EXAMPLES IN THE INVERSION OF SEAFLOOR MAGNETOTELLURIC DATA

F.E.M. Lilley and G.S. Heinson

1. INTRODUCTION

Much modern understanding of Earth's interior comes from marine geophysics; the study of various physical properties of Earth beneath the ocean floor. The present paper concerns a recently-developed marine method known as seafloor magnetotelluric (MT) measurement. The method, for which new interpretational procedures are still being developed, exploits the phenomenon of natural electromagnetic induction which occurs in Earth on a global scale. Fluctuations in the natural electric and magnetic fields are measured on the seafloor, and the ratio of the electric signal to the magnetic signal is termed the MT impedance. This quantity is frequency dependent, complex, a tensor, and a function of the electrical conductivity beneath the observing site and of the salt water in the ocean above.

The inversion of such MT data, to obtain electrical conductivity values for the material beneath the seafloor, then becomes central to determining geophysical information. This paper presents and compares inversions of a single data set (derived from actual seafloor observations) by four different, and published, methods. The methods are those of Parker's D^+, D^+ layered, Fischer and Le Quang, and Constable et al. Each method approaches the inversion problem from a different viewpoint, and it is therefore of substantial significance that the results of the inversions are in agreement, especially concerning those characteristics of most importance to geophysics. The agreement gives confidence that such data can be inverted to give reliable information on the electrical conductivity structure beneath the seafloor. In particular, such one-dimensional inversions indicate that an asthenospheric layer of partial melt, with high electrical conductivity, at depth of order 100 km in the Earth, may be resolvable from seafloor magnetotelluric data.

2. DATA

The particular example to be studied in this paper comes from site TP3 of the Tasman Project of Seafloor Magnetotelluric Exploration (TPSME) or Tasman Experiment (Filloux et al. [7], Lilley et al. [10]). The MT impedance values were obtained by Ferguson [5] (and see Ferguson et al. [6]). Site TP3 is at geographic position (38°54'S, 159°50'E) in mid-Tasman Sea, approximately half-way between Australia and New Zealand, as shown in Fig. 1.

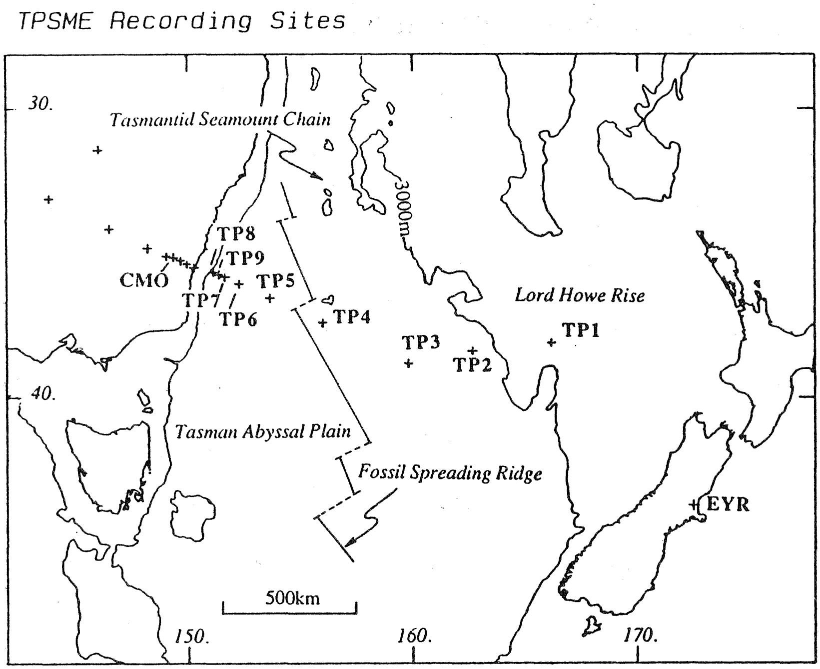

Fig. 1 The location of the seafloor Tasman Project. Site TP3 is shown in the centre of the Tasman Sea.

The data inverted in this paper are for the component of the MT impedance which corresponds to alignment of the electric field along the major axis of the Tasman Sea (approximately north-south). It is this component of the MT impedance which is most reasonably inverted on the basis of a one-dimensional conductivity structure.

3. ONE-DIMENSIONAL INVERSION

For the problem of an electrically conducting layered half-space, the relationship between the observed magnetotelluric quantities and the unknown conductivity profile is non-linear. This consequence of the physics of electromagnetic induction renders the magnetotelluric inverse problem non-linear, and raises many important points in its execution (see, for example, Anderssen [1], [2]).

To introduce the actual inversions carried out in this paper, it is instructive to first review the mathematical questions of a) existence of solutions, b) uniqueness, and c) construction of a suitable model.

3.1 Existence

It is necessary to decide whether a given data set is compatible with a mathematical model, in this case the model of electromagnetic induction in a one-dimensional conductivity structure. The question of existence concerns finding any model which can adequately satisfy the observations. Weidelt [15] expressed the conditions necessary for existence in nineteen inequalities involving the real and imaginary parts of a frequency-dependent complex parameter $c(\omega)$, defined

$$c(\omega) = -E_x/(i\omega B_y)$$

where E_x and B_y are the measured components of electric and magnetic field (orthogonal to each other), ω is the frequency, and $i = \sqrt{-1}$. Weidelt [15] expressed $c(\omega)$ as a Stieltjes integral

$$c(\omega) = \int_0^\infty da(\lambda)/(\lambda + i\omega)$$

Parker [11], [12] used a discrete formulation of $c(\omega)$,

$$(1) \qquad c_j = \int_0^\infty da(\lambda)/(\lambda + i\omega_j) \qquad j = 1,2, \ldots N$$

where the ω_j are N discrete angular frequencies. It can be shown that if there are any solutions to equation (1) there must be one in which $a(\lambda)$ consists of a function which is constant except at J points of discontinuity, and $J \leq 2N$. In practice, the integral is approximated by a summation. If a solution does exist, then there will be one which generates a conductivity profile as a series of delta functions,

$$(2) \qquad \sigma(z) = \sum_{k=1}^{J} \tau_k \delta(z - z_k) \qquad k = 1,2, \ldots J$$

where z is depth into Earth, τ_k is the electrical conductance of the spike at $z = z_k$, and $\delta(z - z_k)$ is a delta function at $z = z_k$. This class of solution is called D^+ and its existence is thus a necessary and sufficient condition for the existence of a solution of the inverse problem, for some given set of data.

The question of acceptable agreement between theory and observation must be considered. Parker [12] introduced a chi-squared misfit, in which if F_j are the model values and D_j are observations with independent Gaussian random variances at N discrete frequencies, then

$$\chi^2 = \sum_{j=1}^{N} (D_j - F_j)^2/S_j^2 \tag{3}$$

where S_j is the uncertainty of D_j. The problem of existence is then reduced to finding the model with χ^2 as small as possible; if this model is considered unacceptable, in that its misfit is too large, then no other model will be compatible with the data.

Parker and Whaler [13] adapted the existence theory for noisy data, minimizing a function

$$\sum_{j=1}^{N} \left| c_j - \int_0^{\infty} da(\lambda)/(\lambda + i\omega_j) \right|^2 / S_j^2$$

The importance of the papers by Parker [11] and Parker and Whaler [13] is that they show that the D^+ model has the smallest possible χ^2. Thus, if a D^+ model cannot satisfy some observed data, then no other model will be able to.

3.2 Uniqueness

If the response of a one-dimensional model is compatible with a set of observed data, it is pertinent to ask what other models could also accomplish this feat. Bailey [3] showed that for exact data at all frequencies, a solution, if it exists, is unique.

A 95% confidence limit on the misfit between predicted and observed data was introduced by Parker [11] as an "acceptable" level of misfit, dependent on the number of data points used. That is to say, the response of a model could misfit each data point by two standard deviations of the number of data points and still be considered acceptable. Such 95% confidence limits may seem a low level of accuracy, however, it is important that the data not be "over-fitted" by a conductivity structure which is not supported by the number of observations. For N discrete observational frequencies, the 95% confidence limit on the chi-squared misfit is taken as

$$\chi^2 = N + 2\sqrt{2N} \quad (4)$$

If the misfit from the D^+ model is less than this value of χ^2, then an infinite number of models will be compatible with the observed data. If it is larger, then no model exists which will fit the data satisfactorily. This problem is that of non-uniqueness.

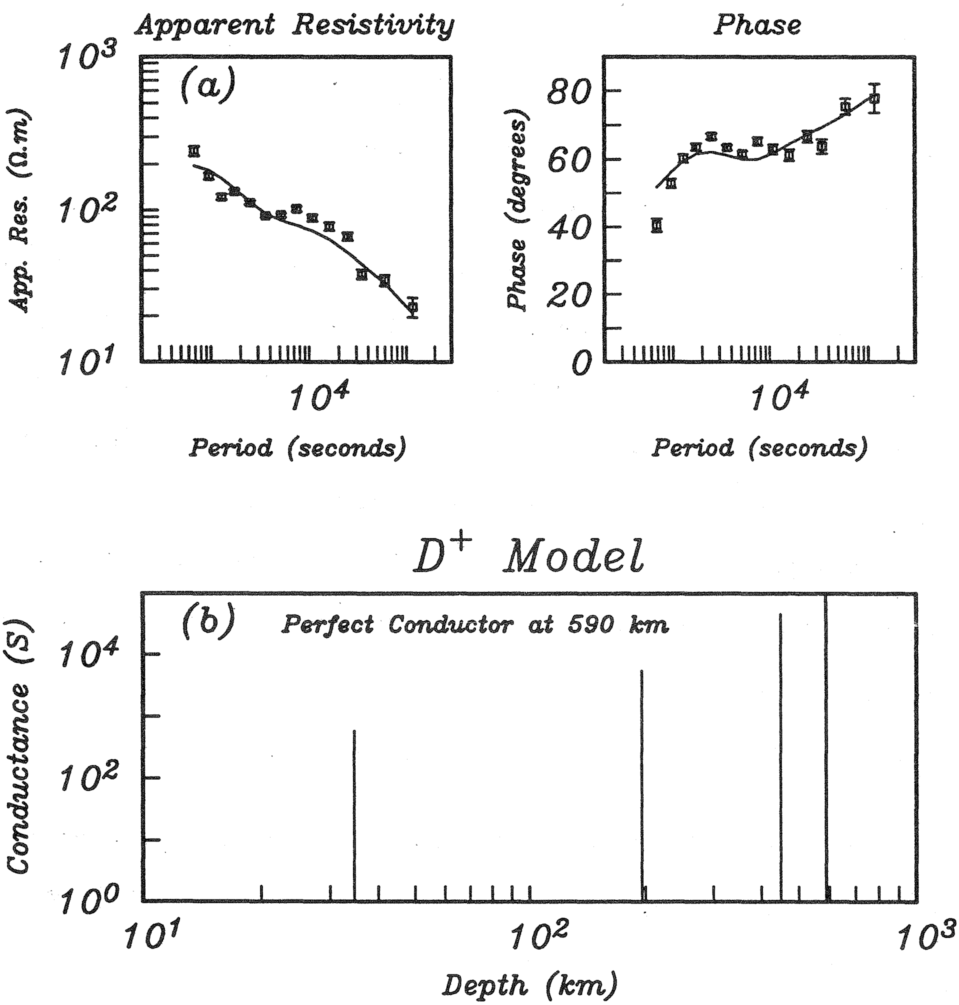

Fig. 2 a) The basic MT data to be inverted are plotted as error bars, with the response of the model in Fig. 2b superimposed as lines. b) The D^+ model obtained by inverting the observed data of Fig. 2a.

3.3 D^+ model for MT data

Figure 2 illustrates the data set described in section 2, and the result of a D^+ inversion performed on this data set, using Parker's algorithm. A statistical χ^2 misfit between the response of the model (solid line) and the observed data (error bars) gives a value of 267. The MT parameters of apparent resistivity and phase are calculated at 14 frequencies, and while these parameters are not strictly independent of each other, in practice, with noisy data, they are usually taken to be so. The 95% confidence limit for this set of data, with an N of 28, should be 43. Hence the observed data are not fitted well, and it must be concluded that no one-dimensional model is compatible with the observed data at this site. Ferguson [5] attributed this phenomenon to electromagnetic induction in the Tasman Sea.

The D^+ model in Fig. 2b is however the best model obtainable for the data, and its response is shown in Fig. 2a as continuous lines.

4. CONSTRUCTION OF MODELS

This section deals with the construction of a suitable one-dimensional model which, if it exists, provides geophysical information. The lack of uniqueness of solutions means that there is always a degree of arbitrariness about any general one-dimensional model, and this factor may place reservations on the usefulness of the model for geophysical interpretation.

Models should, as a rule, be kept as simple as possible; however the idea of simplicity may be subjective. One school of thought regards solutions consisting of a small number of layers separated by discontinuities as being simple. Alternatively, smoothly varying models with smooth gradients are preferred by some geophysicists. There is independent geophysical evidence to suggest that discontinuous layered models are appropriate for the upper crust, while deeper in the Earth, where temperatures primarily influence electrical conductivity, the changes will be relatively smooth.

4.1 Layered models from D^+

A simple layered model may be constructed directly from a D^+ model, by taking the spikes which are of infinite conductivity but finite conductance, and distributing their conductances over suitable depth ranges.

Figure 3 shows a layered profile thus constructed from the D^+ model of Fig. 2. In this case the depth range over which a spike conductance is uniformly distributed is taken to be the midway points between a spike and its upper and lower neighbours. For the uppermost spike, the distance halfway to

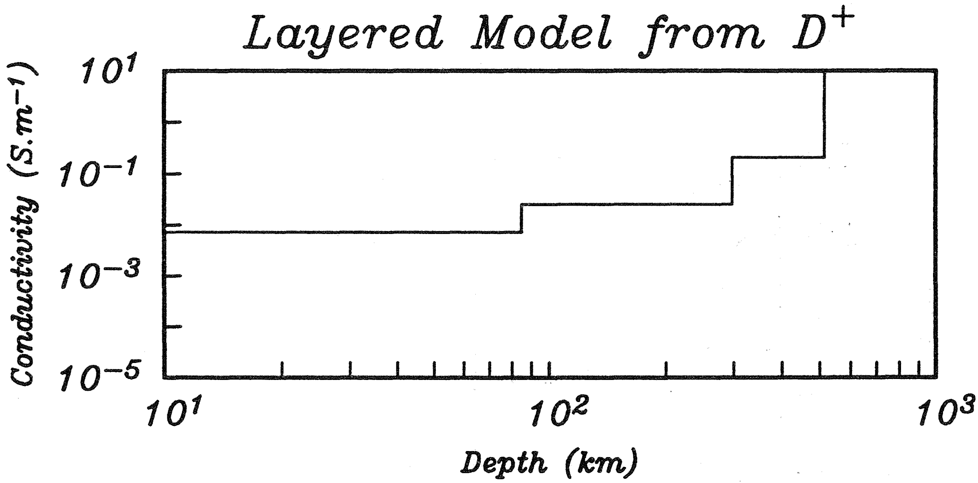

Fig. 3 A layered model constructed from the D^+ model in Fig. 2b.

the surface is used, and for the bottom spike, half the distance to the perfect conductor. Smith and Booker [14] found that such simple layered models were a satisfactory inversion of synthetic data.

4.2 Layered Models of Fischer and Le Quang

Fischer and Le Quang [8],[9] proposed an elegant layered modelling scheme, in which the least number of layers required to adequately model a set of data is found by a systematic search procedure. For that desired minimum number of layers, the best-fitting model is sought by a ridge regression scheme.

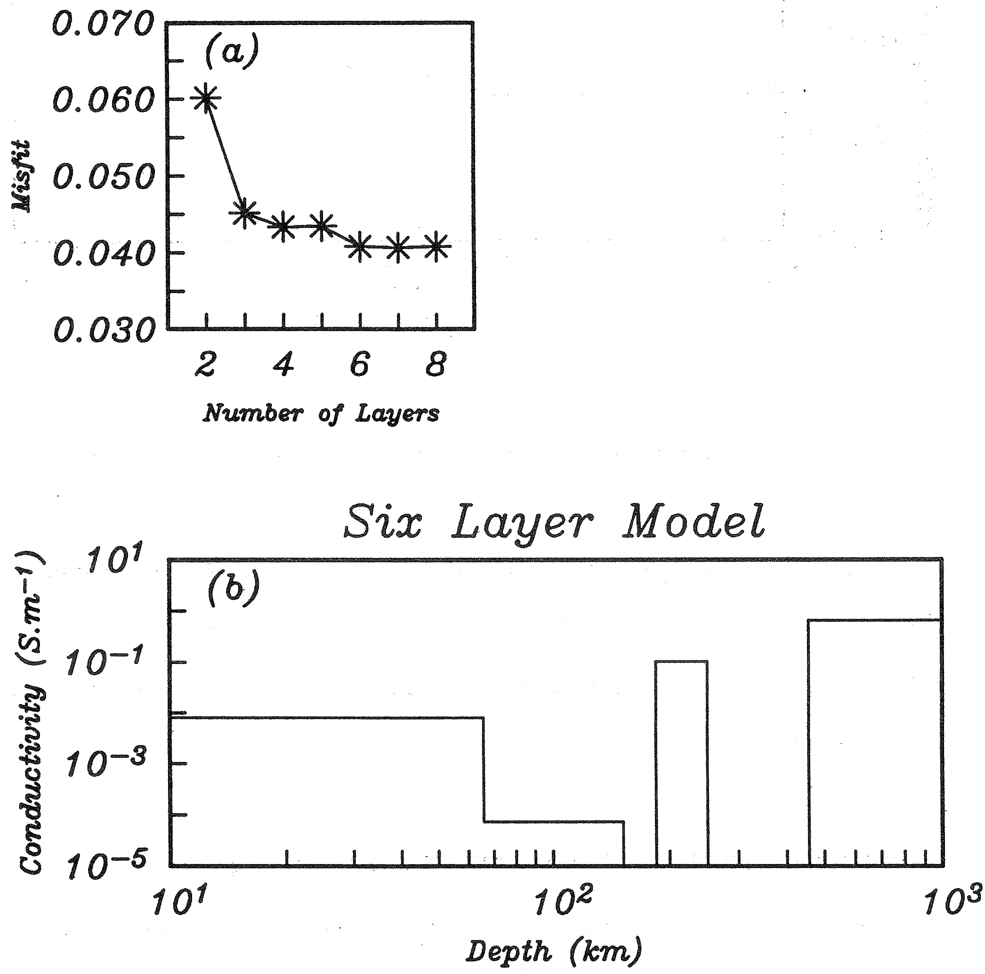

Fig. 4 a) Fischer and Le Quang results for a series of layered start models. The six-layer model shows the best misfit, whilst not introducing unnecessary layers that are not required by the data. b) The resulting six-layer model from Fischer and Le Quang's modelling scheme.

An arbitrary starting model of a fixed number of n layers is first constructed, with (2n - 1) parameters of thickness and conductivity (as the bottom layer is always indefinitely thick). A ridge regression search is then undertaken in parameter space, so that the optimum parameters are found. The resulting conductivity profile will have the minimum chi-squared misfit with the observed data, for that number of layers. Checks are made to ensure that a global and not a local minimum has been located by the search routine.

By altering the number of layers in the start model, the resulting misfit may be examined. Figure 4a illustrates this relationship for the data set of this paper. The model with the chosen number of layers must show a significant improvement in chi-squared misfit over models with less layers, but insignificant improvement over models with more layers. A statistical technique, such as an F-Test, may be used to quantify such a judgement. Thus, in Fig. 4a, a six-layer model is chosen as the simplest model to fit the data adequately; and this six-layer model is shown in Fig. 4b.

4.3 Smooth models

A very different approach to the problem of determining a simple model has been developed by Constable et al. [4]. This modelling scheme is known as "Occam inversion", after the

William of Occam ideal of simplicity (as expressed in "Occam's razor"). The line of thought followed is that as layered models rely on parameterization, such parameterization must be appropriate before a layered-model solution will reflect true Earth structure. Further, excessively complex parameterization, as might be suggested generally by geological well-logging for example, may not be constrained by the observed data in any particular instance. The objective is therefore to construct models which reflect the limitations of the observing experiment.

Constable et al. [4] allow their model to be as flexible as possible, but the complexity is suppressed explicitly by defining a term (R_1) known as the roughness of the model,

(5) $$R_1 = \int_0^\infty (dm(z)/dz)^2 \, dz$$

where m(z) is the electrical resistivity or its logarithm. The strategy is to find the solution agreeing with the measurements which has some desired level of roughness, a procedure of regularization which is common in the solution of ill-posed problems.

The roughness in the present case can be written in the discrete form, for n layers,

(6) $$R_1 = \sum_{i=2}^{n} (m_i - m_{i-1})^2$$

and for N observations D_1, D_2, ... D_N with associated errors S_j the chi-squared misfit can be written as in equation (3)

(7) $$\chi^2 = \sum_{j=1}^{N} (D_j - F_j(m))^2/S_j^2$$

where the model value $F_j(m)$ is written as a functional of the geophysical structure.

The optimisation proceeds by first defining a desired level of misfit, χ^{*2}. The constraint equation on the misfit is rearranged to form an expression equal to zero, which is then multiplied by a Lagrange multiplier, and added to the roughness value R_1, which must also be minimized.

The original function is a minimum where the new one, U, is stationary without constraint. This circumstance may be expressed

(8) $$U = R_1 + (1/\mu)(\chi^2 - \chi^{*2})$$

where $1/\mu$ is the Lagrange multiplier. For any value of m, this functional of m is stationary when $\nabla_m U$ the gradient of U with respect to m, vanishes. The Lagrange multiplier must be selected so that the desired χ^2, namely χ^{*2}, is obtained.

The quantity μ may be regarded as a smoothing parameter. As μ tends to zero, the roughness is of little significance and the solution will attempt to satisfy the observed data at the expense of smoothness. When μ is large, the solution is

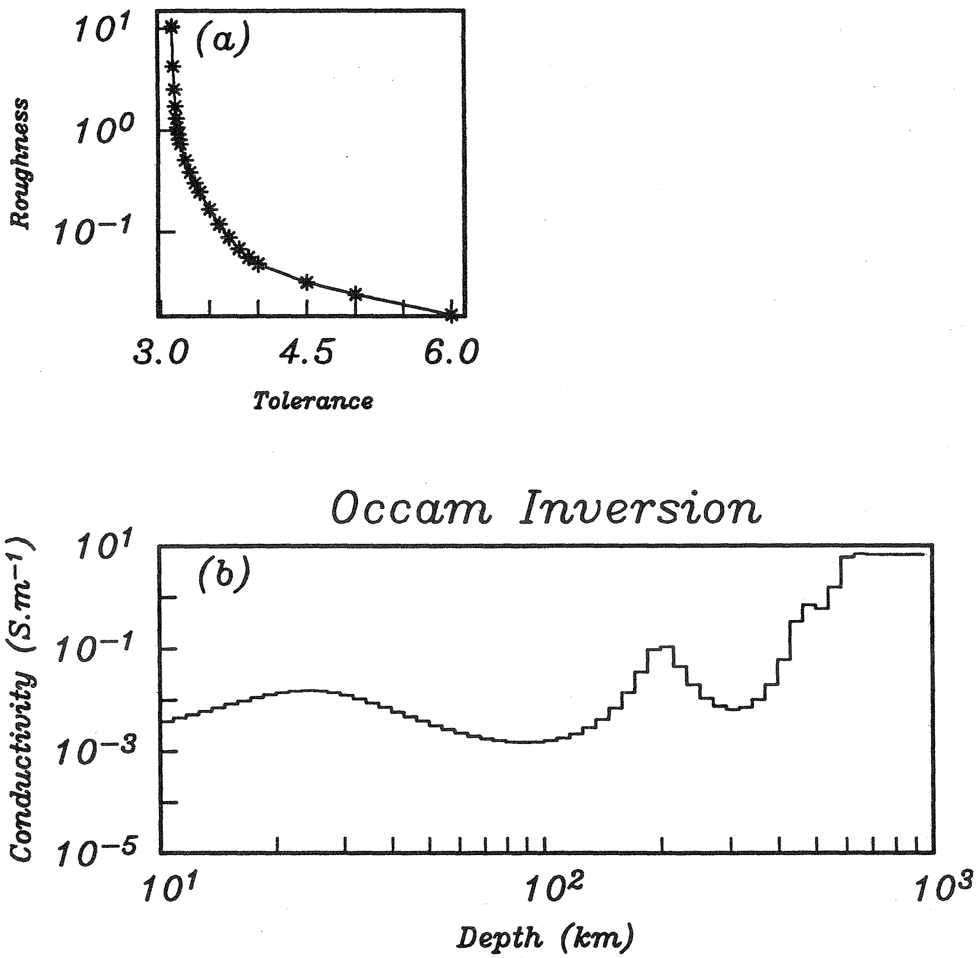

Fig. 5 a) Trade-off curve between the tolerance (reckoned as the normalized chi-squared misfit) and the corresponding roughness, for Occam inversion of the MT data. b) Resulting Occam inversion of the MT data.

influenced little by the data misfit, and the resulting profile is very smooth (in the limit, a uniform half-space).

Figure 5 illustrates an Occam inversion for the MT data described in section 2. Sixty discrete layers, increasing in log thickness, form a starting model. Ideally the desired level of misfit χ^{*2} could be set at the 95% confidence limit for the number of data points, however the D^+ inversion in section 3.3 showed that no one-dimensional model was compatible with the data. Thus some higher level of misfit must be chosen. A trade-off curve between desired misfit and roughness can be constructed and is shown for the present in Fig. 5a.

As the desired misfit is reduced, the roughness is increased. If the thickness of the layers tends to zero, the smallest misfit will correspond to the D^+ model. At the other extreme, with large desired misfits, the roughness tends to zero, and the model to a uniform half space.

The Occam model presented in Fig. 5b falls somewhat arbitrarily between these two extremes. Although smooth models are mathematically correct and avoid the *a priori* parameterization of layered models, they can be bland, and offer little insight into the geoelectric structure of Earth. Electromagnetic energy obeys the diffusion equation in the Earth, so that the effects of sharp geophysical boundaries will be smoothed over some depth scales. It is for this reason that smooth models are perhaps the best interpretation of the information which can be recorded at Earth's surface.

5. CONCLUSIONS

The dichotomy between the philosophies of layered and smooth models cannot easily be resolved. The best approach appears to be to perform both types of modelling, and compare the information each provides. Good agreement will indicate features which are required by the data, as opposed to features which are artifacts of a modelling algorithm.

Figure 6 shows superimposed the results of the four inversions: D^+, layered D^+, minimum layers, and Occam. The agreement is generally excellent and shows the relative strengths of the different techniques. It is interesting to note that D^+, which is physically unrealistic (specifying spikes of conductivity), nevertheless provides a layered model which is consistent with the others.

In geophysical terms the set of inversions indicates the electrical conductivity of the upper mantle to be less than $10^{-2.5}$ $S.m^{-1}$ down to depth 100 km. Below that depth an increase in conductivity commences of some two orders of magnitude. There is then a reduction in conductivity again before a deeper increase to greater than 10° $S.m^{-1}$ at depth 500 km. Beyond this depth the method does not penetrate.

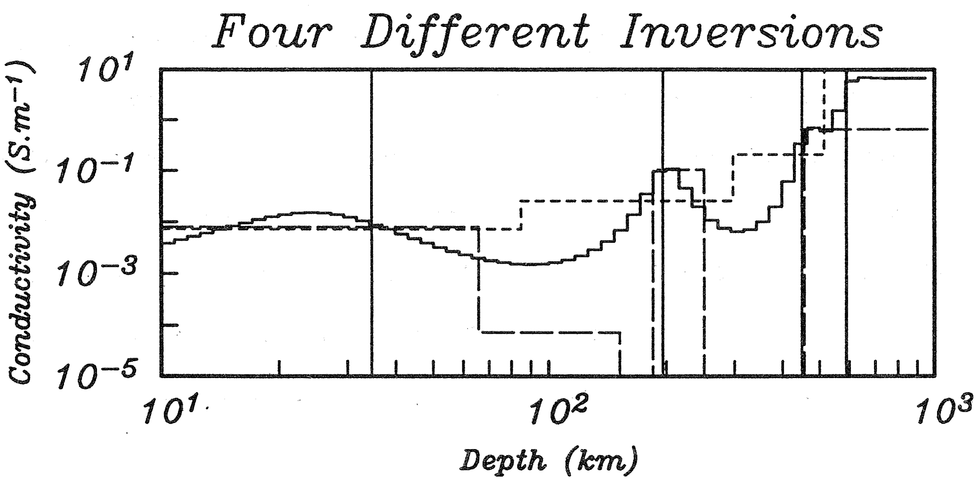

Fig. 6. The models obtained by the four different inversion procedures superimposed for comparison. These are the models presented individually in Figs 2b, 3, 4b and 5b.

The deeper increase is found commonly in global analyses of magnetic variation data, while the increase which commences at depth of order 100 km may be more local to the Tasman Sea region. Such a feature may be of major significance to geophysics, as it occurs in the depth range of the postulated asthenosphere, a zone in the Earth of reduced shear strength. That such a zone might be indicated by high electrical conductivity has been a major driving factor in seafloor magnetotelluric studies.

The inversion exercises presented in this paper have been on the basis of one-dimensional structure. That the original data set did not satisfy the one-dimensional criterion set up for it is an example of the commonplace nature of such departures of observed data from ideal models. In the wider study of the magnetotelluric inverse problem, understanding and allowing for more complicated induction geometries is a major present frontier. "Thin-sheet" modelling of ocean effects is one line of research currently being actively pursued.

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Research School of Earth Sciences
Australian National University
GPO BOX 4, Canberra 2601
Australia