THE MAGNETIC RELIEF PROBLEM

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1. Introduction

Airborne magnetic surveys represent a fast and inexpensive reconnaissance method of gathering information about the main signatures of the magnetic field caused by geological structures in the subsurface.

In regions of sedimentary basins, the variations in the magnetic field are mainly due to the igneous basement rocks and can be considered as arising from two causes: from lateral changes in the magnetisation of the rocks, and from the relief of the basement structure. Experimental evidence (see Nagata [1953]) seems to indicate that, for most igneous rock masses, the magnetisation can be taken to be parallel to the inducing field. One may therefore assume that the variations in the magnetic field are predominantly due to the relief of the basement structure. The following problem, which we address here, therefore arises:

> The determination of the depth to the basement rocks, their relief, and the occurrence of steep gradients in the relief, from collected airborne magnetic data.

Work in this area started a few decades ago, and, because of its great importance in exploration geophysics, has attracted renewed attention more recently.

During the preparation of this workshop, the authors noticed that the only explicit formulation of the (restricted two-dimensional) problem in the literature had been derived incorrectly (Peters [1949]). As a consequence, our first priority became to derive a (three-dimensional) mathematical formulation of the problem from first principles. This resulted in an integral equation for the magnetic field; the field depends linearly on the derivatives of the relief and nonlinearly on the relief itself.

At the presentation to the workshop, we restricted attention to this new formulation and some associated approximations. At that stage, little thought had been given to solution techniques or numerical evaluations. The first aim of the workshop for us was to obtain a better understanding of the presented mathematical formulation of the problem in order to arrive at a mathematically tractable inverse problem as well as preserving a physically realistic model. In a general sense, the discussions of the workshop helped to improve the presentation of this report. The more explicit progress of the workshop is presented in Section 4.

2. Statement of the Problem

2.1 Airborne magnetic surveys

There are estimates that, between 1961 and 1978, about 36 million line-kilometres of aeromagnetic data have been collected by various organisations around the world. At present, nearly all of Australia is covered by airborne magnetic surveys; between 1951 and 1985, about 3.5 million line-kilometres of aeromagnetic data were collected by government organisations. Government airborne surveys in Australia are usually made from fixed wing aircraft flying at the altitude of 150m above the ground along lines spaced about 1500-3000m apart (see Figure 1). The pilots try to keep a constant altitude relative to the ground terrain by following its topography, but in rugged terrain the altitude of the aircraft can vary by as much as 100m.

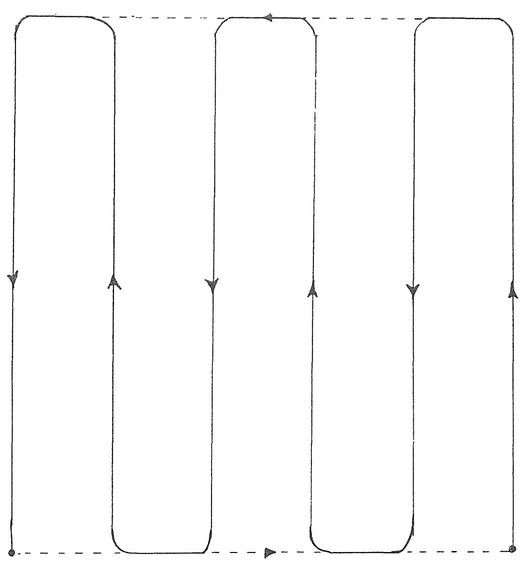

Figure 1. Typical flight path during an aeromagnetic survey.

The amplitude of the total magnetic field is sampled very densely in the direction of the flight (every sec). This gives a sampling density of approximately 50-60m along flight lines (depending on the speed of the aircraft). The accuracy of the magnetic field measurements varies from 0.1 - 1nT (the Earth's field varies from 30,000 - 70,000nT).

The location of the position, at which the measurements are made, is recovered from aerial photographs and Doppler navigation. The accuracy of determining the location along flight lines varies from 1 - 5m. However, the accuracy of recovering the position in the direction perpendicular to the flight lines (see Figure 1) is much less precise and varies from 50m up to 300m in exceptional cases.

A typical aeromagnetic survey covers an area of 1 degree latitude and 1.5 degrees longitude and contains about 12,000 line-kilometres covered by about 200,000 data points. The data are characterised by three numbers: 2 for the position and 1 for the amplitude of the magnetic field.

The first step in an interpretation of the measured data is to preprocess the raw data. This includes reducing the magnetic data by removing diurnal changes, by levelling the data and by gridding it onto a regular grid. There exists a whole spectrum of different preprocessing techniques, but these are beyond the scope and interest of the present discussion. Different approaches will provide a more or less accurate set of values of the magnetic field; from hereon we shall simply assume that such values are available together with some estimate of the likely error.

2.2 The derivation of the relief equation

We select a cartesian co-ordinate system with the x and y axis along the geographic north and east directions respectively, and with the z axis pointing vertically downward. For notational convenience we assume that the measurements take place in the plane $z = 0$ at points $(x,y,0)$. Points in the magnetic rocks in the subsurface are denoted by (α,β,γ) (see Figure 2).

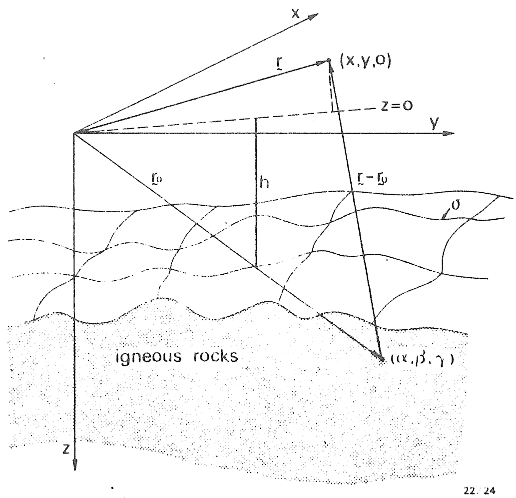

Figure 2. The co-ordinate system used in the derivation of the full three-dimensional relief equation.

The potential U at $(x,y,0)$ induced by a magnetic body of volume V is given by

$$(1) \qquad U(x,y,0) = \int_V J(\alpha,\beta,\gamma)\ \nabla_0 \left[\frac{1}{r-r_0}\right] dv$$

where

$$r-r_0 \equiv [(x-\alpha)^2 + (y-\beta)^2 + \gamma^2]^{1/2} ,$$

$$\nabla_0 \equiv \mathbf{i}\frac{\partial}{\partial\alpha} + \mathbf{j}\frac{\partial}{\partial\beta} + \mathbf{k}\frac{\partial}{\partial\gamma} ,$$

and $\mathbf{J}$ denotes the magnetisation vector in V . The magnetic field vector $\mathbf{H}$ is related to the potential by

(2) $$\mathbf{H}(x,y,0) = - \nabla U(x,y,0) ,$$

where

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z} .$$

Substituting (1) into (2) leads to

(3) $$\mathbf{H}(x,y,0) = - \int_V \mathbf{J}(\alpha,\beta,\gamma)\ \nabla_0\ \nabla \left[\frac{1}{r-r_0}\right] dv ,$$

since $\nabla\ \mathbf{J} = 0$ and $\nabla\ \nabla_0 = \nabla_0\ \nabla$.

If one assumes that $\mathbf{J}$ satisfies $\mathrm{div}_0(\mathbf{J}) = \nabla_0\ \mathbf{J} = 0$ (see also the comments made in the introduction about $\mathbf{J}$), (3) becomes

(4) $$\mathbf{H}(x,y,0) = -\int_V \nabla_0 \left[\mathbf{J}(\alpha,\beta,\gamma)\ \nabla\left[\frac{1}{r-r_0}\right]\right] dv$$

Applying Green's lemma to replace the volume integral in (4) by a surface integral over the surface S of the body V gives

(5) $$\mathbf{H}(x,y,0) = -\int_S \mathbf{J}(\alpha,\beta,\gamma)\ \mathbf{n}(\alpha,\beta,\gamma)\ \nabla(\frac{1}{r_0-r})\ ds$$

where $\mathbf{n}$ denotes the outward unit vector normal to the surface S . As most rock formations are very large and extend deep into the subsurface, the integral in (5) can be approximated by an integral over the top surface σ of the rock formation:

(6) $$H(x,y,0) = -\int_{\sigma} J(\alpha,\beta,\gamma)\ \mathbf{n}(\alpha,\beta,\gamma)\ \nabla(\frac{1}{r-r_0})\ ds\ .$$

Thus the field H can be expressed as a surface integral over the relief function, as the latter represents the top surface σ (see Figure 2).

Let

(7) $$dH(x,y,0) = -J(\alpha,\beta,\gamma)\ \mathbf{n}(\alpha,\beta,\gamma)\ \nabla(\frac{1}{r-r_0})\ ds$$

denote the contribution to the field from the point (α,β,γ) on the surface σ . To derive a formulation for dH in terms of the relief function we now consider the individual terms in (7). Let h denote the average depth from the plane $z = 0$ to the basement relief, and let f denote the deviation of the relief from h . A point r_0 on the surface σ is then given by $r_0 = (\alpha,\beta,h+f(\alpha,\beta))$, and the surface σ is described by the equation

$$\sigma(\alpha,\beta) = h + f(\alpha,\beta)\ .$$

For such a surface, the equation of the outward normal $\mathbf{n}$ is given by

(8) $$\mathbf{n} = (f_\alpha^2 + f_\beta^2 + 1)^{-1/2}\ (f_\alpha, f_\beta, -1)$$

(see Spain [1965], ch6.38), where $f_\alpha \equiv \frac{\partial f}{\partial \alpha}$ and $f_\beta \equiv \frac{\partial f}{\partial \beta}$. The change from surface co-ordinates ds to cartesian co-ordinates is given by the transformation.

(9) $$ds = (f_\alpha^2 + f_\beta^2 + 1)^{1/2}\, d\alpha d\beta$$

(see Jeffreys & Jeffreys [1980]).

Substituting (8) and (9) into (7), and recalling that

$$\nabla\left(\frac{1}{r-r_0}\right) = (r-r_0)^{-3}\,(r-r_0) \ ,$$

one obtains

(10) $$dH(x,y,0) = -(J_\alpha f_\alpha + J_\beta f_\beta - J_\gamma)\,\frac{r-r_0}{(r-r_0)^3}\, d\alpha d\beta \ ,$$

where $J(\alpha,\beta,\gamma) = (J_\alpha, J_\beta, J_\gamma)$.

The equation of the magnetic field now becomes

(11)
$$H(x,y,0) = -\iint_{-\infty}^{\infty}(J_\alpha f_\alpha + J_\beta f_\beta - J_\gamma)\frac{[x-\alpha, y-\beta, -(h+f(\alpha,\beta))]}{[(x-\alpha)^2+(y-\beta)^2+(h+f(\alpha,\beta))^2]^{3/2}}\, d\alpha d\beta \ .$$

It is clear from (11) that the magnetic field depends nonlinearly on the relief function f , as well as on the derivatives of f . (In the Appendix a more detailed derivation of the restricted two-dimensional problem is given together with Peters' [1949] derivation.)

3. Related Linear Inverse Problems

3.1 Approximations to the relief equation

An integral equation of the form of equation (11) cannot, in general, be solved directly. No known inversion formulae exist for this type of integral, so its nonlinear nature requires the use of iterative methods of solution (see Ortega & Rheinboldt [1970]). Therefore, instead of attempting to find appropriate solution techniques straight away, we first further analyse (11).

From (11) it is clear that the magnetic field $\mathbf{H}$ is a function of the magnetisation vector $\mathbf{J}$, the depth h to the relief, and the relief function and its partial derivatives. Unless further assumptions are made, the problem is highly underdetermined. Therefore, since our main concern is to recover the relief, we shall assume here that an estimate for the magnetisation is available and that the magnetisation is constant in the region of interest (these assumptions are justified in view of the comments made in Section 1).

For airborne magnetic field measurements, various methods are now available for calculating the (average) depth h to the magnetic sources. The main three approaches commonly used are spectral analysis, error minimisation between the measured field and the field calculated from simple geometric reliefs (see Bhattacharyya [1980]), and finally the more heuristic method of characteristics (see Äm [1972]).

These assumptions about $\mathbf{J}$ and prior knowledge of h reduce (11) to a nonlinear integral equation for the relief function. Some comments concerning possible solution techniques for this equation can be found in section 4.

Sometimes it is advantageous to linearise (11) and to solve the resulting approximate inverse problem instead. A linearisation can be achieved if one assumes that $|f(\alpha,\beta)| \ll h$ (i.e. the variations in the relief are small compared to the average depth from the plane of observation to the igneous rocks). Replacing $|(x-\alpha)^2 + (y-\beta)^2 + (h+f(\alpha,\beta))^2|^{3/2}$ by $|(x-\alpha)^2 + (y-\beta)^2 + h^2|^{3/2}$ in (11) results in the linearised field equation

$$\text{(12)}\quad H'(x,y,0) = -\iint_{-\infty}^{\infty}(J_\alpha f_\alpha + J_\beta f_\beta - J_\gamma)\,\frac{(x-\alpha,y-\beta,-(h+f(\alpha,\beta)))}{[(x-\alpha)^2+(y-\beta)^2+h^2]^{3/2}}\,d\alpha d\beta\ .$$

A further simplification can be obtained if one replaces $h + f(\alpha,\beta)$ by h in the numerator, too:

$$\text{(13)}\quad H''(x,y,0) = -\iint_{-\infty}^{\infty}(J_\alpha f_\alpha + J_\beta f_\beta - J_\gamma)\,\frac{(x-\alpha,y-\beta,-h)}{[(x-\alpha)^2+(y-\beta)^2+h^2]^{3/2}}\,d\alpha d\beta\ .$$

As can be seen, (13) takes the form of a convolution equation with known kernel, and therefore, at least in theory, it may be solved exactly via Fourier deconvolution. This inversion, however, is unstable, since the convolution kernel has an unbounded inverse. Nevertheless the linearised equation is still useful in obtaining approximate bounds on the ill-posedness of the original nonlinear problem (see also Section 3.2 and Section 4).

To conclude this section, it is worth pointing out the relationship between equation (13) and the (geophysically) well-known equation of the downward continuation which relates the magnetic field in a plane of observation to the field in a lower plane. (Note that the following equation is usually just given for the z-component of the field, but it is presented here vectorially to enable comparisons with the above.)

$$(14) \qquad \mathbf{H}^D(x,y,0) = \iint_{-\infty}^{\infty} H_z(\alpha,\beta,h) \frac{(x-\alpha, y-\beta, h)}{[(x-\alpha)^2+(y-\beta)^2+h^2]^{3/2}} \, d\alpha \, d\beta \; .$$

Equation (14) can be obtained from (6) under the assumption that the relief is constant (i.e. $f(\alpha,\beta) \equiv 0$). Here H_z denotes the z component of the field at height h, the component of primary interest to geophysicists. The result is a convolution equation with a symmetric kernel. The solution of such equations is an area of active research, and many different approaches have been attempted including Wiener filtering (see Chittineni [1984]), matrix perturbation methods (see Silva and Hohmann [1984]), Backus-Gilbert techniques (see Huestis & Parker [1979]) and Direct Surface Smoothing (see Koch & Anderssen [1986]).

3.2 Ill-posedness of the inverse problems

To estimate the degree of ill-posedness to be encountered in the field equation (11), we now consider (13) and (14). For reasons of simplicity, only the z-component part of the magnetic fields in each equation, denoted here by h'' and by h^D respectively, are used. We now rewrite (13) and (14) as follows

$$(13a) \qquad h''(x,y,0) = [(c_1 \frac{d}{dx} + c_2 \frac{d}{dy})f(x,y) - c_3] * k(x,y)$$
$$\equiv \mathcal{K}(\mathcal{D}f(x,y) - c_3)$$

$$(14a) \qquad h^D(x,y,0) = h^D(x,y,h) * k(x,y) \equiv \mathcal{K}(h^D(x,y,h))$$

where

$$(15) \qquad k(x,y) = h\ (x^2 + y^2 + h^2)^{-3/2}$$

and c_1, c_2, c_3 are constants ($J = (c_1, c_2, c_3)$).

The spectrum of the linear operator $\mathcal{K}$ can be described explicitly, since $\mathcal{K}$ is given by a convolution kernel and can therefore be written as a multiplication operator in the Fourier domain, i.e.

$$(16) \qquad \mathcal{K} = \mathcal{F}^{-1}\mathcal{M}\mathcal{F} ,$$

where $\mathcal{F}$ is the Fourier transform and

$$(17) \qquad \mathcal{M}\hat{g}(u,v) = \exp[-h(u^2+v^2)^{1/2}]\ \hat{g}(u,v) .$$

The last equation implies that $\mathcal{K}$ has an continuous spectrum, and does not have a bounded inverse. The degree of the ill-posedness of (14a) may now be estimated from the exponential decay of the spectrum of $\mathcal{K}$.

Formally, (13a) involves the same kernel as (14a). One crucial difference between the two equations, however, is that $\mathcal{K}$ is applied to the function directly in the equation of downward continuation, while $\mathcal{H}$ acts on the derivatives of the function to be sought in the case of the field equation (13a). Because of the form of the kernel k, (13a) can be rewritten as a function which involves f only. Integration by parts leads to the following expression for h'', which we call the <u>linear field equation</u>:

$$(18) \qquad h''(x,y,0) = f(x,y) * [(c_1 \frac{d}{dx} + c_2 \frac{d}{dy})k(x,y] - c_3 * k(x,y)$$
$$\equiv (\mathfrak{D}\mathcal{H})(f(x,y)) - \mathcal{H}c_3$$

with

$$(19) \qquad \mathfrak{D}\mathcal{H}(x,y) = 3h(c_1x + c_2y)(x^2 + y^2 + h^2)^{-5/2} .$$

Equation (18) is a convolution equation, and since it is linear in f, $\mathfrak{D}\mathcal{H}$ can be expressed as a multiplication operator in the Fourier domain, i.e.

$$(20) \qquad \mathfrak{D}\mathcal{H} = \mathcal{F}^{-1}\mathcal{M}''\mathcal{F} ,$$

where

$$(21) \qquad \mathcal{M}''\hat{g}(u,g) = (c_1u + c_2v) \exp[-h(u^2 + v^2)^{1/2}] \hat{g}(u,v) .$$

We first note that the operator $\mathfrak{D}\mathcal{H}$ is not positive, although it is still symmetric (a property it shares with the positive operator $\mathcal{H}$). Equation (21) furthermore shows that $\mathfrak{D}\mathcal{H}$ is not invertible, since zero belongs to its spectrum. A comparison between the two operators (and hence between the downward continuation and the linear field equation) shows that both spectra decay exponentially, but that the spectrum of $\mathfrak{D}\mathfrak{H}$ decreases slightly more slowly.

4. Summary of Workshop Discussions

4.1 Interpretation of the magnetic field measurements

During the first part of the workshop discussions, a clearer understanding of the actual physical measurements made in practice crystalised. We therefore start by summarising this and outlining its relevance to the field equation (11).

Natural rocks in the subsurface of the Earth usually possess the properties of paramagnetic material (i.e. they are weakly magnetic materials with a small, but positive, susceptibility). In the presence of an external (primary) magnetic field (here the existing magnetic field H_0 of the Earth), a paramagnetic body acquires a magnetic moment and an induced magnetisation J (i.e. a magnetic moment per unit volume). The induced magnetisation J is defined over the volume of the magnetic body and is proportional to the primary or inducing field, namely $J = kH_0$, where k denotes the (dimensionless) susceptibility and $k \in [10^{-5}, 10^{-3}]$. Let H denote the anomalous field arising from the magnetisation J. At an observation point r, the anomalous field H depends on J and the distance between r and the magnetic body; it decreases as this distance increases, and as $r \to r_0$, $H(r) \to J(r_0)$ (see Figure 2 for notation).

Close to a magnetic body, the total magnetic field becomes

$$\mathbf{B} = \mathbf{H}_0 + \mathbf{H} \ , \tag{22}$$

while $\mathbf{B} \sim \mathbf{H}_0$ a long distance away from the magnetic body. Note that $\mathbf{B}$ is sometimes also referred to as the magnetic field strength, the magnetic intensity or the magnetic induction.

In aeromagnetic surveys, usually the total magnetic field $\mathbf{B}$ is measured. Generally, one is not interested in this total field, but rather in the small variations caused by magnetic bodies (such as ore bodies) which are placed in the primary magnetic field $\mathbf{H}_0$ of the Earth. The magnitude of $\mathbf{B}$ is about 25.000 - 70.000 nT, while contributions from the anomalous field range from only a few nT up to several thousand nT.

In practice, one often assumes that the susceptibility of the magnetic bodies is constant in the region of interest. Furthermore, the direction and magnitude of the primary magnetic field $\mathbf{H}_0$ of the Earth is assumed to be known. As part of the preprocessing (see Section 2.1), the Earth's magnetic field is subtracted from the survey measurements. The remaining anomalous field (see (22)) is then used for further analysis.

From the above comments, it follows that the field on the l.h.s of (11) is to be interpreted as the anomalous field. Equation (11) therefore represents the functional relationship between the anomalous field, the induced magnetisation of the body, its relief, and lastly the distance between the magnetic body and the observation point.

It is easy to check that the anomalous field satisfies the requirements made at the beginning of this section:

(i) As $|r-r_0| \to \infty$, where r_0 denotes points on the relief surface σ , $|H(r)| \to 0$, because of the term $|r-r_0|^{-2}$.

(ii) As $r \to r_0$, note that $(r-r_0)/(r-r_0)^3$ tends to the δ-function in three dimensions. It follows that $|H(r)| \to$ $|J_\alpha f_\alpha + J_\beta f_\beta - J_\gamma|$.

Note that the anomalous field converges to the weighted magnetisation, where the weights are given by the direction of the normal n on the surface σ (see also (6) & (8)). This is to be expected, since we have assumed that the magnetisation is nearly constant and that the relief function is the main cause of the anomalous field.

A similar interpretation of the equation of the downward continuation (equation (14)) can also be made. Equation (14) relates the anomalous fields in different planes. Since it can be shown that the anomalous field in the plane $z = h$ is the magnetisation (recall that a constant relief in the plane $z = h$ is the underlying assumption in downward continuation), the equation of the downward continuation also relates the anomalous field at an arbitrary height to the magnetisation J of the magnetic body.

4.2 Some computational aspects

In airborne magnetic surveys, a typical data set would be collected over an area covering about 500 x 600 km, thus measurements are restricted to this region. Let $R = [-a,a] \times [-b,b]$ denote such a surveyed region. In principle, one would like to determine the relief over the same region R at average depth h. If

$$\pi_R(x,y) \equiv \begin{cases} 1 & \text{if } (x,y) \in \mathbb{R} \\ 0 & \text{if } (x,y) \notin \mathbb{R} \end{cases}$$

denotes the window function for R, we denote by g_R the restriction of a function $g \in L^2(\mathbb{R})$ to the region R, i.e.

$$g_R(x,y) \equiv \pi_R(x,y)\, g(x,y) \; .$$

Thus in practice (13a) is replaced by the following equation

$$h''_R = \pi_R \mathfrak{D}\mathfrak{H}(f) \; . \tag{23}$$

This is now an equation over finite regions and the associated operator is now a compact operator [Baker, 1977]; that is, it has a spectrum consisting of an infinite but discrete set of eigenvalues, as opposed to the continuous spectra of $\mathfrak{H}$ and $\mathfrak{D}\mathfrak{H}$. If f is expanded as an infinite sum of eigenfunctions of $\pi_R \mathfrak{D}\mathfrak{H}$ only coefficients of components corresponding to eigenvalues that are significantly greater than the error levels in the data will be accurately recovered. The remaining coefficients will be hopelessly corrupted by data errors.

Fortunately it is possible to estimate the distribution of the eigenvalues in terms of the transform of the kernel (given in (21)) and the region R. In particular they depend on the depth h and on the area of R (see Newsam and Barakat, 1985).

However (23) is still not an accurate statement of the practical problem as, while measurements have been restricted to a finite region, the equation does not impose any limits on the extent of reconstructions of the relief function f. Obviously it is not possible to determine the relief function everywhere from survey data on a finite region. Therefore one would like to replace (23) by

$$h''_R = \pi_R \mathcal{D}\mathcal{K} \pi_S (f) \tag{24}$$

where $S = [-a',a'] \times [-b',b']$ is the region over which f is to be reconstructed.

For this replacement to be valid S must be chosen so that the difference between the operators $\pi_R \mathcal{D}\mathcal{K}$ and $\pi_R \mathcal{D}\mathcal{K} \pi_S$ is of the same order as the noise level in the data. However, the question of exactly which sets satisfy this requirement is as yet unsettled. In particular, while one would like to take $S = R$, it is not clear that this choice will not introduce unacceptably large truncation errors.

Nevertheless, given the size of R and S and the average depth, one can calculate the decay rate of the eigenvalues of the finite problem (24), and hence obtain an estimate of the ill-posedness of the associated computational problem that is to be solved. It is clear that the inverse problem becomes more ill-posed the greater the distance h becomes. Usually, the variations in the relief function are small compared to the depth h. However if steep gradients occur, more informative are upper and lower bounds for the decay of the eigenvalues. Such estimates are obtained by replacing the average depth h by $h^{\pm} = h \pm \max_{S} |f(\alpha,\beta)|$ respectively.

The average depth to the magnetised rocks varies from very shallow rocks (tens of meters) up to a few km. Knowledge of the average depth leads to information about the resolution: in practice, the critical scale on which features can be resolved from the observed data appears to be ~1.5h.

Another issue discussed at the workshop concerned possible solution techniques. Because of the nonlinear nature of the relief equation, it seems most appropriate to employ iterative techniques. As a starting point for the iteration, the relief function f can be chosen to consist of a number of prisms with flat tops (i.e. the relief function is approximated by step functions in two dimensions). A similar model was employed by Bhattacharyya [1980], however he used an assortment of fixed prisms in order to find the magnetisation in these prisms (i.e. he worked with the equation of the downward continuation over a surface consisting of prisms).

In summary, the relief equation is nonlinear and ill-posed, making it a very hard problem to solve. For this reason, it seems more promising to solve the linear relief equation (equation (13) or (18)) first - possibly by starting with the two-dimensional problem. One may then hope that the solution to the linear problem might be an acceptable solution to the nonlinear problem, or that at least it would serve as a good starting point for iterative solution for the nonlinear problem.

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Appendix

Under the assumption that the magnetisation is constant in the y-direction, its contribution to the field can be integrated out. This, formally leads to the following two-dimensional analogue of equation (6):

$$\text{(A1)} \qquad \mathbf{H}(x,0) = \int_{\sigma} J(\alpha,\gamma)\ \mathbf{n}(\alpha,\gamma)\ \nabla\left(\frac{1}{r-r_0}\right) d\ell\ ,$$

where $d\ell$ now denotes the line element along the curve f (see Figure 3), and $r-r_0 = [(x-\alpha)^2 + (h+f(\alpha))^2]^{1/2}$. The contribution to $\mathbf{H}$ at the point $(x,0)$ from an infinitesimal element on f now becomes

$$\text{(A2)} \qquad d\mathbf{H}(x,0) = -2J(\alpha,\gamma)\ \mathbf{n}(\alpha,\gamma)\ \frac{\mathbf{r}-\mathbf{r}_0}{(r-r_0)^2}\, d\ell\ .$$

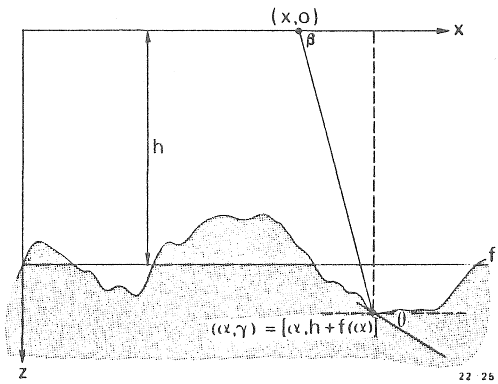

Figure 3. The co-ordinate system used in the derivation of the simplified two-dimensional relief equation.

Expressions for $\mathbf{n}$ and $d\ell$ in terms of the relief function f now become $d\ell = (1+(f')^2)^{1/2}\, d\alpha$ and $\mathbf{n} = (1+(f')^2)^{-1/2}\,(f',-1)$, with $f' \equiv \frac{d}{d\alpha} f$. Substitution into (A2) yields the following equation for the field in two dimensions:

$$\text{(A3)} \qquad \mathbf{H}(x,0) = -2\int (J_\alpha f'(\alpha) - J_\gamma)\frac{(x-\alpha, h+f(\alpha))}{(x-\alpha)^2+(h+f(\alpha))^2}\, d\alpha \ .$$

To obtain a more geometric interpretation of (A2), substitute

$$\sin\beta = \frac{h+f(\alpha)}{r-r_0}$$

into (A3), noting that f is positive in the positive (downward) z-direction as indicated in Figure 3. The components of dH may now be written as

$$dH_x(x,0) = -2J(\alpha,\gamma)\ n(\alpha,\gamma)\ \frac{\cos\beta}{r-r_0}\ d\ell$$

(A4)

$$dH_z(x,0) = -2J(\alpha,\gamma)\ n(\alpha,\gamma)\ \frac{\sin\beta}{r-r_0}\ d\ell\ .$$

Furthermore, let θ denote the angle between the line element $d\ell$ on f and the x-axis, so that $\tan\theta = f'$. Since $n = (1+(f')^2)^{-1/2}(f',-1)$, equation (A4) becomes

$$dH_x(x,0) = -2(J_\alpha \sin\theta - J_\gamma \cos\theta)\ \frac{\cos\beta}{r-r_0}\ d\ell$$

(A5)

$$dH_z(x,0) = -2(J_\alpha \sin\theta - J_\gamma \cos\theta)\ \frac{\sin\beta}{r-r_0}\ d\ell\ .$$

The expression dH_z is given in this form as (1) in Peters [1949]. The latter author then claims to derive the following integral equation for H_z.

$$H_z(x,0) = -2\int[J_\alpha(h+f(\alpha)) + J_\gamma(x-\alpha)]\ \frac{f'(\alpha)}{(x-\alpha)^2 + (h+f(\alpha))^2}\ d\alpha$$

(see (3) in Peters [1949]). A simple calculation shows, however, that the last equation is derived from an equation slightly different from equation A5, namely

$$dH_z(x,0) = -2[J_\alpha \sin\beta - J_\gamma \cos\beta] \frac{\sin\theta}{r-r_0} d\ell \ .$$

(Note the roles, but not the meaning, of β and θ have been interchanged.)

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