2}}\mathcal{F}_{abc}\hat D^c\psi^a\lambda^b + c.c.\Big)\ ,$$

where

$$\left\{\begin{array}{l} \hat D^a \equiv -\frac{\sqrt{2}}{4}g^{ab}\left(\mathcal{F}_{bcd}\psi^d\lambda^c + \mathcal{F}^*_{bcd}\bar\psi^d\bar\lambda^c\right) \\ \hat F^a \equiv \frac{i}{4}g^{ab}\left(\mathcal{F}^*_{bcd}\bar\lambda^c\bar\lambda^d - \mathcal{F}_{bcd}\psi^c\psi^d\right)\end{array}\right. .$$

Here we have defined the covariant derivative as $\mathcal{D}_m\Psi^a \equiv \partial_m\Psi^a - \frac{1}{2}f^a_{bc}v^b_m\Psi^c$ for $\Psi^a \in \{A^a,\psi^a,\lambda^a\}$, and $v^a_{mn} \equiv \partial_m v^a_n - \partial_n v^a_m - \frac{1}{2}f^a_{bc}v^b_m v^c_n$.

§3. Analysis of vacua

The scalar potential $V = -\mathcal{L}_{\rm pot}$ determines the vacua. Let us examine for concreteness the case with

$$\mathcal{F} = \sum_{k=0}^{n}\mathrm{tr}\frac{g_k}{k!}\Phi^k\ , \tag{8}$$

then the the vacuum condition $\partial\mathcal{L}_{\rm pot}/\partial A^a = 0$ is solved by

$$\langle\mathcal{F}_{00}\rangle = \frac{-e \pm i\xi}{m} \tag{9}$$

where $\langle\circ\rangle$ denotes the vacuum expectation value (vev) of $\circ$. Without loss of generality we may choose $+$ sign in (9). By examining the vev of (6), it is revealed that the Nambu–Goldstone fermion exists in the overall $U(1)$ part of $U(N)$ gauge group,

$$\left\langle \delta_{\mathcal{N}=2}\left(\frac{\lambda^0-\psi^0}{\sqrt{2}}\right)\right\rangle = -2im(\eta_1+\eta_2),\ \left\langle \delta_{\mathcal{N}=2}\left(\frac{\lambda^0+\psi^0}{\sqrt{2}}\right)\right\rangle = 0.$$

As seen from $\langle \mathcal{L}_{\text{Yukawa}}\rangle$, $\frac{\lambda^0-\psi^0}{\sqrt{2}}$ is massless, and thus is the Nambu–Goldstone fermion which is included in the overall $U(1)$ part of the $\mathcal{N}=1$ $U(N)$ vector superfield.

In order to examine general vacua, let us denote indices $a=(i,r)$ where i (r) label the (non) Cartan generators of $u(N)$, as depicted in the figure 1. We obtain three types of $\mathcal{N}=1$ supermultiplets in the case

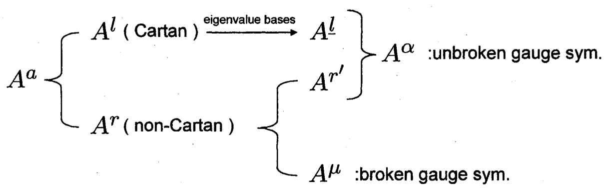

Fig. 1. index labelling

of partial breaking of $U(N)$ gauge symmetry, $U(N)\to\prod_{i=1}^n U(N_i)$. We

field	mass	label	# of polarization states
v_m^α	0	A	$2d_u$ $(d_u\equiv\dim\prod_i U(N_i))$
v_m^μ	$\frac{1}{\sqrt{2}}\lvert f_{\mu\underline{i}}^\nu\langle A^{\underline{i}}\rangle\rvert$	C	$3(N^2-d_u)$
$\frac{1}{\sqrt{2}}(\lambda^\alpha\pm\psi^\alpha)$	0	A	$2d_u$
$\frac{1}{\sqrt{2}}(\lambda^\alpha\mp\psi^\alpha)$	$\lvert m\langle g^{\alpha\alpha}\rangle\langle\mathcal{F}_{0\alpha\alpha}\rangle\rvert$	B	$2d_u$
λ_I^μ	$\frac{1}{\sqrt{2}}\lvert f_{\mu\underline{i}}^\nu\langle A^{\underline{i}}\rangle\rvert$	C	$4(N^2-d_u)$
A^α	$\lvert m\langle g^{\alpha\alpha}\rangle\langle\mathcal{F}_{0\alpha\alpha}\rangle\rvert$	B	$2d_u$
$\mathcal{P}_\mu^{\mu}A^\mu$	$\frac{1}{\sqrt{2}}\lvert f_{\mu\underline{i}}^\nu\langle A^{\underline{i}}\rangle\rvert$	C	N^2-d_u

Table 1. table of the mass spectrum

find the following three types of $\mathcal{N}=1$ supermultiplets. The fields labelled as A in the table form the massless $\mathcal{N}=1$ vector multiplets of spin $(1/2,1)$ composed of fields. The Nambu–Goldstone vector multiplet is contained in the overall $U(1)$ part. Those labelled as B form the massive $\mathcal{N}=1$ chiral multiplets of spin $(0,1/2)$ with masses $\lvert m\langle g^{\alpha\alpha}\rangle\langle\mathcal{F}_{0\alpha\alpha}\rangle\rvert$.

Those labelled as C form two massive multiplets of spin $(0, 1/2, 1)$ with masses $\frac{1}{\sqrt{2}}|f^{\nu}_{\mu\underline{i}}\langle A^{\underline{i}}\rangle|$. The zero modes of A^{μ} are absorbed into v^{μ}_{m} as the longitudinal modes to form massive vector fields.

§4. Description in harmonic superspace formalism

Harmonic superspace [12] provides a manifestly $\mathcal{N}$=2 supersymmetric formulation of $\mathcal{N}$=2 supersymmetric theories in terms of off-shell $\mathcal{N}$=2 unconstrained superfields. In [11], we gave a manifestly $\mathcal{N}=2$ supersymmetric formulation of the $\mathcal{N}=2$ $U(N)$ gauge model discussed above by using $\mathcal{N}$=2 vector multiplets V^{++} in harmonic superspace. The kinetic term of V^{++} is given by

$$S_V = -\frac{i}{4}\int d^4x (D)^4 \mathcal{F}(W) + c.c. \tag{10}$$

where W^a is the curvature of V^{++}. We note the electric FI term $S_e = \int d^4x \xi^{AB} D^0_{AB}$, where D^a_{AB} is the auxiliary field contained in V^{++}, causes an imaginary shift of the auxiliary field contained in the dual vector multiplet $\tilde{V}^{++}$ of $\tilde{W}^a = \mathcal{F}_a$. So we introduce the magnetic FI term S_m so as to shift the auxiliary field in V^{++} by an imaginary constant: $D^a \to \boldsymbol{D}^a = D^a + 4i\xi_D\delta^a_0$, so that $S_V + S_m = S_V|_{D\to\boldsymbol{D}}$. See [11] for detail. These electric and magnetic FI terms cause $\mathcal{N}=2$ supersymmetry to be broken spontaneously to $\mathcal{N}=1$.

In addition, we generalize this gauge model to one coupled with $\mathcal{N}=2$ hypermultiplets, q^+ and ω, in adjoint representation of $U(N)$. Examining vacua of the model, we show that this model also describes partial spontaneous supersymmetry breaking.

§5. Decoupling limit of Nambu–Goldstone fermion

In [8], we derive the $\mathcal{N}=1$ action expanding the $\mathcal{N}=2$ $U(N)$ gauge model around the vacua and taking the limit in which the Nambu–Goldstone fermion is decoupled from other fields. Let us return to the $\mathcal{N}=1$ superfield notation used in sections 2 and 3. We consider the case that $U(N)$ gauge symmetry is unbroken at vacua. This is the case for $d_u = \dim \prod_i U(N_i) = N^2$ so that $N^2 - d_u = 0$. It means that there is no $\mathcal{N}=1$ massive vector supermultiplets (" C " in table 1). Let us examine the case with $\mathcal{F}$ given in (8). The fermions ψ^a and λ^a are to be mixed and the scalar fields A^a are to be shifted by its vacuum expectation value. We define

$$\lambda^{-a} \equiv \frac{1}{\sqrt{2}}(\lambda^a - \psi^a), \quad \lambda^{+a} \equiv \frac{1}{\sqrt{2}}(\lambda^a + \psi^a), \quad \tilde{A}^a \equiv A^a - \langle A^0\rangle\delta^a_0 \ .$$

Substitute these into (7), we obtain the $\mathcal{N}=1$ $U(N)$ gauge action after spontaneous breaking of $\mathcal{N}=2$ supersymmetry,

$$\mathcal{L}_{\mathcal{N}=1} = \tilde{\mathcal{L}}_{\mathrm{kin}} + \tilde{\mathcal{L}}_{\mathrm{pot}} + \tilde{\mathcal{L}}_{\mathrm{Pauli}} + \tilde{\mathcal{L}}_{\mathrm{Yukawa}} + \tilde{\mathcal{L}}_{\mathrm{fermi}}, \tag{11}$$

with

$$\begin{aligned}
\tilde{\mathcal{L}}_{\mathrm{kin}} &= -\tilde{g}_{ab}\mathcal{D}_m\tilde{A}^a\mathcal{D}^m\tilde{A}^{*b} - \frac{1}{4}\tilde{g}_{ab}v^a_{mn}v^{bmn} - \frac{1}{8}\mathrm{Re}(\tilde{\mathcal{F}}_{ab})\epsilon^{mnpq}v^a_{mn}v^b_{pq}\\
&\quad + \Big(-\frac{1}{2}\tilde{\mathcal{F}}_{ab}\lambda^{-a}\sigma^m\mathcal{D}_m\bar{\lambda}^{-b} - \frac{1}{2}\tilde{\mathcal{F}}_{ab}\lambda^{+a}\sigma^m\mathcal{D}_m\bar{\lambda}^{+b} + c.c.\Big),\\
\tilde{\mathcal{L}}_{\mathrm{pot}} &= -\frac{1}{8}\tilde{g}^{ab}\tilde{\mathfrak{D}}_a\tilde{\mathfrak{D}}_b - \tilde{g}^{ab}\tilde{\partial}_a\widetilde{W}\tilde{\partial}_{b^*}\widetilde{W}^*,\\
\tilde{\mathcal{L}}_{\mathrm{Pauli}} &= i\frac{\sqrt{2}}{8}\tilde{\mathcal{F}}_{abc}\lambda^{+c}\sigma^m\bar{\sigma}^n\lambda^{-a}v^b_{mn} + c.c.\ ,\\
\tilde{\mathcal{L}}_{\mathrm{Yukawa}} &= \left(-\frac{i}{4}\tilde{\mathcal{F}}_{abc}\tilde{g}^{cd}\tilde{\partial}_d\widetilde{W} - \frac{1}{2}\tilde{\partial}_a\tilde{\partial}_b\widetilde{W}\right)\lambda^{+a}\lambda^{+b}\\
&\quad - \frac{i}{4}\tilde{\mathcal{F}}_{abc}\tilde{g}^{cd}\tilde{\partial}_{d^*}\widetilde{W}^*\lambda^{-a}\lambda^{-b}\\
&\quad + \left\{-\frac{1}{4\sqrt{2}}\tilde{\mathcal{F}}_{abc}\tilde{g}^{cd}\tilde{\mathfrak{D}}_d + \frac{1}{\sqrt{2}}\tilde{g}_{ac}\tilde{k}^{*c}_b\right\}\lambda^{+a}\lambda^{-b} + c.c.\ ,\\
\tilde{\mathcal{L}}_{\mathrm{fermi}} &= \tilde{g}_{ab}\check{F}^a\check{F}^{*b} + \frac{1}{2}\tilde{g}_{ab}\check{D}^a\check{D}^b + \Big(-\frac{i}{8}\tilde{\mathcal{F}}_{abcd}\lambda^{+c}\lambda^{+d}\lambda^{-a}\lambda^{-b}\\
&\quad + \frac{i}{4}\tilde{\mathcal{F}}_{abc}\check{F}^{*c}\lambda^{+a}\lambda^{+b} + \frac{i}{4}\tilde{\mathcal{F}}_{abc}\check{F}^c\lambda^{-a}\lambda^{-b} + \frac{1}{2\sqrt{2}}\tilde{\mathcal{F}}_{abc}\check{D}^c\lambda^{+a}\lambda^{-b} + c.c.\Big)
\end{aligned}$$

where

$$\begin{aligned}
&\tilde{\mathcal{F}}(\tilde{\boldsymbol{A}}) \equiv \mathcal{F}|_{\boldsymbol{A}=\tilde{\boldsymbol{A}}+\langle A^0\rangle t_0},\ \tilde{\mathcal{F}}_a \equiv \partial\tilde{\mathcal{F}}/(\partial\tilde{A}^a),\ \tilde{\mathcal{F}}_{ab} \equiv \partial^2\tilde{\mathcal{F}}/(\partial\tilde{A}^a\partial\tilde{A}^b), \cdots,\\
&\tilde{g}_{ab} \equiv \mathrm{Im}\tilde{\mathcal{F}}_{ab}\ ,\ \check{D}^a \equiv -\frac{\sqrt{2}}{4}g^{ab}\mathcal{F}_{bcd}\lambda^{+c}\lambda^{-d} - \frac{\sqrt{2}}{4}g^{ab}\mathcal{F}^*_{bcd}\bar{\lambda}^{+c}\bar{\lambda}^{-d},\\
&\tilde{\mathfrak{D}}_a \equiv -i\tilde{g}_{ab}f^b_{cd}\tilde{A}^{*c}\tilde{A}^d\ ,\ \check{F}^a \equiv \frac{i}{4}g^{ab}\mathcal{F}^*_{bcd}\bar{\lambda}^{-c}\bar{\lambda}^{-d} - \frac{i}{4}g^{ab}\mathcal{F}_{bcd}\lambda^{+c}\lambda^{+d},\\
&\widetilde{W} \equiv (e-i\xi)(\tilde{A}^0 + \langle A^0\rangle) + m\tilde{\mathcal{F}}_0,\quad \tilde{\partial}_a \equiv \frac{\partial}{\partial\tilde{A}^a}.
\end{aligned}$$

As a result, the action (11) agrees with the action (7) except for the superpotential term and FI D-term. There is no FI D-term in (11), and the superpotential $W = eA^0 + m\mathcal{F}_0$ get shifted to $\widetilde{W} = (e-i\xi)\tilde{A}^0 + m\tilde{\mathcal{F}}_0$. Component fields $(\tilde{A}^a, \lambda^{+a})$ form massive $\mathcal{N}=1$ chiral multiplets $\tilde{\Phi}^a$. Other component fields (v^a_m, λ^{-a}) form massless $\mathcal{N}=1$ vector multiplets $\tilde{V}^a$. The Nambu–Goldstone fermion λ^{-0} is contained in the overall $U(1)$ part of $\tilde{V}^a$.

Reparametrizing as

$$g_k = \frac{g'_k}{\Lambda}(k \geq 3)\ ,\ (e,\ m,\ \xi) = (\Lambda e',\ \Lambda m',\ \Lambda \xi')$$

and taking the limit $\Lambda \to \infty$, the action (11) is converted into

$$\begin{aligned}(12)\qquad \mathcal{L} =& \mathrm{Im}\left[\frac{-e+i\xi}{m}\left(2\int d^4\theta \mathrm{tr}\tilde{\Phi}^+ e^{\tilde{V}}\tilde{\Phi} + \int d^2\theta \mathrm{tr}\tilde{\mathcal{W}}^\alpha \tilde{\mathcal{W}}_\alpha\right)\right] \\ &+ \left(\int d^2\theta \widetilde{W}(\tilde{\Phi}) + c.c.\right),\end{aligned}$$

where

$$\begin{aligned}\widetilde{W} \equiv& m\sum_{k=1}^{n-2} \frac{h_k}{k+1}\mathrm{tr}\tilde{\boldsymbol{A}}^{k+1}, \\ h_k \equiv& \frac{(k+1)}{\sqrt{2N}} \sum_{\ell=0}^{n-2-k} \frac{g_{k+\ell+2}}{(k+\ell+1)!}\begin{pmatrix} k+\ell+1 \\ \ell \end{pmatrix}\left(\frac{\langle A^0\rangle}{\sqrt{2N}}\right)^\ell .\end{aligned}$$

Here $\tilde{\mathcal{W}}$ is the field strength of $\tilde{V}$. Note that the Nambu–Goldstone fermion λ^{-0} , which is contained in the overall $U(1)$ part of $\mathcal{N}=1$ $U(N)$ vector superfields $\tilde{V}$, is decoupled from other fields in (12). However $\mathcal{N}=2$ supersymmetry is broken to $\mathcal{N}=1$ because of the presence of the superpotential. We have seen that a general $\mathcal{N}=1$ action (12) called a "softly" broken $\mathcal{N}=1$ action can be derived as a spontaneously broken $\mathcal{N}=2$ action. We conclude that the fermionic shift symmetry in [2] is related to the decoupling limit of the Nambu–Goldstone fermion.

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