# EARTHQUAKE LOCATION AS AN INVERSE PROBLEM

*B. L. N. Kennett*

The location of earthquakes in space and time is an important practical inverse problem which may be used to illustrate features common to a wide range of situations.

*The basic problem is:*

Given the arrival times of seismic waves at a number of different receivers, deduce the origin time and spatial location of the hypocentre of the earthquake (i.e. the point at which radiation is initiated).

*Incidental problems are:*

i) The identification of the seismic phases whose arrival time is measured.

ii) The choice of earth model used to calculate the theoretical passage times for the seismic waves using ray theory.

We will assume that the earth model is known and that we have $N$ observations

$t_i$ - the arrival times of identified seismic phases at seismic receivers

Several different phases can often be recognised at the same receiver corresponding to different ray paths through the earth model or alternatively to different wave types. The specification of the location of the earthquake requires the determination of four parameters

$t_h$ - the origin time of the seismic disturbance

$x_h, y_h, z_h$ - the spatial coordinates of the hypocentre.

For the $i$th seismic phase we calculate the travel time $t_{ri}\,(x_s,y_s,z_s)$ for a source at $(x_s,y_s,z_s)$ to the requisite receiver. This will be determined by ray tracing in a particular earth model. From the travel times we can construct estimates of the arrival times of the phases for an assumed origin time $t_s$ as

$$t_{ci}\,(x_s,y_s,z_s,\,t_s) = t_s + t_{ri}\,(x_s,y_s,z_s)$$

and these values are to be compared with the measurements $t_i$. We note that there is a separation between the dependence of the estimated arrival time $t_{ci}$ on the spatial and temporal components of the estimated location.

The conventional treatment due to Geiger (1910) is to adopt a least-squares measure C for the misfit between the observed and calculated travel times

$$C = \sum_i [(t_i - t_{ci}) / \sigma_i]^2$$

where the $\sigma_i$ are estimates of the variance of the observations. The estimates $(x_s, y_s, z_s, t_s)$ are then assumed to be close to the true location. Linearising about the estimated location generates a set of equations of the form

$$G\ (x_s, y_s, z_s)\ \Delta\mathbf{h} = t_i - t_{ci}\ (x_s, y_s, z_s, t_s)$$

where $\mathbf{h}$ is the hypocentre 4-vector $(x_h, y_h, z_h, t_h)$, and $G$ depends on the derivatives of the travel times with respect to the source parameters. These equations can be solved for the update to the hypocentral location $\Delta\mathbf{h}$ by introducing a generalised inverse to $G$. The mixture of dimensions in $G$ causes some problems because the relative sizes of the terms can be rather different. We note that $\partial t_{ci}/\partial t_s = 1$, whilst the spatial derivatives are often much smaller. Once a new estimate for the hypocentral location is found the linearisation is repeated and the process iterated to convergence.

Convergence can generally be obtained if the errors in the arrival times are small and the assumed earth model gives a good representation of the travel times in the region. However, there is a strong trade-off between the depth estimate $z_{he}$ and the estimated origin time $t_{he}$ that is best resolved if observations of the different wavetypes P and S can be made. The properties of this style of inversion are discussed in detail by Buland (1976).

With the advent of faster computers it is now feasible to calculate the travel times afresh for each postulated source location rather than rely on linearisation. This more flexible formulation allows the introduction of different (and better) representations of the expected misfit distribution between observed and calculated arrival times.

The statistics of such travel time residuals can often be well represented by a distribution suggested by Jeffreys (1932) which consists of a Gaussian superimposed on a slowly varying pedestal function. For regional seismic observations, at least, the narrower Gaussian represents the distribution of picking errors and the broader background arises from the differences between the real earth and the times predicted from a simplified model. A further contribution to the pedestal will come from major blunders in the assignment of arrival times, e.g. identification of the wrong minute.

We have noted earlier the separation of the dependence of the arrival times for the seismic phases on the estimate of the origin time $t_s$ and the spatial location of the earthquake source $(x_s, y_s, z_s)$. This separation can be exploited in a non-linear inversion scheme in which the location of the minimum of the misfit function is approached by a directed spatial grid search with bracketing of the origin time. This scheme due to Sambridge & Kennett (1986) does not require any derivatives to be evaluated and works well e.g. for regional phases where the costs of recalculating travel times are low. The same algorithm can be used with different measures of the misfit between observed and calculated arrival times. Much better results can be achieved by using a maximum likelihood estimator based on the Jeffreys distribution than a least squares measures of misfit.

A slightly less elegant method works well for the location of distant earthquakes where the costs of calculating individual travel times are larger. The travel times for the various phases at the different receivers are calculated for a fixed set of sources on e.g. a 7x7x7 spatial grid. The current misfit measure is then calculated for a suite of origin times using the existing array of source locations. The minimum misfit measure found during the discrete search in space and time is then used as the centre of a new and somewhat smaller 4-dimensional mesh. The process of shrinking the mesh size is then continued until prescribed tolerance levels on the location are met. The convergence of this procedure can be readily tracked by monitoring the variation in the misfit values across the 4-dimensional meshes.

Such a procedure cannot of course guarantee that a global minimum for the misfit function will be found via the discrete search. However the results obtained with different starting meshes are generally very consistent. Since, once again, no differentiation is involved any reasonable measure of the misfit between observed and calculated arrival times can be used.

Figure 1 illustrates the sensitivity of earthquake locations to the earth model employed. The data set consisted of 42 P phase arrival times from world wide seismic stations for an underground explosion in East Kazakhstan, USSR for which accurate source location information has been published. Eight different models are compared, we see that depth errors vary more than time errors and the geometry of the recording sites has lead to a displacement of the estimated hypocentres from the true location by about 10 km in longitude. All the earth models assumed spherical symmetry and 3-dimensional structure within the earth will contribute to the location error.

In the absence of external information about the seismic source, the accuracy of the hypocentral location has to be estimated from the information available in the inversion. The implied precision of the estimates is therefore model based and can be distinctly misleading to the error in the location of the true hypocentre (as would be the case for the Soviet event in fig 1).

Confidence intervals for the hypocentral location can be deduced by examining the spatial distribution of

$$\Gamma(\mathbf{h}) = C(\mathbf{h}) - C(\hat{\mathbf{h}})$$

where C is the misfit criterion and $\hat{\mathbf{h}}$ is the best estimate for the location of the hypocentre. Within the nonlinear schemes it is also possible to examine the influence of individual data on the location procedure. One such approach which can shed light on biases in the inversion procedure is to map the number of data points for which

$$|(t_i - t_{ci})/\sigma_i| < \varepsilon$$

for some prescribed $\varepsilon$, as a function of location on the 4-dimensional grid. Commonly there will be a cluster of locations for which a limited number of the observations are well satisfied and the need to satisfy further constraints forces a migration of the estimated hypocentre.

REFERENCES:

Buland R. (1976) The mechanics of locating earthquakes, *Bull Seism Soc Am*, **66**, 173-187.

Geiger L. (1910) Herdbestimmung bei Erdbeben aus den Ankunftzeiten, *K. Gessell. Wiss. Goett.* **4**, 331-349.

Jeffreys H. (1932) An alternative to the rejection of observations, *Proc R Soc Lond*, **137A**, 78-87.

Sambridge M.S. & Kennett B.L.N. (1986) A novel method of hypocentre location, *Geophys J R astr Soc* **87**, 313-331.

Research School of Earth Sciences
Australian National University
GPO Box 4, Canberra
ACT-2601, Australia

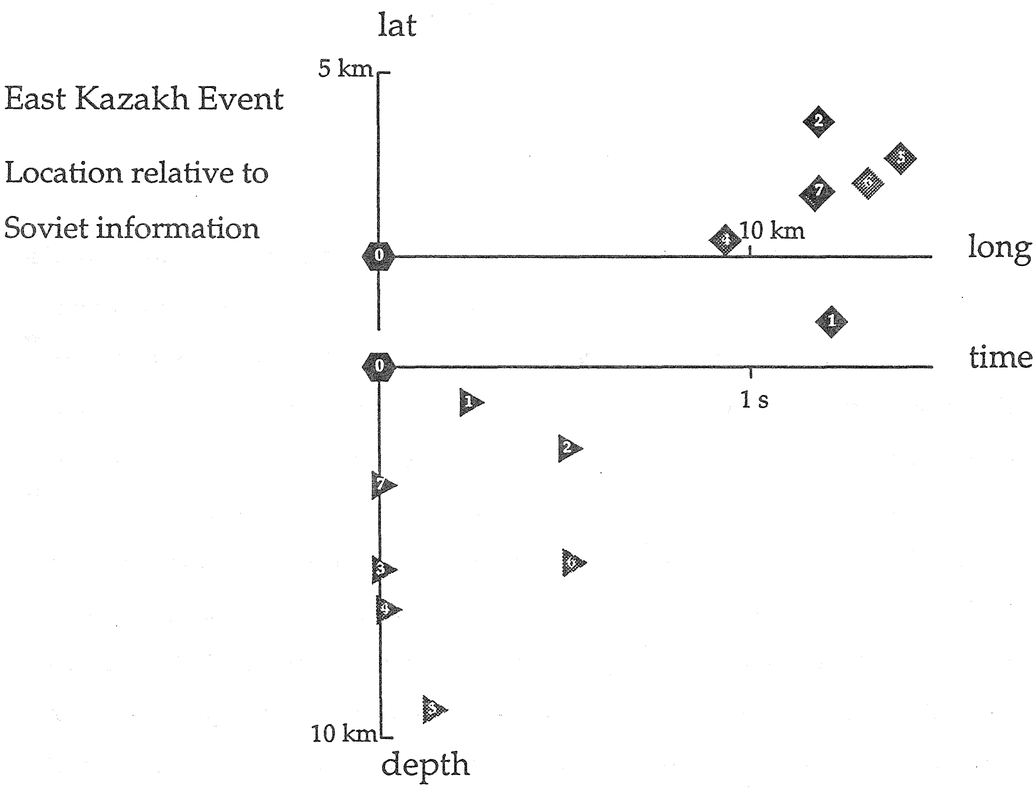
lat
5 km
East Kazakh Event
Location relative to
Soviet information
2
5
7
5
4
10 km
long
0
1
time
0
1 s
1
2
7
3
6
4
5
10 km
depth

Figure 1