LACUNAE IN BOUNDARY LAYER MODELLING

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1. INTRODUCTION

Lacuna: A gap, an empty space, a cavity [OED].

This paper examines two gaps in our understanding of basic boundary layer physics that are obstacles to climate modelling and, in the spirit of this meeting, emphasizes their mathematical aspects. The particular facets of boundary layer dynamics we shall discuss are flow over steep topography and transfer processes in boundary layers with strong stability; the corresponding mathematical disciplines are the theories of dynamical chaos and hydrodynamical stability. The first part of this paper will be devoted to showing how these problems arise naturally when we attempt to devise practical rules for sub-grid scale averaging of land surface processes, a fundamental requirement in setting boundary conditions for global climate models (GCM's).

The problems of forecasting future climate fall roughly into four classes. The first concerns the generic predictability of climate itself, whether it has chaotic tendencies at relevant time scales and whether global climate models (GCM's) can capture the essentials of this property. The other three concern the details of the GCM's themselves. GCM's are schemes to numerically integrate sets of coupled, partial differential equations describing, primarily, the behaviour of oceans and atmosphere. The second class of problems, therefore, concerns the stability and nature of the numerical integration schemes employed. The third class is the parameterization of physical processes that occur on scales too small to be resolved by the solution grid. Important atmospheric examples are cloud formation and dissipation, gravity wave momentum transfer and turbulent mixing in the boundary layer. The last group of problems is concerned with the specification of the boundary conditions, particularly the surface boundary conditions. Less attention need be paid to the allied question of setting initial conditions as this is much less crucial over the long integration times used for climate prediction (typically decades to centuries) than in numerical weather prediction. There, essentially the same equations are integrated but for a relatively short time so that prediction errors are conditioned primarily by uncertainty in initial conditions (Garratt [25]).

We shall concern ourselves here with the surface boundary conditions over land, a subset of the fourth class of problem but in the process will be led to consider the parameterization of sub-grid scale turbulent mixing.

2. SUB-GRID VARIABILITY AND LOCAL CLIMATES

The sensitivity of GCM predictions to planetary boundary layer (PBL) and land surface processes has recently been reviewed by Garratt [25]. His study showed that although few experiments on the global response to surface parameterization have actually been performed, those that have been done confirm the overall sensitivity of the predicted climate to the way that surface roughness and evapotranspiration are parameterized. At a regional level the dependence is even more marked. One of the most detailed studies looked at the change in regional climate following a hypothetical deforestation of the Amazon basin (Dickinson and Henderson-Sellers [10]). They observed significant increases in near surface temperature (3–5K) and decreases in evapotranspiration when the jungle was replaced by grassland. Their model coupled a detailed representation of the vegetation canopy (BATS - Dickinson et al. [9]) with the NCAR Community Climate Model run in full 3d mode.

Representation of surface boundary conditions cannot be divorced from simulation of the PBL. Indeed, by definition, the PBL is the layer of the atmosphere directly influenced by radiant energy partitioning and momentum absorption at the ground and responding, as a result, to the diurnal cycle of solar heating. Exchange processes at the surface are not independent of the (mainly turbulent) dynamics of the PBL, which forms the buffer between the surface and synoptic scale atmospheric processes developing on time scales longer than a day. This interdependance ensures that local climates are responsive to local surface conditions to a greater or lesser degree and has several important consequences for our present purpose.

First of all it means that, if a GCM is run with homogeneous surface boundary conditions, then its prediction of near-surface conditions will not correspond to any observable local climate. This leads to the "top down" problem of surface parameterization: how to interpret GCM output at local scale. Its solution is crucial in verifying GCM model output against actual measurement. Secondly, it means that the detailed information available on surface properties, which satellites such as Landsat now provide, cannot be averaged into homogeneous boundary conditions without taking into account the dynamic response of the PBL. For example, the evapotranspiration from a particular patch of surface is determined not only by the character of that patch but by the nature of upwind surfaces that have contributed to the properties of the PBL flowing above it. Combining local surface properties into a homogeneous boundary condition (bc) is the "bottom up" problem. In what follows we intend to make a virtue of necessity and exploit this interdependence between the PBL and surface exchange to attack in a rational way the question of sub-grid scale averaging in the horizontal.

Current GCM's have typical grid square resolution of $5^\circ \times 5^\circ$, which is roughly 500 km by 500 km near the equator, although the generation of models now coming into use generally doubles this resolution to 250 km x 250 km. Such a large area of land surface can contain a range of significantly different surface types. There may be variety in surface cover from bare soil or sand through grass and crops up to forest canopies. Surface elevation may also vary through coastal plains, plateaux, hills, mountains and open water bodies.

Vertical resolution is provided by as few as four or as many as thirty grid levels, although most present "production model" GCM's have about nine levels in the atmosphere (Garratt [25]). Temporal resolution is determined by the time step, whose upper limit in turn is set by the requirement of numerical stability. Physically, this choice depends on the time that the fastest moving resolved disturbance (wave) takes to traverse a grid cell. Typical time steps vary from a few minutes in models that resolve the PBL (e.g. Suarez et al. [53]) to one day in coarser models that cannot, therefore, resolve the diurnal cycle.

The boundary conditions must adequately represent the exchange of energy and momentum over an entire grid cell by a single set of parameters. In the bottom up problem we must find rules to combine the descriptors of exchange at each patch of surface into an average description that produces the correct net exchange for the whole cell. In the top down problem we must do the reverse; we must find rules to translate the single set of values of wind speed, temperature, scalar concentration and so on, produced by the GCM at each grid cell, into correct local values. An ideal scheme would be symmetrical and could be used in both the upward and downward mode. Let us formalize these statements and, at the same time, introduce some concepts and notation that we shall use throughout.

The energy balance at the earth's surface can be written:

$$(1) \qquad (1 - \alpha)S_d + \epsilon L_d - \epsilon \sigma T_0^4 - G = H_0 + \lambda E_0$$

where α is the surface albedo, S_d is the downward component of short wave radiation, L_d the downward long wave radiation, ϵ is the emissivity, σ the Stefan–Boltzmann constant, T_0 the surface temperature, G the heat flux into the ground, H_0 the surface flux of sensible heat, λ the latent heat of evaporation and E_0 the surface flux of water vapour. We use the shorthand term "flux" here but the components of equation (1) are formally flux densities. On a homogeneous patch of surface the sensible and latent heat fluxes may be parameterized as:

(2) Sensible heat: $$\frac{H}{\rho c_p} = \frac{T_0 - T(z)}{r_a(z)}$$

$r_a(z)$ being the aerodynamic resistance to transfer between the surface at temperature T_0 and the reference level at height z, where the temperature is T(z). ρ is air density and c_p the specific heat of air at constant pressure. Note that subscript $_0$ always denotes surface values.

(3) Latent heat: $$\frac{\lambda E}{\rho \lambda} = \frac{Q_0 - Q(z)}{r_a(z)} ,$$

where Q is the specific humidity of air. The resistance to transfer between the surface and level z is affected both by turbulent dynamics and by molecular diffusion through the unsteady laminar boundary layers on leaves. However, despite this it is possible to use the same aerodynamic resistance for both heat and water vapour because the appropriate Lewis Number is close to 1 in air.

Equations (1) – (3) contain the surface concentrations T_0 and Q_0. It is possible to eliminate these by a set of now standard assumptions and manipulations to produce the combination equation (also called the Penman-Monteith equation):

(4) $$\lambda E = \frac{eA + \rho \lambda D(z)/r_a(z)}{e+1 + r_s/r_a(z)}$$

with

(5) $$A = (1 - \alpha)S_d + \epsilon L_d - \epsilon \sigma T_0^4 - G$$

e is the dimensionless rate of change of the saturated specific humidity, Q_{sat} with temperature, viz $e = (\lambda/C_p)\, d\, Q_{sat}(T)/dT$; D is the specific saturation deficit, $D(z) = Q(z) - Q_{sat}((z))$. r_s is called the surface resistance and A is the 'available energy', available, that is, to evaporate water. r_s represents the resistance to transfer of water vapour from its source within plant tissue or the soil to the surface. When applied to plants, the classical interpretation of r_s is the resistance imposed by leaf stomata to vapour transfer between the saturated air in substomatal cavities and the leaf surface.

Derivations and various equivalent forms of equation (4) can be found in Monteith [39], McNaughton and Jarvis [36] or Finnigan and Raupach [23]. The combination equation neatly encapsulates the driving forces for evaporation, which are a supply of radiant energy A or of dry air D and the controls on evaporation, which are the rate of diffusion of water vapour away from the surface, parameterized by r_a, and the availability of water vapour at the surface, expressed as r_s. The combination equation can be used to describe evaporation from individual plant leaves or from areas of vegetation, in which case it is often called the "big leaf" model. We shall make it the central pillar of our surface description for the simple reason that any point on the land that receives regular and sufficient rainfall supports some plant cover. Seen in this light there has been misplaced emphasis in the past on complex soil evaporation schemes as the land boundary conditions for GCM's.

When equation (4) is applied to a single leaf, r_s has a precise interpretation as a physiological property of the plant. However, when equation (4) is integrated through the depth of the plant canopy and used to characterize the evapotranspiration of the whole canopy and the underlying soil, the interpretation of r_s is less simple. In this big leaf mode, r_s can no longer be regarded as a purely physiological or biological property of the vegetation but depends also upon the distribution of wind speed, radiation and D throughout the canopy. Raupach and Finnigan [47] discuss these questions and show how to relate individual leaf resistances to whole canopy values.

Similar problems arise when we wish to assign whole canopy values to r_a or to albedo and emissivity, and although vertical averaging of r_a is also discussed by Raupach and Finnigan [47] the relationship between the leaf surface temperature and whole canopy values of α and ϵ still awaits an adequate treatment. The approach of this paper, however, will be to assume that this vertical averaging can be performed in a rational way and the surface properties controlling evapotranspiration condensed into four variables: α, ϵ, r_a and r_s, so that we may concentrate instead on the equally vexing problem of horizontal sub-grid averaging.

Raupach [46] actually presents a complete scheme for performing the vertical average, drawing heavily on the techniques of Raupach and Finnigan [47] and his own work on modelling of turbulent transport in canopies, but a somewhat different approach has been taken in the only complete vegetation models to have actually been employed in GCM's. These are SiB (Sellers et al. [51]) and BATS (Dickinson et al. [9]). Both models employ a detailed description of the canopy and the underlying soil in the vertical, requiring many parameters to be specified. In a commentary on one of the SiB papers, McNaughton [35] argued that this type of land-surface model faces three difficulties: critical biological and aerodynamic processes are handled crudely (mainly by bulk resistances), making irrelevant a detailed physical model of other parts of the system such as the lower soil layers; it is impossible in practice to measure the required number of parameters for every land-surface grid point in a GCM, so most of the parameters will necessarily be guessed; and even if the model did work at one point in space with properly measured parameters, it includes no method for spatially averaging the soil and vegetation parameters over the many vegetation and other surface types within a single grid cell in a GCM.

Assuming that the necessary vertical averaging has been performed to obtain values of α, ϵ, r_a and r_s that characterize the whole canopy and underlying soil, we can formalize this sub-grid horizontal averaging problem. Let $\langle\rangle$ denote a horizontal average over a grid cell. Cell average values of the four parameters above are defined as

$$
(6)\quad
\begin{aligned}
&a \quad \hat{\alpha} = \langle\, \alpha(\underline{x})\ \varphi_{\alpha}(\underline{x}) \,\rangle \\
&b \quad \hat{\epsilon} = \langle\, \epsilon(\underline{x})\ \varphi_{\epsilon}(\underline{x}) \,\rangle \\
&c \quad \hat{r}_s = \langle\, r_s(\underline{x})\ \varphi_{r_s}(\underline{x}) \,\rangle \\
&d \quad \hat{r}_a = \langle\, r_a(\underline{x})\ \varphi_{r_a}(\underline{x}) \,\rangle \ .
\end{aligned}
$$

The problem is to find weighting functions φ such that:

$$
(7)\qquad \langle\lambda E(\underline{x})\rangle = \frac{\epsilon \langle A\rangle + \rho\ \lambda\ \hat{D}/\hat{r}_a}{\epsilon+1 + \hat{r}_s/\hat{r}_a}
$$

$$
(8)\qquad \langle A\rangle = (1 - \hat{\alpha})\hat{S}_d + \hat{\epsilon}\ \hat{L}_d - \hat{\epsilon}\ \sigma\ \hat{T}_0{}^4 - \hat{G}
$$

where $\hat{D}$, $\hat{S}_d$, $\hat{L}_d$, $\hat{G}$ and $\hat{T}_0$ are the single values produced by the GCM for that grid cell.

The problem can be simplified somewhat if we assume that the surface of the cell is covered with n discrete homogeneous patches, each of area f_i and with attributes σ_i, ϵ_i, r_{ai}, r_{si}. The averaging operator <> can now be replaced by a simple sum and the weighting functions φ suitably redefined. Equation (6) thus becomes:

$$
(9)\quad
\begin{aligned}
&a \quad \hat{\alpha} = \sum_i \alpha_i\ f_i\ \varphi_{\alpha i} \\
&b \quad \hat{\epsilon} = \sum_i \epsilon_i\ f_i\ \varphi_{\epsilon i} \\
&c \quad \hat{r}_s = \sum_i r_{si}\ f_i\ \varphi_{r_{si}} \\
&d \quad \hat{r}_a = \sum_i r_{ai}\ f_i\ \varphi_{r_{ai}} \ .
\end{aligned}
$$

In equations (7) and (8) we have grossly simplified the averaging problem for the radiation balance in order to focus attention on the PBL dynamics. We should acknowledge, however, that $\hat{L}_d$ should strictly be replaced by $\langle L_d\rangle$, which must take account of sub-grid scale variability in water vapour content, particularly as clouds. Problems also arise because of the nonlinearity of the Stefan-Boltzmann term, which requires T_0 be replaced by $\langle T_0{}^4\rangle$ and ensures that this 'radiation' surface temperature is potentially different from the 'evaporation' surface temperature that appears in equation (2). These are questions at least as difficult as the ones we shall address below and the fact that we do not discuss them in detail should not be interpreted as casting doubts on their importance.

Comparable problems arise when we consider the transfer of momentum rather than evapotranspiration or radiation. Because the wind speed at the surface is identically zero we can write for a homogeneous patch of surface:

(10) $$\tau_0 = \rho\, C_d\, U^2(z)$$

where τ_0 is the surface shear stress, U(z) a reference velocity at height z and C_d is the drag coefficient. A new difficulty arises here because the aerodynamic drag on the surface is parallel to the near surface wind, which in turn is parallel to the large scale pressure gradient (the geostrophic gradient over the flat ground). Above the surface layer, however, the mean wind is directed at an angle to the surface wind, turning through the 'Ekman spiral' to attain the geostrophic balance between pressure gradient and Coriolis force above the boundary layer. Deviation of the vertically averaged wind in a convective mixed layer from the surface direction is typically 10° (Deardorff [8]) but it may be much larger than this in stable layers. Various procedures can be suggested to cope with this problem, see for example Deardorff [8].

Given a range of drag coefficients appropriate to each surface patch, the horizontal averaging problem is to find a weighting function $\varphi_m(x)$ defined by

(11) $$\hat{C}_d = \langle C_d(\underline{x})\, \varphi_m(x) \rangle$$

such that

(12) $$\langle \tau_0 \rangle = \rho\, \hat{C}_d\, \hat{U}^2$$

where, as before, $\hat{U}$ is the output of the GCM, or, discretizing the procedure as in equation (9),

(13) $$\hat{C}_d = \sum_i C_{di}\, f_i\, \varphi_{mi}.$$

3. THE PLANETARY BOUNDARY LAYER AND SURFACE EXCHANGE PROCESSES

It is conventional to divide the planetary boundary layer into two regions: the surface layer, where turbulent motion is dynamically constrained by the proximity of the ground, and the rest. The depth h of the PBL is defined in different ways according to whether the surface heat flux is positive (unstable), negative (stable) or zero (neutral). The clearest definition occurs in unstable conditions when a convectively driven 'mixed' layer develops above the surface layer and extends to a capping density inversion at height h. The surface layer then occupies about the lowest 10% of the PBL. Within the well mixed region of this convective boundary layer (CBL) scalar properties like temperature and humidity are essentially constant, as is the wind speed magnitude, although changes in wind direction of order 10^{o} are typical, as we have already mentioned. The depth of the CBL increases through the day, rapidly at first then levelling off until the evening "collapse", which occurs as the surface heat flux weakens. Typical values for afternoon CBL height in mid-latitudes are 1-2 km.

The height of the stable boundary layer that supersedes the CBL is much less, typically of ~ 100 m. A recent diagnostic formula for h in neutral and stable conditions encapsulating much earlier work is that of Zilitinkevich [60]

$$(14) \qquad h = \frac{u_*}{|f|} \left[\frac{1}{\Lambda} + \frac{\mu^{\frac{1}{2}}}{kC_h} \right]^{-1}$$

where u_* is the friction velocity given by $u_* = (\tau_0/\rho)^{\frac{1}{2}}$, f is the Coriolis parameter (~ 10^{-4} s^{-1} in mid-latitudes), k is Von Kármán's constant (= 0.4), Λ and C_h are constants of order 0.3 and 1.0 respectively (Garratt and Pielke [26]) and μ is defined by:

$$(15) \qquad \mu = \frac{-k^2 H_{v0}}{|f| \, u_*^2} ,$$

where H_{vo} is the surface buoyancy flux:

$$(16) \qquad H_{vo} = H + 0.07 \lambda E .$$

The contributions of both water vapour and temperature to density fluctuations are combined in the buoyancy flux.

The depth of the stable boundary layer is clearly affected by the magnitude of H_{vo}. In daytime convective conditions the boundary layer depth is even more strongly coupled to H_{vo}. At the same time the boundary layer depth exerts an important influence on the magnitude of H_{vo}. This close coupling or feedback between the state of the whole PBL and the surface fluxes is a vital element which must be modelled when a parameterized PBL is interposed between resolved, GCM outputs and surface characterizations in a GCM.

In 1972 Deardorff [8] presented a complete scheme to do this. His approach remains an excellent point of departure for more recent efforts. He began by using standard, stability-dependent Monin-Obukhov surface layer formulae to model transfer from the ground to the top of the surface layer at height z_a. Between z_a and the top of the PBL at h, he employed empirical, stability-dependent formulae to describe the departures of virtual temperature θ_v and wind speed U from their values at z_a. The virtual temperature θ_v, which bears the same relationship to H_v as H does to T, is defined by:

(17) $$\theta_v = T + 0.61Q$$

He then matched the two descriptions to eliminate the unknown values of θ_v and U at z_a and obtained formulae valid throughout the whole PBL. These relationships were of the form,

(18)

a $$\frac{u_*}{U_m} = f_1(\frac{h}{L}, \frac{h}{z_o})$$

b $$\frac{\theta_{v*}}{\theta_{vm} - \theta_{vo}} = f_2(\frac{h}{L}, \frac{h}{z_o})$$

where z_0 is the aerodynamic roughness length, $\theta_{v*} = H_{v0}/\rho c_p u_*$ and L is the Obukhov length defined as:

(19) $$L = - u_*^3 \left[\frac{k\ g}{\theta_{vm}} \frac{H_{v0}}{\rho c_p}\right]$$

where g is the acceleration due to gravity and subscript $_m$ denotes values in the 'well mixed' PBL above the surface layer.

The actual forms of the diabatic influence functions and bulk PBL formulations used by Deardorff have been largely superseded now with the advent of better data but the essential point remains unchanged; formulae (18)a and b parameterize the surface fluxes u_* and θ_{v*} in terms of stability L, surface roughness z_0 and boundary layer depth h. In order to close the description of the surface fluxes, therefore, a specification of h must be added. In stable or neutral conditions, equation (14) could be used. (Deardorff actually used a simpler formula.)

In the more rapidly varying daytime CBL a prognostic equation is necessary. Deardorff used

$$(20) \qquad \frac{\partial h}{\partial t} = \hat{W}(h) - \hat{\underline{V}}(h) \cdot \nabla h + S + \nabla \cdot (K \nabla h)$$

where $\hat{W}(h)$ is the vertical velocity at level h obtained from the GCM, $\hat{\underline{V}}(h) \cdot \nabla h$ is the advective term, $\hat{\underline{V}}(h)$ being the horizontal wind vector, also from the GCM, and ∇ is the horizontal gradient operator. S is the source term associated with penetrative convection through the inversion that caps the CBL, and the last term, involving an eddy coefficient K, represents (at least partly) effects of sub-grid-scale lateral diffusion of h. Except for the last term, this equation expresses the idea that an average fluid particle, initially located at z = h, remains at h unless entrainment causes h to increase by means of the term S.

Simpler, one-dimensional "slab" models of the convective boundary layer ignore the first, second and fourth terms on the right hand side of equation (20) and parameterize S in terms of the surface buoyancy flux and the gradient of synoptic virtual temperature above h. See, for example, Tennekes [56] and Tennekes and Driedonks [57].

Deardorff's approach was extended by Suarez et al. [53] who made two important advances. They rewrote the system of equations in terms of pressure based 'sigma' coordinates, a common device in large scale meteorology, enabling the top of the PBL to be identified with the first grid level of the GCM. This practice avoids an awkward matching problem that is encountered when h lies between GCM grid levels. More fundamentally, they incorporated cloud physics into their boundary layer model, specifically the formation and breakup dynamics of a stratocumulus deck and of cumulus towers above the inversion. When their PBL parameterization was coupled with the UCLA GCM (Suarez et al. [53]), cloud dynamics were seen to play a significant role in the evolution of climate.

Raupach's [46] treatment of the coupled CBL-surface layer system differed from that of Deardorff and Suarez et al. in two important respects. Firstly, he employed the combination equation as the basis of the evapotranspiration boundary condition. This had the immediate consequence of avoiding the necessity of specifying T and Q at the surface, replacing them by the quasi-physiological parameter r_s and providing a link between models of the biosphere and the atmosphere. Secondly, by comparing the dynamic and thermodynamic response times of the coupled system he obtained expressions for the weighting functions φ of equation (9) in two important limits. To illustrate these points we will outline the relevant aspects of Raupach's 'SCAM' (simple canopy-atmosphere model).

The first step is to specify the reference level for equations (2) and (3) as within the mixed layer, where scalar quantities are assumed to be constant with height. The aerodynamic resistance, therefore, describes the resistance to transfer imposed by turbulent mixing rates across the entire surface layer. Next, by assuming a simplified form for the structure of the CBL, equations are written for the mixed layer concentrations. For a scalar C these are:

$$(21) \qquad \frac{dC_m}{dt} = F_{co} + \left[\frac{\Delta C}{h}\right] \frac{dh}{dt}$$

or

$$(22) \qquad \frac{d}{dt}(h\,\Delta C) = \gamma_c\, h \frac{dh}{dt} - F_c \quad ,$$

(Tennekes and Driedonks [57] where $\Delta C = C_+(h) - C_m$, ΔC being the jump in concentration at the top of the PBL, F_{co} the flux of C at the surface and $\gamma_c(z) = \left.\frac{dC}{dz}\right|_{z>h}$.

With F_{co} given by

(23) $$F_{co} = \frac{C_o - C_m}{r_a}$$

and C_+ given by the GCM output, a further equation is needed to close the system. This is the equation for h, equation (20), which, for a simple one-dimensional slab CBL, reduces to

(24) $$\frac{dh}{dt} = S$$

S can be specified if $H_v(h)$, the downward flux of buoyancy caused by penetrative convection at the top of the mixed layer, is known or inferred. A common way to do this is to assume that this entrainment flux is a constant fraction of the surface buoyancy flux (Tennekes and Driedonks [57]), hence

(25) $$H_v(h) = - B\, H_{vo}$$

where $B \triangleq 0.2$ is a constant.

Equation (25) provides the required closure assumption enabling the surface fluxes and synoptic concentrations to be coupled through a combination of surface layer (equation (23)) and mixed layer (equations (21) and (24)) dynamics. r_a is obtained by adopting Monin-Obukhov formulations for surface layer transfer. Standard methods for obtaining r_a in this way can be found in Monteith [39] or Finnigan and Raupach [23] They require, as one might expect, that momentum transfer be computed in parallel with evapotranspiration and that surface roughness, z_o, therefore, be specified.

As we mentioned previously, Raupach avoided the need to specify surface concentrations, when C represented temperature or humidity, by using the combination equation (4) and employing the quasi-physiological surface resistance r_s. To make the problem mathematically more tractable he also worked with new variables, which were linear combinations of T and Q. This had the important effect of making the boundary conditions separable but the features of the SCAM model that we are interested in can be illustrated without this refinement.

The *thermodynamic* equilibrium of the coupled PBL-surface system can be defined as the achievement of a constant rate of evaporation and constant values of T_m and Q_m when A, r_s, r_a and synoptic concentrations are held fixed. The precise time taken to achieve this equilibrium depends on the nature of the closure hypothesis and growth model chosen for the PBL - equations (24) and (25) for example - as well as on the nature of the surface as characterized by r_s and r_a. For any plausible choice of CBL growth model, however, Raupach showed that the 'e folding time' of the (approximately exponential) approach to equilibrium exceeded 18 hours for lush grass surfaces, 21 hours for forests and 56 hours for open water. In other words, over a diurnal cycle, evaporation from the land surface never approaches thermodynamic equilibrium.

The *dynamic* equilibrium of the convective boundary layer is a different concept. By this we mean that the turbulent exchange processes in the surface and mixed layer reflect local values of surface buoyancy flux. It can be characterized by the distance X one would have to go downwind of a step change in surface conditions before changes in turbulent quantities no longer occurred. To estimate X we note that the characteristic turbulent velocity scale in the mixed layer is w_*, where

$$(26) \qquad w_* = \left[\frac{g}{\theta_{vm}} \frac{H_{vo}}{\rho Cp} h\right]^{1/3}.$$

Then simple scaling arguments invoked by Raupach [46] show that for step changes in H_{vo}, both positive and negative, we can write:

$$(27) \qquad X \simeq \frac{U_m h}{w_*} ,$$

where, for definiteness, we can take h and w_* as the means of values far upwind and downwind of the change. h is of order 1000 m while w_* is of order 2 m s^{-1} so that the adjustment time scale L/w_* is like 8 minutes. This great disparity between the timescales of thermodynamic and dynamic equilibria effectively decouples the specification of the transfer process from that of the controls upon evaporation rate.

Noting this, Raupach uses X as a scale to order the heterogeneity of the surface. Three classes are identified:

a) $\surd f \ll X$

b) $\surd f \sim X$

c) $\surd f \gg X$

where $\surd f$ is the linear scale of a typical patch of homogeneous surface in the grid square. Case (a) defines microscale heterogeneity. Here, the convective boundary layer does not have time to react to each individual patch of surface. Consequently, each patch, f_i, is overlaid with the same well-mixed layer. The total flux of C from the grid square is therefore:

(28) $$\langle F_c \rangle = \frac{\langle C_o \rangle - C_m}{\langle r_a \rangle} = \sum_i \frac{C_{oi} - C_m}{r_{a_i}} f_i$$

(29) hence $$\frac{1}{\langle r_a \rangle} = \sum_i \left[\frac{f_i}{r_{a_i}}\right]$$

(30) and $$\langle C_o \rangle = \langle r_a \rangle \sum_i \left[\frac{f_i \, C_{oi}}{r_{a_i}}\right] .$$

Equation (29) defines the $\varphi_{r_{ad}}$ function of equation (9d). Equation (30) defines an analogous relationship, which, when used in the combination equation (4) to substitute for C_o (with C = Q or T), produces an equivalent φ_{r_s} function for the surface resistance (equation (9c).

Case (c) defines macroscale (or mesoscale) heterogeneity. Each surface patch f_i must now be regarded as having its own convective boundary layer so that we must write:

(31) $$\langle F_c \rangle = \frac{\langle C_o \rangle - \langle C_m \rangle}{\langle r_a \rangle} = \sum_i \frac{(C_{oi} - C_{mi})}{r_{a_i}} f_i .$$

There is now no obvious unique choice of weighting for r_a but, if we adopt equation (29) as in the previous case, then equation (30) continues to apply not only to $\langle C_o \rangle$ but also to $\langle C_m \rangle$:

(32) $$\langle C_m \rangle = \langle r_a \rangle \sum_i \left[\frac{f_i \, C_{mi}}{r_{ai}}\right] .$$

The formula yielding $<r_s>$ is now a complex dynamical expression involving solution of the coupled system of equations (21), (23), (24) and (25) with only the synoptic values of concentration being common across the grid square. In particular the weighting functions are time dependent as each individual CBL evolves at its own rate. Fortunately, with $\sqrt{f_i} >> X$ we can only accommodate a few patches within a grid square, probably less than 10, so that the computing costs of running independent slab models is probably not excessive.

Case (b), where $\sqrt{f_i} \sim X$, is the most difficult to handle because the coupled adjustment of the surface layer and convective boundary layer in two or three dimensions must be considered. No suitable treatment of this problem exists at present (at least not to this author's knowledge). First and second order closure models of the surface layer are not uncommon (e.g. Philip [44] but non-equilibrium mixed layers are rarely considered. A combination of the two presents special problems in the matching of three distinctly different modes of turbulent mixing: diabatically modified shear-driven turbulence in the surface layer, free convection in the mixed layer and entrainment at the capping inversion. Indeed, little experimental data for this situation exist and we could justifiably identify this as the first of our lacunae.

In moderately stable or neutral conditions the same approach as we have outlined for the convective boundary layer may be adopted. This brings with it both simplifications and complications. It is usually considered unnecessary to use a prognostic equation for stable layer height; although such creatures exist (Arya [1]), this is a level of complexity which certainly exceeds our needs. It is clear from equation (14) that, near neutrality, h is determined mainly by momentum transfer and that coupling between the synoptic state and surface fluxes of heat and moisture is driven by shear generated turbulence. In determining X, the appropriate mixing velocity scale, therefore, is u_*, which in stable layers would be of order $0.1\ m\ s^{-1}$ or less while $h_{(stable)}$ would also be of order $0.1\ h_{(convective)}$. Hence the readjustment distance of stable layers on flat ground is typically of the same order as the CBL although now, surface roughness z_o rather than r_s may be the prime determinant of a change in surface conditions. Instead of steady state solutions of the CBL equation set (21-25) similarity relationships of the Deardorff [8] form (equations (17) and (18)) or more up-to-date variants may be used (e.g. Nieuwstadt [40]). When conditions are strongly stable, more subtle descriptions, of surface layer exchange at least, are needed with radiative flux divergence and gravity-wave forced, intermittent turbulence explicitly accounted for. A stable boundary layer exhibiting the full range of these extra complications was studied by Finnigan et al. (1984).

A more intractable problem concerns the resolved synoptic output from the GCM. After the evening collapse of the CBL, the PBL contracts to about 10% of its convective depth in a very short time. Fifteen minutes to achieve the evening transition is a typical period. We recall the phenomenological definition of the PBL as the layer responding directly to surface exchange. There remains, however, above this new, thin PBL, a deep layer of previously convective turbulence that is now decoupled from the ground and is decaying towards synoptic conditions. The rate of this convergence towards the grid average depends on the local state of the CBL before transition and is governed by comparatively poorly understood dynamics in which wave-turbulence interaction features prominently.

A comment is appropriate here on the relative importance of daytime versus nocturnal, stable evapotranspiration and drag. Under stable conditions turbulent exchange of any property is strongly damped, although, as we shall note more fully in §5, horizontal transport may be enhanced by density driven flows. If daytime exchange is so predominant, therefore, it may well be asked why parameterization of the nocturnal case is so important? The main reason lies in the expressions for CBL height, the prognostic equations (20) or (24). Because we are able to link only the rate of change of the CBL height to surface exchange, specification of the initial conditions at sunrise can determine the prediction of CBL evolution through a large part of the day. These initial conditions are, of course, set by the nocturnal PBL behaviour. Although this is the main reason for our interest, other motivations can certainly be found. Respiration by plants is but one example of an exchange process that continues through the nighttime hours. Dew and frost formation, processes which often feature prominently when the agricultural impacts of Greenhouse warming are discussed, are others.

Let us summarize the story so far. Given a GCM, which we must assume resolves the synoptic atmospheric state above the PBL, and a surface that is heterogeneous on a scale that the GCM cannot resolve, we find that the weighting functions necessary to perform sub-grid scale averaging of the surface descriptors involve parameterizing the PBL. In fact, we exploit simplified PBL dynamics to connect the unresolved surface to the resolved synoptic conditions. The PBL parameterizations we employed were, however, derived for the simplest situation: flat, homogeneous surfaces with most attention paid to convective conditions. We now wish to ask how applicable these formulae are to the range of conditions encountered in the real world. Furthermore, throughout this section we have assumed that each sub-grid scale surface patch f_i can be assigned homogeneous surface descriptors r_{si}, r_{ai}, z_{oi}, σ_i, ϵ_i, C_{di}. However, particularly for the small patches of microscale inhomogeneity, a local advection problem must be solved to find equivalent r_s and r_a values for a given patch. At a much smaller scale (on a single leaf) this problem was addressed by Cowan [7]. For the larger patches of cases (b) and (c) the region where advective changes in r_a and r_s occur occupies only a small fraction of the total area and can essentially be ignored. But local advection is only one of the complicating factors and in the sections that follow we shall ask whether such quasi-one-dimensional surface descriptions can, in fact, be achieved in two common circumstances: neutral or unstable flow over steep topography and very stable flow over moderate or flat topography.

4. MOMENTUM AND SCALAR TRANSFER OVER COMPLEX TOPOGRAPHY

Let us restrict our attention to a sub-grid scale patch covered by a range of hills, whose relief can be considered statistically homogeneous on the scale of the patch. Physical examples are not difficult to find. Most of the Great Dividing Range running up the eastern margin of the Australian continent falls under this description. Any one-dimensional or homogeneous characterization of such a surface entails averaging properties over the patch. To simplify matters let us stipulate that the patch be sufficiently large that we can ignore edge effects.

In flat, horizontally homogeneous conditions, the integrated Monin-Obukhov similarity laws for velocity and a scalar C take the form of logarithmic profiles (Yaglom [59]):

(33) velocity: $$U(z) = \frac{u_*}{k}\left[\ell n\left[\frac{z}{z_0}\right] + \psi_m(z/L)\right]$$

(34) scalar C: $$C(z) - C_o = \frac{c_*}{k}\left[\ell n\left[\frac{z}{z_c}\right] + \psi_c(z/L)\right]$$

where $c_* = F_{c0}/u_*$, ψ_m and ψ_c are diabatic influence functions (Paulson [42]), z_0, the surface roughness length, can be regarded as a measure of the capacity of the surface to absorb momentum, while z_c is the equivalent roughness length for C. The relationship between equation (33) and equation (10) is obvious and leads to an expression linking C_d and z_0:

(35) $$C_d = k^2\left[\ell n(z/z_0) - \psi_m(z/L)\right]^{-2} .$$

These equations have been extended to boundary layers on hills by defining extended influence functions that depend on parameters describing flow distortion as well as stability (Finnigan [19]). We consider momentum only for brevity and equation (33) becomes:

$$(36) \qquad U(z) = \frac{u_*}{k}\left[\ell n\left[\frac{z}{z_0}\right] + \psi_m(z/L,\ z/R,\ z/La,\ z/\sigma)\right] \quad .$$

The 'log law' is now expressed in orthogonal, physical streamline coordinates where x is distance along a streamline and z and y, distances along two orthogonal trajectories to the streamline. z corresponds roughly to the surface normal coordinate and y to the transverse coordinate parallel to the surface. R is the local radius of curvature of the streamline and La the local 'e folding distance' of streamwise acceleration so that $1/La = 1/U\ dU/dx$. $1/\sigma$ is streamline torsion which is probably an important parameter in general three-dimensional flows but about which we have little empirical information. The derivation of this coordinate system is described in Finnigan [16] for the two-dimensional case and Finnigan et al. [24] and Finnigan [20] for three-dimensional flow.

Two important restrictions apply to the modified log law, equation (34), Firstly it is confined to a relatively thin layer of depth ℓ, where ℓ is defined by

$$(37) \qquad \frac{\ell}{\lambda}\ \ell n\left[\frac{\ell}{z_0}\right] \simeq 0.025 \ ,$$

λ being the distance between successive hill crests. Because of the dependence of ℓ upon z_0, we find that over hills covered with tall vegetation, ℓ may be of the same order as the heights of the roughness elements (plants) so that no local logarithmic dependence is observed at all. The second restriction applies behind hills with separation bubbles or where flow separation is imminent. In these regions the connection between surface stress ρu_*^2 and the velocity shear remote from the surface is tenuous or totally absent. In such cases, the basic assumptions upon which equations (33) and (36) are based are no longer tenable and the log law may not be applied.

The consequence of this is that only over very gentle, relatively smooth topography is it feasible to average the local, topographically modified log law (36) across the grid patch to yield average descriptions of momentum transport or C_d. Over steeper, rougher terrain, this description is discontinuous and another approach is necessary.

A method adopted by several workers has been to treat the hills themselves as roughness elements and to write a new log law of the form

$$\langle\langle U \rangle\rangle = \frac{\langle\langle u_* \rangle\rangle}{k}\left[\ln \frac{z}{\langle\langle z_0 \rangle\rangle} + \psi\left[\frac{\langle\langle z \rangle\rangle}{\langle\langle L \rangle\rangle}\right]\right] \quad (38)$$

where we have used << >> to distinguish an areal average over the sub-grid patch from the full grid square average. Compatible definitions of $\langle\langle \tau_0 \rangle\rangle$, $\langle\langle u_* \rangle\rangle$, $\langle\langle U \rangle\rangle$ and $\langle\langle z_0 \rangle\rangle$ must now be defined, and different ways of doing this have been proposed by Mason [38], Taylor [54] and Taylor et al. [55]. We shall not describe their methods here but ask instead, what intrinsic limits are there to the applications of the log law? In fact, there are two essential prerequisites for a relationship of the form of equations (33), (36) or (38). The first is that u_* or $\langle\langle u_* \rangle\rangle$ characterize the transfer of momentum down the mean velocity gradient. The second is that a scale separation of 100 to 1000 exist between the boundary layer shear depth δ and the length scale (or scales) characterizing the surface roughness. We have distinguished here between the relevant depth scale δ, the height above which shear is dynamically insignificant, and our earlier value of h because in convective conditions δ should strictly be taken as the surface layer depth, i.e. $\delta \doteq 0.1$ h, although in neutral or stable PBL's we can equate δ and h. In the case of the average log law (38) it is this second requirement that is crucial.

We have already discussed h at length. Order of magnitude values for h are ~ 1000 m for the CBL and 100 m for the stable PBL. If hills are to be regarded as roughness elements, two obvious length scales suggest themselves as characterizing the surface. These are the hill height and the hill spacing. Less obvious scales involving parameters such as the roughness element frontal area per unit ground area have also been employed to determine the dependence of z_0 on roughness geometry (Raupach et al. [49]) but this does not affect the present argument.

Broadly speaking, the height a determines the amplitude of the disturbance caused by an isolated hill in a flow but the hill length scale (usually taken as ~ $\lambda/4$ for a single hill) determines the height to which the hill's influence is felt. This conclusion follows from simple arguments based upon potential flow theory (Hunt [28]) and has been employed in several successful asymptotic analyses of flow over low hills, for example, Hunt et al. [30]. Since, even in very rugged terrain, it is rare to observe λ/a smaller than 3 or 4, the requirement that δ/λ = 100 to 1000 is the crucial one.

This condition would be automatically satisfied if the greater momentum absorption of rugged topography could increase the boundary layer depth sufficiently. This does not happen for two reasons. First of all, in convective conditions, h is determined by the buoyancy rather than the momentum flux. The mechanism by which hilly topography absorbs more momentum than flat country with the same ground cover is the form or pressure drag that develops around the hills. There is no counterpart to this in scalar transfer and the limited experimental evidence suggests that area-averaged, convective heat flux is fairly insensitive to topography. Conversely, in times of strong stability, there is an essential decoupling of the flow below some level from the flow aloft. This is called the 'dividing streamline effect' (Hunt and Snyder [29]). Flow above the dividing streamline continues to behave as if the hills were three-dimensional obstacles and flows both over and around them, while flow below the dividing streamlines goes around the hills. The result is that the surface looks substantially smoother to the boundary layer above than in neutral or convective conditions. Even in neutral conditions, $\langle\langle u_* \rangle\rangle$, which determines the increase in h through equation (14) ($h = \Lambda_0 u_*/|f|$), does not increase rapidly enough with a and λ to maintain the required scale separation.

Evidence for the failure of the average log law concept over large scale topography can be found, ironically, in publications that attempt to prove its usefulness but where the data have to be culled so drastically and arbitrarily to fit the log-law mould that the opposite conclusion can more easily be drawn. See, for example, Kustas and Brutsaert [32].

The attraction of an extended log law such as equation (38) is that it makes maximum use of well tried concepts and formalisms to derive the drag coefficient C_d, through equation (35) or the equivalent scalar transfer coefficient. The alternatives are an appeal to empiricism or a search for a different paradigm in other branches of fluid mechanics.

One well studied field with many parallels to the present case is the flow in plant canopies. It has been recognized for a considerable time that in a 'roughness sublayer' extending about three canopy heights from the surface, adjustments must be made to the standard influence functions ψ of Monin-Obukhov theory to account for the proximity of the flow to the roughness elements themselves and the changed character of the turbulent motion in the plant-air layer. Furthermore, the spatial averaging required for equation (38) is a technique which has reached its maximum sophistication in the plant canopy context (Raupach and Shaw [48]; Finnigan [17]).

The main difference is that, while in fairly dense plant canopies the character of the turbulence is affected by eddies shed in the wakes of individual plants, it is dominated by the much larger eddies that result from a global instability in the inflected velocity profile of the areally averaged flow field (Raupach et al. [50]). When the 'canopy' is very sparse or consists of a range of hills this, is no longer true. The areal average is then a purely abstract concept and can have no dynamical significance and the areally averaged mixing is dominated by the unconnected separation regions behind individual plants or hills.

It seems unlikely, therefore, that formulae describing the enhancement of the area averaged diffusivity can be carried over directly from plant canopies to ranges of hills. It is equally improbable that studies of random rough surfaces at the bottom of laboratory boundary layers have much to offer us for the reasons cited earlier, and we are forced to take a closer look at the flow characteristics of ranges of hills themselves.

Here we forcefully encounter our second lacuna; field, wind tunnel and theoretical attacks on this problem have been very few indeed and what conclusions there are tend to be contradictory. We shall illustrate this briefly by comparing a relatively sophisticated turbulence closure model of flow normal to a range of sinusoidal hills (Taylor et al. [55]) with a field study of a similar situation (Bradley and Coppin [2]). Bradley and Coppin measured turbulence fluxes on 60 m towers placed on the crests of a series of ridges in rugged but homogeneous terrain, consisting of parallel N-S ridges in a westerly airflow. The height a of individual ridges was ~ 200 m and the spacing $\lambda \sim 2000$ m. The ridges were thickly wooded. Measurement of turbulent momentum flux $\rho\overline{u'w'}$ at the 60 m level on the ridge crests amounted to roughly one half the total, areally averaged drag to be expected from such hills according to the model of Taylor et al. [55]. (Here, u' and w' are the horizontal and vertical components, respectively, of the velocity fluctuation.)

This discrepancy could result from three different causes: the turbulence model could be inappropriate for the field situation or simply inaccurate; the hill-top turbulent flux $\rho\overline{u'w'}$ could be a poor representation of the areally averaged value $\langle\langle\rho\overline{u'w'}\rangle\rangle$; or there may be another mechanism of momentum transfer not accounted for by $\langle\langle\rho\overline{u'w'}\rangle\rangle$. Treating these in turn, the three models cited by Taylor et al. [55] agree reasonably well with each other when predicting flow and drag over hills of rather small steepness. However, over steep hills with real separation bubbles, such as those studied by Bradley and Coppin, only the most sophisticated of the models used by Taylor et al. [55] is applicable and this has not been reliably calibrated against field or wind tunnel measurements. The possibility of prediction error cannot, therefore, be dismissed. Turning next to the question of how well $\rho\overline{u'w'}$ (60) at the hill crest matches the areally averaged value, we can say with confidence that, while $\rho\overline{u'w'}$ does vary as a hill is traversed, it attains its maximum value above the crest (Finnigan [18]). In other words, the measured value of momentum flux on the ridge crest can only overestimate the areally averaged turbulent flux.

The third possibility is particularly interesting. The areally averaged momentum flux to the surface can be written as:

$$(39) \qquad \langle\langle\tau\rangle\rangle = -\langle\langle\rho\overline{u'w'}\rangle\rangle - \rho\langle\langle\overline{u}'\overline{w}'\rangle\rangle \quad ,$$

where the velocity field, whose horizontal component is u and vertical component w, has been decomposed into its time average $(\overline{u},\overline{w})$ and fluctuation (u',w') at a point:

$$u = \overline{u} + u'$$

$$(40) \qquad w = \overline{w} + w'$$

while the time average has been further decomposed into its areal average $\langle\langle\overline{u}\rangle\rangle$, $\langle\langle\overline{w}\rangle\rangle$ and local departure therefrom $\overline{u}''$, $\overline{w}''$:

$$\overline{u} = \langle\langle\overline{u}\rangle\rangle + \overline{u}''$$

$$(41) \qquad \overline{w} = \langle\langle\overline{w}\rangle\rangle + \overline{w}'' \quad .$$

In other words, correlated spatial variations in the mean flow can transmit momentum, a circumstance that has long been recognized in plant canopy studies, where it is called the dispersive flux (Finnigan and Raupach [23]).

Wavelike disturbances in the mean flow that are stationary with respect to the underlying hills (lee waves) have been suggested as the source of a similar case of 'extra momentum transfer', observed in the SESAME boundary layer study (Lenschow et al. [33]).

Over more complex topography or in convectively unstable conditions, such simple, wave-like structures in the velocity field have not been observed. Nevertheless, over hills, complex, spatially constant patterns in the mean flow are probably the rule rather than the exception. Two particular candidates may be catalogued. The first is the streamwise vortex rolls that may form on the concave, upwind faces of two-dimensional ridges or the flanks of axisymmetric hills. See, for example, Finnigan et al. [24] for comment on the former case and Hunt and Snyder [29] and Jenkins et al. [31] for model and full scale examples of the latter. Separate, though occasionally coupled to such features, is the three-dimensional separation bubble that forms behind both two-dimensional ridges and three-dimensional hills if they are steep enough. In the two-dimensional case, instability of the detached shear layer ensures that the bubble breaks up into distinct cells.

Perry and Steiner [43] showed that three-dimensional separation bubbles behind bluff bodies in laminar streams have complex topological signatures, in particular, for kinematic reasons they cannot be closed and so have mean inflow and outflow. (See also Tobak and Peake [58].) Furthermore, there is some evidence, both from the behaviour of integral length scales (Finnigan [18]) and turbulence budgets (Pronchik and Kline [45]), that the energy containing turbulence in a separation bubble is of small scale relative to the bubble dimension, being formed in the highly strained, reattaching shear layer behind the bubble and entrained back into the bubble by the weakly unsteady mean flow. In such cases it is valid to treat the mixing or transfer process across the bubble as being made up of a mean advective component and a smaller scale, turbulent component, as a low (turbulent) Prandtl or Peclét Number flow in fact.

Detailed consideration of flow fields of this kind are generally avoided in the atmospheric boundary layer literature but have recently been tackled head on by the wider fluid mechanics community in the study of dynamical chaos in fluids. The complex (chaotic) advection behind steep hills can be termed, in the modern jargon, "Lagrangian turbulence", that is, steady but spatially random motion, and the diffusive component as conventional Reynolds turbulence. The Reynolds turbulence itself would usually have high molecular Prandtl and Peclêt numbers.

In flows of this kind, statistical methods have generally failed to provide the universal scaling laws which could form a basis for interpreting experiment, while case by case treatments have produced little of universal utility (Gibson [27]). As a result, an attempt has been made by Ottino [41] to produce a new paradigm for mixing in these complex situations. It is based upon the topology of the flow, where 'flow' now takes a precise meaning as the spatial pattern of streamlines in the time averaged velocity field. We should note at once that relatively simple, three-dimensional steady velocity fields can result in essentially chaotic streamline patterns (Dombre et al. [11]; Finnigan [20]).

Ottino's paradigm equates efficient mixing with the presence of connections between the stable and unstable manifolds of hyperbolic fixed points in the flow. If the connection is between the stable and unstable manifolds of different hyperbolic points, we speak of a transverse heteroclinic point; of the same hyperbolic point, a transverse homoclinic point.

This topological structure has the effect of stretching and folding stream or iso-concentration surfaces and taking them to their original, or an equivalent, location. The process is the physical realization of the Smale horseshoe map (Smale [52]) and is a signature of dynamical chaos. Ottino's approach differs a little from standard treatments of chaos in that he consistently deals with rate processes rather than long term, asymptotic behaviour. Hence his formalism emphasises mixing efficiencies rather than, for example, Lyapunov exponents as indicators of chaos, although correspondence is regained at long times.

Topological classification of three-dimensional flows in terms of fixed points has been developed to a substantial degree by Chong et al. [6], and Ottino [41] discusses the features of diffusion across separation regions described in this way. It attains further generality and applicability in the present context when it is cast in intrinsic or streamline coordinates. Finnigan [20] shows how to do this and has proposed a heuristic condition for characterizing streamline chaos that traces the evolution of the 'wrinkliness' or folding of stream surfaces in steady velocity fields. In this theory, chaos depends, suggestively in the light of Ottino's work, on the presence of hyperbolic points in the tangent map of the flow. A further conclusion of Finnigan's analysis is that chaos requires a component of vorticity in the flow direction such as inevitably occurs on the flanks of isolated hills, where the vortex lines of the mean flow wrap around the obstruction.

This is a rapidly developing field which may provide a much needed fresh approach to the analysis of mixing in complex, topographically forced flow fields. At the very least, we might expect an analysis based upon these concepts to suggest new experiments to resolve and quantify the relative roles of Reynolds turbulent fluxes and dispersive fluxes in transfer of both momentum and scalars to steep topography.

5. MOMENTUM AND SCALAR TRANSFER IN TIMES OF STRONG STABILITY

The third lacuna that we shall discuss is really a very wide gap furnished with occasional stepping stones and bridges. As a result we can only touch briefly on some of the many topics that could reasonably fall under this heading. We will treat the subject under two sub-headings: the processes that govern the noctural decay of previously convective turbulence towards the synoptic state and the determination of r_a values at times of strong stability. When stability is weak or moderate, let us say when local Richardson Numbers are between 0 and 1/4, established theory is quite sufficient. The dynamics of the whole stable PBL are adequately described by turbulence closure models such as that of Brost and Wyngaard [4] or the local similarity scheme of Nieuwstadt [40]. It is at times when the average Richardson Number in the lowest 1000m of the atmosphere is larger than 1, conditions typically coinciding with light winds and clear skies, that real difficulties occur.

At such times, the atmospheric dynamics are often dominated by propagating internal gravity waves that have length scales much longer than those of turbulent eddies but whose periods may easily coincide with turbulent eddy timescales. The ubiquity of these structures is typified by the recent study of one month's contiguous data by Einaudi et al. [13]. Using records from the Boulder Atmospheric Observatory in Colorado, USA, they found short period waves (1-5 min) during 40% of the nights between mid-March and mid-April, while longer period waves (10-20 min) were present 95% of the time. Basing his comments on European data, Bull (personal communication) has suggested that these figures represent lower limits on the frequency of occurrence.

Propagating internal waves of this kind can be generated by a variety of mechanisms. Marht [37] has identified inertial oscillations, directional shear coupled to downslope drainage, 'shooting flows' and Ekman gravity flows as possible sources of the waves while Einaudi and Finnigan [15] (and references therein) have studied a large number of cases of Kelvin-Helmholtz waves that were generated on elevated shear layers or low level jets.

A feature of these disturbances, however generated, is that they couple strongly with existing turbulence or generate turbulence ab-initio and maintain it at a significant level despite background Richardson Numbers much larger than $\frac{1}{4}$. This coupled wave-turbulence field can display some or all of the following distinguishing features: it may be intermittent on time scales of several or many wave periods; it may drive counter-gradient heat and momentum fluxes in horizontally homogeneous conditions; it may stratify into a layered structure of alternating strong and weak shear, strong shear being associated with high turbulence intensities. The height scale of these layers is typically 10–50 m (Einaudi and Finnigan [15]; Finnigan et al. [22]; Li et al. [34]). An understanding of the details of this coupled wave–turbulence system is essential to predict the rate at which conditions in the lower atmosphere approach higher-level synoptic values, as it is these unique dynamics that drive the necessary diffusion processes, as well as to extend the simple models of PBL behaviour we used in §3 to times of strong stability.

The fundamental problem is to characterize the behaviour of inhomogeneous turbulence maintained by unsteady forcing against a strongly stable buoyancy gradient. Two theoretical approaches, which have shown some promise, are presently being pursued. The first is the extension of rapid-distortion (or linearized turbulence) theory to unsteady, stably stratified and inhomogeneous flows. The applicability of rapid-distortion theory, a field having much in common with classical, small-perturbation theory, follows from the fact that wave periods often match the periods of energy-containing eddies so that wave straining is rapid compared to turbulent relaxation times. Characteristic signatures of this situation were noted by Finnigan and Einaudi [21] in a near neutrally stratified, wave-perturbed boundary layer.

A second approach is the use of higher order closure schemes to model the relaxation of the turbulence field under wave straining. This has been done, using a $1\frac{1}{2}$ order, k–ϵ model, by Einaudi et al. [12]. The challenge now is to combine this with a realistic treatment of the near surface behaviour of the gravity wave so as to quantify an idea proposed by Jones and Hooke (personal communication). They suggested that the vertical structure of the multiple shear layers observed in the lower atmosphere (most clearly by remote sensing) corresponds to the vertical wavelength of the 'viscous wave' that, in linear perturbation descriptions of gravity waves, must be added to the inviscid solution to satisfy a no-slip condition at the ground. See, for example, Einaudi and Finnigan [14]; Jones and Hooke (personal communication). If the viscosity used in the model solution is the molecular viscosity, then the vertical wavelength λ_z of the viscous wave is only a few centimetres but if a realistic eddy viscosity is used, λ_z grows to tens of metres and provides a reasonable match to the observations. Unfortunately, a temporally constant eddy viscosity does not describe wave-turbulence coupling well and so the use of a more sophisticated approach is clearly indicated. The hope is that the vertical wavelength λz might provide the elusive scaling parameter that would not only describe the turbulence locally, but also respond to both wave forcing and the presence of the ground.

Turning to our second sub-heading, the specification of r_a in very stable times, much of what we have said above remains relevant. We can draw attention particularly to the times of counter-gradient flux, which would produce negative r_a values, and the complicating effects of multiple shear layers. However, two further aspects of stable layer behaviour must now be considered. The first of these is the possibility of local advection driven by katabatic effects such as gravity currents. A catalogue of some important classes of these flows on the boundary-layer scale may be found in Marht [37]. We must also consider the role of non-propagating gravity waves or lee waves. (Strictly, such waves propagate upwind at the mean wind speed but they are stationary with respect to the ground.) Lee waves have been widely studied at large scale as mountain waves. See, for example, Bretherton [3]. Here we wish to point out the possible importance of these waves on the PBL scale. Two features should be noted. The amplitude of the lee wave component of the momentum flux $\langle\langle\rho\bar{u}''\bar{w}''\rangle\rangle$ is proportional to a^2 so that, as the height a of the hills increases, the wave stress soon exceeds the Reynolds stress component $<<\rho \overline{u'w'}>>$ of momentum flux, even over moderate terrain (Chimonas and Nappo [5]). Secondly, the direction of the shear stress vector associated with the lee waves is perpendicular to the ridges generating those waves, not necessarily parallel to the mean, near-surface flow (Chimonas and Nappo [5]). This can have important consequences for the resultant wind directions in the middle and upper PBL.

In contrast to the work on propagating waves mentioned earlier, there has been, to this author's knowledge, no work at all done on the interaction between turbulence and lee waves in the PBL. The problems must be very similar to those encountered in propagating waves but, undoubtedly, new features will arise. Finally we should point out that these waves are not merely a theoretical prediction but were measured by aircraft based instruments in the SESAME experiment (Lenschow et al. [33]) and were estimated there to carry a significant fraction of the total momentum flux.

6. CONCLUSIONS

The object of this paper was to reveal some of the problems that occur when we write land surface boundary conditions for GCM's. Specifically we have discussed the way that sub-grid scale, horizontal variability might be handled and have proposed a scheme that relies on using a simple description of the planetary boundary layer as the link between heterogeneous surfaces and the synoptic scales that are resolved by the GCM. This simple description requires, paradoxically, a good understanding of PBL physics in both daytime convective and nighttime stable conditions as well as the ability to attach quasi one-dimensional descriptors to each distinct patch of the inhomogeneous surface.

We have shown that for a significant range of conditions our understanding of PBL dynamics is not equal to this task. In convective conditions, when surface properties, particularly aerodynamic and surface resistancees to heat and moisture transfer, change over length scales comparable to the dynamic equilibrium length scale $\frac{Uh}{w_*}$ of the CBL, we are required to predict the behaviour of an advective CBL. At present no suitable models exist to do this.

When the boundary layer flows over an extensive region of hilly topography, then established theory is only capable of predicting r_a values in very restricted circumstances: in even moderately steep and rugged terrain it fails completely. The understanding we lack is of the nature of momentum and scalar transfer over regions of separated flow. Modern dynamical chaos theory has produced some quite new ways of looking at this problem which may allow valuable insights in the future.

Under very stable conditions, specification and parameterization of atmospheric exchange processes requires a good understanding of the various modes of wave-turbulence interaction. At present this is a field poorly served by theory; most studies of this problem have involved analysis of field data while the voluminous literature on wave dynamics almost all avoids mention of turbulence and indeed nonlinearity of any kind.

It is clear that the construction of GCM's and the interpretation of GCM output at local or regional scale cannot be viewed as a problem in efficient codification or interpretation of existing knowledge. There remain significant gaps in our understanding of boundary-layer processes that pose real obstacles to matching GCM's to the real world. To fill these gaps will require sophisticated mathematical techniques no less than experiment.

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