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Mini Conference on

INVERSE PROBLEMS IN PARTIAL DIFFERENTIAL EQUATIONS

(Canberra, August 23-25, 1990)

Edited by
Amiya K. Pani and Robert S. Anderssen

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PREFACE

As part of the special year devoted to the *Application and Numerical Solutions to Partial Differential Equations*, the Centre for Mathematical Analysis at the Australian National University, Canberra, hosted a Mini-conference on *Inverse Problems for Partial Differential Equations*, on August 23–25, 1990. The primary aim was to stimulate strong interaction between practitioners (scientists, industrialists and engineers) with specific inverse problems and mathematicians (pure, applied and computational) working on the analyais and approximate solution of such problems. The specific points of focus for the conference were regularization for nonlinear problems, tomography, geophysical inverse problems and inverse eigenvalue problems. In addition, the Mini-conference aimed to foster the interest of younger colleagues in research connected with these problems. A number of Australian and overseas speakers were invited to participate. They were:

Dr. Garry Newsam DSTO, Adelaide	Uniqueness and Stability Results for Transmissivity Identification
Dr. Tony Jakeman CRES, ANU	Practical Solution of Some Inverse Problems in Hydrology
Prof. Tolik Yagola Moscow State University, Moscow	Inverse Problems in Astrophysics
Dr. Brian Kennett RSES, ANU	Assessing Quality of Solutions of Inverse Problems: Earthquake Locations
Dr. Beverley Moore University of Technology, Sydney	Inverse Problems in Seismic Surveys
Dr. Geoff Latham CMA, ANU	Diffusion Tomography
Prof. Frank Natterer Universitat Münster	Tomography

Prof. Lee White University of Melbourne	Some Inverse Problems from Colloid Science
Dr. Mark Lukas Murdoch University	Methods for Choosing the Regularization Parameter
Dr. Bob Anderssen CSIRO & CMA, Canberra	Inverse problems in Vibration
Dr. Mike Englefield Monash University	Darboux Transformation and Inverse Scattering
Dr. Ted Lilley RSES, ANU	The Inverse Problem of Electromagnetic Induction in the Earth
Dr. David Hurley University of Western Australia	A First Order Correction to the Theory for the Electromagnetic Responce of a Thin Conducting Sheet
Dr. Tim Monks BHP Melbourne Research Laboratories	Multigrid Techniques for Emission Tomography
Dr. Mark Westcott CSIRO, Canberra	Statistical Methodology for Inverse Problems
Dr. Amiya Pani IIT, Bombay	A Finite Element Galerkin Procedure for Parameter Identification in Parabolic Problems
Dr. David Suter La Trobe University	Coupled Derivative/Mixed Finite Element Approach to Visual Reconstruction
Dr. Sum Chow City Polytechnic, Hong Kong	Resolving the Transmissivity Zonation in a Confined Aquifer
Dr. Ian Enting CSIRO, Canberra	Inverse Problems in Greenhouse Gas Modelling

This volume contains the proceeding of the Mini-conference. The papers are arranged in their order of presentation at the conference. Alternatively, the papers could have been organised in terms of the focus they gave to deliberations about the application and numerical solution of inverse problems. In particular, such a reorganization would fall naturally under the following headings: inverse problems from physics, inverse problems in hydrology, tomography and methodology (theoretical and computational).

The challenge of inverse problems is their improperly posedness.

Inverse Problems in Physics. Inverse problems arise naturally in a great variety of physical applications including astrophysics, geophysics, seismology, colloidal and surface science, marine geophysics, ionospheric physics, atmospheric science and geophysical exploration.

Astrophysics deals with immensely remote objects such as stars and galaxies, the properties of which can only be measured by observations from the Earth's surface or spacecraft in orbit. Hence, inverse problems must be solved in order to interprete the observed data. Professor Yagola discussed such problems connected with the interpretation of the observed properties of binary systems, and problems in vibrational spectroscopy.

Earthquake location is an important practical inverse problem. Professor Kennett addressed the problem of determining the origin-time and spatial location of the hypocentre of an earthquake from data on the arrival time of the seismic waves generated by the shock.

Seismic surveys are commonly used in oil and natural gas exploration to help picture the subsurface structure in the region of interest. One way of displaying recorded data is by means of an unmigrated stacked section. However, this gives a distorted picture of the subsurface substructure, because of the refractive effects produced by the variations in velocity. Therefore, it is necessary to determine a good coarse-scale velocity model for use in migrating the data to its true location. Dr. Moore discussed a generalised inversion technique for deriving a coarse-scale velocity model.

Much of our modern understanding related to the Earth's interior comes from the determination of the electrical conductivity of rocks beneath the oceanfloor. Drs. Lilley and Heinson presented and compared the inversion of seafloor magntolelluric measurement data by four published methods in order to obtain reliable information on the electrical conductivity structures in the subsurface.

In climate studies, the influence of the greenhouse gases such as CO_2,

NO_2 and CH_4, on the Earth's radiation budget must be estimated from indirect measurements. The underlying inverse problems and methods for their solution was examined by Dr. Enting. In particular, he examined the problem of estimating the atmospheric CO_2 budget.

Inverse Problems in Hydrology. Of the three papers presented in this category, the first discusses theoretical issues like existence, uniqueness and stability in transmissivity identification, while the second addresses the special features and difficulties associated with the practical solutions of inverse problems in hydrology. Finally, the third paper considers transmissivity zonation in a confined aquifer and related stability issues.

Parameter identification plays a crucial role in the study of ground water flow in aquifers. Dr. Newsam discussed the transmissivity identification problem arising from the study of steady-state flow of groundwater in a confined aquifer. Because the solutions can be very sensitive to errors in measurements of the piezometric head or to errors in the model, the choice of computational methods need to be preceeded by a rigourous mathematical analysis in order to ensure their reliability. Such issues were addressed by Dr. Newsam.

Dr. Jakeman *et al.* outlined the major features of hydrological systems and their mathematical implications. In the later part of their paper, they addressed practical considerations and major difficulties arising in environmental modelling. Finally, using three case studies, they showed that hydrologically-useful solutions of the forward and inverse problems for flow and transport could often be obtained even when there were high levels of uncertainty.

For aquifers having a zonation structure, with the transmissivity varying smoothly and slowly over each zone, a common approach in determining transmissivity is to presume a known zonation structure and seek a constant approximation to the transmissivity in each zone. However, this procedure is not always acceptable as it may lead to instability in the estimation process. By adapting the linear functional strategy proposed by Anderssen and Dietrich (1987), Drs. Anderssen and Chow discussed how to determine simultaneously the zonation structure alongwith a piecewise constant representation of the transmissivity. They also examined the sensitivity of the estimated transmissivity with respect to different choices of test functions, and related stability issues.

Tomography. There are two papers in this category. One is related to diffusion tomography, while the other is related to emission computed tomography. Both these problems have applications in diagnostic medical imaging.

Diffusion tomography attempts to reconstruct the internal properties of a body from external measurements such as in X-ray computed tomography (X-ray CT). Diffusion tomography is a model of emission tomography where the model incorporates the paths of the scattered diffused radiation. Dr. Latham *et al.* proposed a probabilistic model for the passage of the radiation through a discrete lattice of pixels, in which the radiation was allowed to scatter from the pixels in only certain fixed directions. The forward and inverse problems for this model were discussed alongwith some numerical results.

Emission computed tomography (ECT) aims to reconstruct a specific *in vivo* metabolic activity from measurements of the emission from the radioactive pharmaceutial being used to track the activity. From the point of view of image reconstruction, a major difference between ECT and X-ray CT is the number of detected photons. In ECT, it is much less than for X-ray CT. Thus,the measured emission is quite noisy. For such reasons, statistically-based methods, rather than the usual inverse Radon transform, have become popular, such as the EM (Estimate and Maximize) algorithm. Unfortunately, such algorithms are not necessarily well-behaved, usually converge very slowly and tend to give poor reconstructions. Dr. Monks examined the use of multigrid methods for improving the efficiency and speed of convergence of the EM algorithm.

Methodology. This category is subdivided into Theoritical Methods and Computational Techniques.

(i) Theoretical Methods. There are four papers in this subcategory. Dr. Englefield examined the application of the Darboux transformation to inverse scattering problems such as the Schrödinger equation and the Korteweg-de Vries equation.

When the depth of penetration of the primary magnetic field is much greater than the thickness of a thin conducting sheet, the eddy-currents are induced in the sheet by a sinusoidally varing primary field of low frequency.

For such problems, Dr. Hurley discussed perturbation technique in terms of a small parameter, which was the ratio of the thickness of the sheet to the length scale of the primary magnetic field.

Dr. Westcott examined the following two statistical aspects of inverse problems: (a) how statistical ideas and methods link with some of the existing formulations for inverse problems; (b) if there was a natural setting for a problem, which would often include appropriate constraints, this should be exploited in any formulation or analysis.

A popular approach to the solutions of inverse problems, which have a natural operator equation setting, is to stabilize the original problem using Tikhonov regularization with an appropriate choice of regularizor and regularization parameter. Dr. Lukas discussed and compared four important methods for choosing the regularization parameter: the unbiased risk estimation, the discrepancy principle, generalized cross-validation, and generalized maximum-likelihood.

(ii) Computational Techniques. Computational solutions play a crucial role in understanding mathematical models. The direct application of Lanczos type method to a discretized inverse Sturm-Liouville problem is not useful since the eigenvalues of the discrete problem behave quite differently asymptotically from the eigenvalues of the continuous problem. Professor Natterer in his paper proposed a multiplicative asymptotic correction for the discrete equation and discussed an algorithm similar to the Lancoz algorithm alongwith numerical results.

In multigrid methods, the aim is to use a system of multi-level grids to increase the rate of convergence of the iterative methods of solution being applied to the discretization of the given problem. Dr. Monks examined the choice of grids for the implementation of a multigrid method to improve the EM algorithm in image reconstruction.

By their nature, because the observational data is two-dimensional, visual reconstruction problems, where one aims to recover three-dimensional information, are improperly posed. Dr. Suter examines the use of Tikhonov regularization to stabilized the recovery of information in visual reconstruction using a finite element approach.

Drs. Anderssen and Chow in their paper discussed the implementation of the linear functional strategy to determine simultaneously the zonation structure and transmissivity in a confined acquifer. This implementation results in a highly adaptive and parallelizable procedures. Further, a generalization of the linear functional strategy using a Petrov-Galerkin interpretation was also discussed along with numerical results.

In the literature, the output-error-criteria procedure has been proposed and analysed for the solution of inverse problems. Such procedures are iterative in nature. There in no *a priori* criteria for determining when to stop the iteration. In addition, such procedures tend to converge very slowly. One way around such difficulties is to obtain a good independent initial estimate of the solution. This is the point of focus of the paper by Drs. Pani and Anderssen. They construct a non-iterative numerical procedure for the initial estimate, examine the use of the finite element method for the parameter identification, when the problem is parabolic, and derive *a priori* error estimates.

In conclusion, the editors would like to thank the participants in the Mini-conference for their contributions, and in particular, the speakers, and authors of the papers in this Proceeding. The editors also wish to acknowledge the support and assistance of the Centre for mathematical Analysis. Finally, a special thanks to Professor Neil Trudinger (the Director of the Centre), Marilyn Gray and Jillian Smith, for their contributions in ensuring the success of the Mini-conference.

Amiya Pani, Department of Mathematics, IIT, Bombay

Bob Anderssen, DMS, CSIRO (CMA, ANU, Canberra)